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[4]It is possible to obtain an arbitrary η^0 to π ratio by taking a mixture of the two $\{8\}$ representations for the parent particle. This would require a near cancellation of the two $\{8\}$ - representation matrix elements for the π mode of decay.

BROKEN SYMMETRIES AND MASSLESS PARTICLES*

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In a recent note Klein and Lee[1] have discussed the Goldstone theorem[2,3]: that any solution of a Lorentz-invariant theory that violates an internal symmetry operation of that theory will contain a massless scalar particle. They showed that this theorem does not necessarily apply in nonrelativistic theories and they implied that their work cast doubt upon the original theorem. In this they were mistaken. The theorem fails, trivially, in the nonrelativistic case for reasons which cannot affect the relativistic version.

Relativistic theories. —The Goldstone theorem can be deduced from the behavior of the generator of the internal symmetry. Since this generator is related to a conserved four-vector current, we can gain enough information about the structure of the Fourier transform of a commutator of the conserved current with field quantities to prove the theorem. If the symmetry operation yields a conserved current, j_μ, such that

$$i[\int d^3x\, j_0(x), \varphi_1(y)] = \varphi_2(y) \tag{1}$$

is part of the statement of the symmetry operation, where φ_1 and φ_2 are scalar or pseudoscalar quantities formed from the field operators (there is an appropriate relationship changing φ_2 into φ_1), and the violation of symmetry in the solution is that the vacuum expectation value of φ_2 shall not vanish, then

$$i\langle[\int d^3x\, j_0(x), \varphi_1(y)]\rangle = \langle\varphi_2(0)\rangle \neq 0. \tag{2}$$

We may write the most general form for the structure of the Fourier transform of the vacuum expectation of the commutator of j_μ with φ as

$$\text{F. T.} = i\int dx\, e^{ikx}\langle[j_\mu(x), \varphi_1(0)]\rangle$$
$$= \epsilon(k_0)k_\mu\rho_1(k^2) + k_\mu\rho_2(k^2). \tag{3}$$

k_μ occurs because the commutator must be a four-vector. In the structure (3), k_μ is actually the total energy-momentum of the intermediate states that would arise in an expansion of the left-hand side. k_μ must be timelike. The conservation law, $\partial_\mu j^\mu = 0$, requires

$$\epsilon(k_0)k^2\rho_1 + k^2\rho_2 \equiv 0 \text{ for all } k_\mu$$

and therefore

$$\rho_1 = C_1\delta(k^2), \quad \rho_2 = C_2\delta(k^2). \tag{4}$$

Thus, in the relativistic case, the Fourier transform of this commutator must be proportional to $\delta(k^2)$. Such a δ function rises only from a massless intermediate state. The broken symmetry condition (2) states that the weight constant, C_1, is nonvanishing, since

$$C_1/2\pi = \langle\varphi_2\rangle \neq 0.$$

There must arise, in the complete set of intermediate states, massless particle states with the quantum numbers of j_μ and φ_1.

Nonrelativistic theories. —We can imbed a nonrelativistic theory in a covariant framework by introducing an external timelike vector, n_μ [which we will take as (1,0,0,0)]. Then the general form of the Fourier transform of Eq. (3) becomes

$$\text{F. T.} = k_\mu\rho_1(k^2, nk) + n_\mu\rho_2(k^2, nk) + C_3 n_\mu\delta^4(k) \tag{5}$$

in which we no longer need the $\epsilon(k_0)$ if we take the ρ's to be arbitrary functions of nk. The conservation condition now relates the ρ's and yields for (5)

$$\text{F. T.} = k_\mu\delta(k^2)\rho_4(nk)$$
$$+ [k^2 n_\mu - k_\mu(nk)]\rho_5(k^2, nk) + C_3 n_\mu\delta^4(k) \tag{6}$$

as the general structure transverse to k_μ.

Now if we specialize to the commutator of the integral of j_0 with φ_1, each term can contribute to $\langle\varphi_2\rangle$. The first term is a massless particle state, as in the relativistic theories. The second term yields contributions from intermediate states whose energy becomes zero at zero three-momentum.[4] The third term arises from a "spurion" state; when this contributes no theorem holds.

This is the term discussed by Klein and Lee. The introduction of a disconnected vacuum state gives rise to a $\delta^4(k)$ term in the Fourier transform, a constant term in the commutator matrix element, which can be used to break the symmetry. In order to get a finite contribution, the coefficient of this state must have weight $1/V$, the total volume considered, as is the case for Klein and Lee.

An isolated eigenstate of energy-momentum with zero eigenvalues can only contribute non-relativistically. In a covariant theory, the $\delta^4(k)$ term introduced into the commutator by such a state must be multiplied by k_μ, the only vector available, and its contribution is identically zero.[5]

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[4]The second term only contributes if $\rho_5 = \bar{\rho}(k^2, nk)/|k|^2$. $\bar{\rho}$ is still further restricted, because in order that $\int d^3x\, j_0$ be conserved, $\int d^3x\, \partial_i j^i$ must vanish. This requires $\delta^3(k)k_0\bar{\rho}(k^2, nk)$ to vanish, and therefore $\bar{\rho}$ must require k_0 to be zero when $|k|$ is. $\bar{\rho}$ cannot contain a $\delta(nk)$ term alone because k_μ must be timelike.

[5]Another way of phrasing our argument is to observe that the expectation of a vector operator between any vacuum states is identically zero in a relativistic theory, since there exists no vector for it to depend on.

BROKEN SU(3) ⊗ SU(3) ⊗ SU(3) ⊗ SU(3) SYMMETRY OF STRONG INTERACTIONS*

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An ever increasing amount of experimental evidence is being accumulated in favor of the octet scheme[1] of SU(3) invariant strong interactions. We would like to suggest here, taking the SU(3) symmetry as a new starting point, that it is natural, or perhaps necessary, that we should consider a hierarchy of symmetries which are even larger and at the same time more dramatically broken than the SU(3). Specifically we shall enlarge the symmetry up to SU(3) ⊗ SU(3) ⊗ SU(3) ⊗ SU(3) from the following considerations:

(1) It is natural to combine the conserved vector currents in weak interactions with conserved axial vector currents, in which the pseudoscalar-meson octet will play a special role.[2]

(2) One way to understand the "reason" for the baryons to form a tensor (octet) representation is to dissociate the two indices and assign them to separate groups.[3] This would then make nine baryons necessary.

We divide the baryon spinor field into the left-handed and right-handed components $\psi_L = (1+\gamma_5)\psi/2$, $\psi_R = (1-\gamma_5)\psi/2$. Each is a 3×3 matrix $\psi_{\mu\nu}(\mu, \nu = 1, 2, 3)$ in the SU(3) space. Then consider the following distinct operations: $\psi_L \to S_L\psi_L$, $\psi_L \to \psi_L S_L'$, $\psi_R \to S_R\psi_R$, $\psi_R \to \psi_R S_R'$, where each S is in the form

$$S = \exp\left[i\sum_{j=1}^{8}\alpha_j\lambda_j\right]$$

using well known notations.[1] Calling the infinitesimal generators of these four operations G_L, H_L, G_R, H_R respectively, construct

$$F_i^V = G_{Li} + G_{Ri} - H_{Li} - H_{Ri},$$

$$D_i^V = G_{Li} + G_{Ri} + H_{Li} + H_{Ri},$$

$$F_i^A = G_{Li} - G_{Ri} - H_{Li} + H_{Ri} = F_i^V\gamma_5,$$

$$D_i^A = G_{Li} - G_{Ri} + H_{Li} - H_{Ri} = D_i^V\gamma_5, \quad i = 1, \cdots 8,$$

so that we have

$$[F_i^V, F_j^{V,A}] = if_{ijk}F_k^{V,A},$$