to the nucleon mass for zero-energy incident pions. Nucleon exchange at low energies, therefore, appears as a special case of a more general scheme.

The proposed model, therefore, appears to be consistent with a number of other experiments which have been examined, as well as with our results for production of the $\Delta(1920)$ isobar. It would seem worth while to look for its effects in other reactions. Furthermore, it should be useful to consider the interference effects of possible baryon-exchange poles in experiments designed to isolate vector-meson exchange mechanisms.

We would like to thank the Alvarez group, and in particular Dr. Gerald Smith, for providing us with the film and for their helpful cooperation in the experimental study of the problem. We are indebted to Dr. Howard White and his group for providing us with the FOG-CLOUDY-FAIR programs and for their continuous efforts in solving programming problems. We also wish to thank Mr. Ward Schultz for his help in measuring procedure and processing the data. Finally, we wish to acknowledge many helpful and stimulating discussions with Mr. William B. Campbell.

---

*Research supported by the National Science Foundation.

[1]Geoffrey F. Chew, S-Matrix Theory of Strong Interactions (W. A. Benjamin, Inc., New York, 1962), p. 81. References to original papers are also given.

[2]F. Salzman and G. Salzman, Phys. Rev. 125, 1703 (1960).

[3]G. F. Chew and F. E. Low, Phys. Rev. 113, 1640 (1959).

[4]See, for example, A. N. Diddens, E. W. Jenkins, T. F. Kycia, and K. F. Riley, Phys. Rev. Letters 10, 262 (1963).

[5]J. Kirz, J. Schwartz, and R. D. Tripp, Phys. Rev. 130, 2481 (1963).

[6]Zaven G. T. Guiragossián, Phys. Rev. Letters 11, 85 (1963).

[7]L. Bertanza, V. Brisson, P. L. Connolly, E. L. Hart, I. S. Mittra, G. C. Moneti, R. R. Rau, N. P. Samios, I. O. Skillicorn, S. S. Yamamoto, M. Goldberg, J. Leitner, S. Lichtman, and J. Westgard, Phys. Rev. Letters 10, 176 (1963).

[8]L. Stoldolsky and J. J. Sakurai, Phys. Rev. Letters 11, 90 (1963).

[9]J. Button-Shafer, D. Huwe, and J. J. Murray, Proceedings of the International Conference on High-Energy Nuclear Physics, Geneva, 1962 (CERN Scientific Information Service, Geneva, Switzerland, 1962), p. 303.

[10]R. H. Dalitz, Strange Particles and Strong Interactions (Oxford University Press, New York, 1962), p. 97.

[11]B. Kehoe, Phys. Rev. Letters 11, 93 (1963).

[12]H. L. Anderson, E. Fermi, R. Martin, and D. E. Nagle, Phys. Rev. 91, 155 (1953).

POSSIBLE RESONANCE AT 829 MeV IN ΛK^0 PRODUCTION

G. T. Hoff
The Enrico Fermi Institute for Nuclear Studies and the Department of Physics,
The University of Chicago, Chicago, Illinois
(Received 27 April 1964)

Some time ago Bertanza et al.[1] observed an anomalous behavior of the polarization of the Λ particles in the reaction $\pi^- + p \rightarrow \Lambda + K^0$ at an incident pion kinetic energy of 829 MeV. From a polynomial analysis they found that this feature was related to the presence of partial waves higher than P, and since this effect died very fast both below and above this energy the authors suggested that the explanation could be the existence of a ΛK resonance $(Z_1{}^*)$ at about a center-of-mass energy of 1650 MeV. It is the purpose of this Letter to present some rather strong evidence to the fact that there is indeed such a resonance and that its spin-parity assignment should be $\frac{5}{2}^+$. It is then natural to try to identify it with the $F_{5/2}$ resonance in pion-nucleon scattering, which has been assumed up to now to be located at 1688 MeV,[2] and we speculate on this possibility. If this turns out to be correct, our estimated value for the contribution of the $F_{5/2}$ resonance to the ΛK^0 production cross section is in agreement with the prediction of Carruthers's model for the higher meson-baryon resonances based on SU(3),[3] if we assume for the radius of interaction the value estimated by Glashow and Rosenfeld.[4]

A while ago we proposed a model for low-energy ΛK^0 production[5] which gave either excellent or reasonably good fits to the angular distributions, polarizations, and cross sections up to a center-of-mass energy of about 1700 MeV except at the energy of the Bertanza et al. anomaly. In this model it was assumed that the K^* exchange term

Table I. Low-energy approximations of the contributions to the angular distribution and to the polarization times the angular distribution from the interference of the K^* exchange term (g_p) with different partial waves ($|g_p|$ is a monotonically increasing function of $\cos\theta$ at each particular energy).

Partial wave	$d\sigma/d\Omega$	$P(d\sigma/d\Omega)$
$s_{1/2}$	$2g_p \operatorname{Re}(f_0^+)$	0
$p_{1/2}$	$2g_p \operatorname{Re}(f_1^-)\cos\theta$	$-2g_p \operatorname{Im}(f_1^-)\sin\theta$
$p_{3/2}$	$4g_p \operatorname{Re}(f_1^+)\cos\theta$	$2g_p \operatorname{Im}(f_1^+)\sin\theta$
$d_{3/2}$	$2g_p \operatorname{Re}(f_2^-)(3\cos^2\theta - 1)$	$-6g_p \operatorname{Im}(f_2^-)\cos\theta\sin\theta$
$d_{5/2}$	$3g_p \operatorname{Re}(f_2^+)(3\cos^2\theta - 1)$	$6g_p \operatorname{Im}(f_2^+)\cos\theta\sin\theta$
$f_{5/2}$	$3g_p \operatorname{Re}(f_3^-)(5\cos^3\theta - 3\cos\theta)$	$-3g_p \operatorname{Im}(f_3^-)(5\cos^2\theta - 1)\sin\theta$
$f_{7/2}$	$4g_p \operatorname{Re}(f_3^+)(5\cos^3\theta - 3\cos\theta)$	$3g_p \operatorname{Im}(f_3^+)(5\cos^2\theta - 1)\sin\theta$

and a resonance in our channel dominate at the energy of the peak of the cross section (at about 1690 MeV), and it was found unambiguously that the dominant resonance (if any) should be $P_{1/2}$.[6] The main clue was given by the polarization curve, which has different characteristic shapes according to the orbital angular momentum of the dominant partial wave.

We apply here the same kind of analysis to the polarization data at 829 MeV. As we are more than 50 MeV below the position of the $P_{1/2}$ resonance (≈1704 MeV), we ignore for the moment its contribution to the polarization and assume that this is given mainly by the (K^* exchange term)-(unknown resonance) interference. In our previous work, we found the approximate expressions valid at low energy for the contribution from different angular momentum states given in Table I. (The notation is the same as in reference 5.) It is then seen that a P wave gives a polarization that does not change sign in the physical region; a D wave gives a polarization that changes sign once near $\cos\theta = 0$; and an F wave gives a polarization that changes sign twice, which is precisely our case [see Fig. 1(b)].

Moreover, it has been observed persistently that the angular distribution has an upward (downward) concavity for positive (negative) values of the cosine of the scattering angle θ in the center-of-mass energy range 1648 to 1688 MeV. [See Figs. 1(b), 2(c), and 3 as examples.] This is connected to the presence of a cubic term in the polynomial fits.[7,8] In Table I we also give the interference of different partial waves with the K^* exchange term. It is seen (by making a Taylor expansion of the K^* exchange amplitude g_p) that an F wave is precisely what is needed to obtain such a cubic term if we assume that it does not come from the interference with the $P_{1/2}$ resonance.

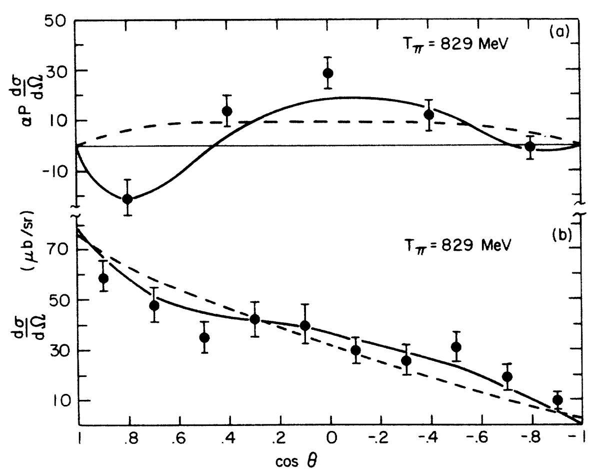

FIG. 1. (a) Polarization (P) times asymmetry parameter (α) times angular distribution ($d\sigma/d\Omega$); (b) angular distribution at an incident pion kinetic energy of 829 MeV (center-of-mass energy of 1648 MeV). The continuous lines represent the fits obtained in the present model and the dashed lines those obtained in our previous model. We have used in the present calculations a value for the asymmetry parameter $\alpha = -0.67$ which falls within the uncertainties of the most recent experimentally determined value (-0.62 ± 0.07). [(In reference 5, a larger value (-0.85) more consistent with the polarization data in our reaction was assumed.] Data from reference 1. [The fits (b) have been improved recently by taking into account the threshold dependence of the $P_{1/2}$ amplitude.]

Both of these effects can be reproduced fairly well (see Figs. 1, 2, and 3), and the over-all fit improved considerably, by adding to our previous model an $F_{5/2}$ resonance located near an incident pion kinetic energy of 829 MeV (≈1647 MeV) and adjusting its parameters properly (total width $\Gamma \approx 10$ MeV and product of partial widths $\Gamma_1\Gamma_2 \approx \Gamma^2/750$).[9] The contribution to the integrated cross section, although small (0.065 mb), improves our previous fit.[10]

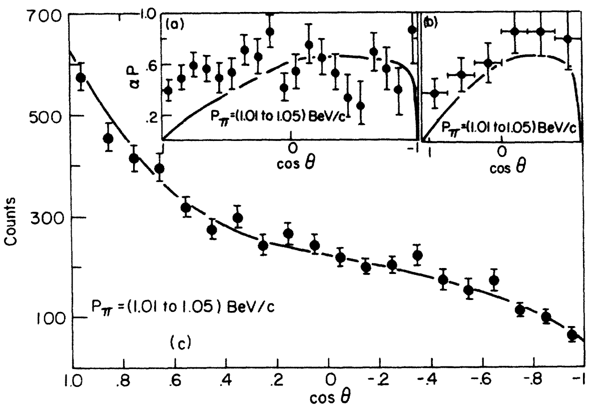

FIG. 2. Fits to (a)(b)αP, (c)$(d\sigma/d\Omega)$ in the interval 1.01 to 1.05 BeV/c of incident pion momentum. It should be noticed that if the value $\alpha = -0.62 \pm 0.07$ is correct, αP should satisfy the inequality $|\alpha P| \leq 0.62 \pm 0.07$. The fits to the polarization were calculated at an incident pion momentum of 1.03 BeV/c (center-of-mass energy of 1688 MeV). The angular distribution was best fitted using an incident pion momentum of 1.015 BeV/c. Data from reference 8.

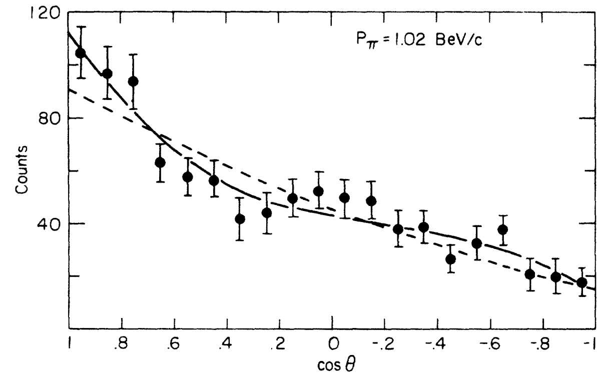

FIG. 3. Fits to the angular distribution at an incident pion momentum of 1.02 BeV/c. Data from reference 7.

It should be mentioned that an $F_{7/2}$ resonance fails to give an improved over-all fit because of the different sign in the expression for the contribution of an $F_{7/2}$ amplitude to the polarization.

If we neglect other possible partial widths (such as $\Gamma_{\eta n}$) and take as the momentum dependence for Γ_1 and Γ_2 the one given by Glashow and Rosenfeld[4] with $l=3$, we obtain $\gamma(\pi N)/\gamma \approx 0$ or $\gamma(\pi N)/\gamma = 0.805$, where the γ's are the reduced widths (momentum dependence factored out). The first corresponds to a practically pure ΛK resonance and the second to a resonance that is mostly πN. In the second case, its contributions to the $T=\frac{1}{2}$ elastic and total pion-nucleon cross sections are, respectively, 32 and 40 mb. These values are very close to the estimates of the contributions from the $F_{5/2}$ resonance to those cross sections.[11] However, what should be observed is a narrow spike at 1647 MeV and not a broad peak near 1688 MeV. As there is some evidence for a wide $P_{1/2}$ resonance slightly above 1688 MeV[11] and there might even exist a D resonance,[11] it is possible that the peak near 1688 MeV is due to these contributions[12] and not to the $F_{5/2}$; but the problem still remains that the sharp peak has not been observed. (The region where this spike should appear is, to our knowledge, the least explored part of the elastic third peak.[13])

However, Cocconi et al.[14] have obtained a shift in the position of the peak corresponding to the third resonance (from 1.65 BeV at $P_\pi = 4.74$ BeV/c to 1.70 BeV at $P_\pi = 8.94$ BeV/c) on inelastic proton-proton scattering. This is analogous to although not as striking as the shift in the position of the second resonance (from 1.40 to 1.51 BeV/c), which has been tentatively explained as due to the existence of a $P_{1/2}$ resonance at 1.40 BeV besides the $D_{3/2}$ resonance at 1.51 BeV.[15] Two resonances (at least) located near 1650 and 1700 MeV provide a similar explanation for the shift on the third peak. If favorable kinematical conditions are chosen and our ideas are correct, more than one peak should be observed in some reactions where the "third resonance" appears as final-state interaction. Maybe the reaction $\pi^- + p \to \pi^+ + \pi^- + \pi^- + p$ where the $P_{1/2}$ resonance near the second peak was observed,[15] is a good process if the experiments are performed at a higher incident momentum.

Measurement of the polarization of the Λ particles at various angles in associated photoproduction at a photon energy of 1056 MeV should also throw some light on the matter. (This energy corresponds to the generally assumed position of the $F_{5/2}$ resonance and is near the approximate location of the $P_{1/2}$ resonance.) The polarization has been measured already at two angles at a photon energy of 1000 MeV (center-of-mass energy of 1660 MeV), and it seems to show a contribution from an $F_{5/2}$ resonance.[16]

The confirmation of our conjecture by these or other experiments is of interest in connection with the model for the higher meson-baryon resonances based on SU(3) proposed by Carruthers[3] in particular and unitary symmetry in general.[4] A value for symmetry-mixing parameter of 0.674 (Carruthers's value which is very close to the value determined empirically by Glashow and Rosenfeld) and the Rosenfeld-Glashow radius of interaction (1/350 MeV^{-1}) give for the contribution of an $F_{5/2}$ resonance located at 1647 MeV to the ΛK^0 production cross section the value 0.078 mb. This should be contrasted with a previous estimate[3] of 0.36 mb (using the same values for the parameters) for the contribution from an $F_{5/2}$ resonance at 1688 MeV, which seems to be definitely ruled out from the ΛK^0 production polarization data in the interval 1.01-1.05 BeV/c of incident pion momentum if our assumption about a predominantly K^* exchange background is correct.[17] [See Figs. 2(a) and 2(b) and Table I.]

Low-energy ΛK^0 production seems to be an ideal reaction to throw some light on the region of the third resonance in pion-nucleon scattering because of the apparent dominance of the K^* exchange term which simplifies the analysis considerably. Unfortunately, the data in the interval 1.05-1.20 BeV/c of incident pion momentum are (to our knowledge) old and scarce. This is a very interesting energy region where a fast change takes place in the polarization, the polarization at 1.12 BeV/c [18] being suggestive of the existence of an important contribution from the imaginary part of a D partial wave.[19] It will be very helpful if the experimentalists provide some accurate measurements in this region, particularly at about 1.08 BeV/c.

A more complete and accurate account of the subject and fits to all the very low-energy ΛK^0 production data available at the present time will be given elsewhere.

We wish to give thanks to Professor Y. Nambu for his interest in our work.

---

*This work was supported by the U. S. Atomic Energy Commission.

[1]L. Bertanza, P. L. Connolly, B. B. Culwick, F. R. Eisler, T. Morris, R. Palmer, A. Prodell, and N. P. Samios, Phys. Rev. Letters 8, 332 (1962).

[2]Actually it has never been rigorously proved that there is an $F_{5/2}$ resonance in pion-nucleon scattering. What has been shown is that there seems to be a large $J=\frac{5}{2}$ contribution (see for instance reference 19) or that there is consistency between a $J=\frac{5}{2}$ resonance and the data available in 1962 (no polarization measurements) if contributions from all the other partial waves up to $F_{7/2}$ are allowed (reference 11).

[3]P. Carruthers, Phys. Rev. Letters 12, 259 (1964).

[4]S. L. Glashow and A. H. Rosenfeld, Phys. Rev. Letters 10, 192 (1963).

[5]G. T. Hoff, Phys. Rev. 131, 1302 (1963).

[6]A possible $T=\frac{1}{2}\,P_{1/2}$ resonance in pion-nucleon scattering near this energy has been reported by Feld and Layson. See B. T. Feld and W. Layson, Proceedings of the International Conference on High-Energy Nuclear Physics, Geneva, 1962, edited by I. Prentki (CERN Scientific Information Service, Geneva, Switzerland, 1962), p. 147. This resonance should not be confused with another $T=\frac{1}{2}\,P_{1/2}$ resonance near 1400 MeV reported more recently [L. D. Roper, Phys. Rev. Letters 12, 340 (1964)].

[7]J. Keren, Phys. Rev. 133, B457 (1964).

[8]F. S. Crawford, Proceedings of the International Conference on High-Energy Nuclear Physics, Geneva, 1962, edited by J. Prentki (CERN Scientific Information Service, Geneva, Switzerland, 1962), p. 270. M. H. Alston, J. A. Anderson, P. G. Burke, D. D. Carmony, F. S. Crawford, N. Schmidtz, and S. E. Wolf, Proceedings of the Tenth Annual International Rochester Conference on High-Energy Physics, 1960, edited by E. C. G. Sudarshan, J. H. Tinlot, and A. C. Melissinos (Interscience Publishers, Inc., New York, 1960), p. 378.

[9]We approximated the $F_{5/2}$ resonant contribution by a Breit-Wigner formula with the correct threshold behavior which is very strong for an F wave in our final momentum range. The constant-widths approximation was used for the $P_{1/2}$ resonant amplitude as in reference 5.

[10]At 829 MeV, the experimental value for the cross section is 0.43 ± 0.04 mb. (There is a slight shoulder at this energy.) We previously obtained 0.36 mb. The addition of the $F_{5/2}$ contribution gives 0.425 mb.

[11]W. M. Layson, Nuovo Cimento 27, 718 (1963).

[12]A calculation for the $P_{1/2}$ contribution to the $T=\frac{1}{2}$ pion-nucleon scattering elastic and total cross sections based on our value for the product of partial widths $(\Gamma_1\Gamma_2=\Gamma^2/36)$ show that they are either negligible (practically pure ΛK resonance) or 12.5 and 14 mb, respectively. These values are much too small to account for the height of the third peak, so another nearby resonance is needed.

[13]The effect on the inelastic cross section should be more like a shoulder due to the stronger momentum dependence of the final partial widths. A shoulder can be noticed in the data shown by Roland Omnes and George Valladas in Proceedings of the Aix-en-Provence Conference on Elementary Particles, 1961, edited by E. Cremieu-Alcan, P. Falk-Vairant, and O. Lebey (C. E. N. Saclay, France, 1961), p. 472.

[14]G. Cocconi, E. Lillethun, J. P. Scanlon, C. A. Stahlbrandt, C. C. Ting, J. Walters, and A. M. Witherell, Phys. Letters 8, 134 (1964).

[15]P. Bareyre, C. Bricman, G. Valladas, G. Villet, J. Bizard, and J. Seguinot, Phys. Letters 8, 137 (1964).

[16]See Shigeo Hatsukade and Howard J. Schnitzer, Phys. Rev. 132, 1301 (1963).

[17]Carruthers suggested, however, that the radius might be smaller (about one-third of Glashow and Rosenfeld's radius). In that case an $F_{5/2}$ resonance at 1688 MeV gives a contribution of 0.045 mb but still should affect the polarization in a detectable way. Also Rimpault [M. Rimpault, Nuovo Cimento 31, 56 (1964)] has shown consistency between a large $F_{5/2}$ resonant ampli-

tude at 1688 MeV (constant-widths approximation) and the experimental data if contributions from all the other partial waves up to $F_{5/2}$ are allowed for. In this paper the anomalous behavior of the polarization and shoulder at 829 MeV seems to be related to an $S_{1/2}$ resonance.

[18]J. Steinberger, Proceedings of the 1958 Annual International Conference on High-Energy Physics at CERN, edited by B. Ferretti (CERN Scientific Information Service, Geneva, 1958), p. 147

[19]Some other authors have found indications for the existence of two different $J=\frac{5}{2}$ important contributions in pion-nucleon scattering. See, for instance, Burton J. Moyer, Rev. Mod. Phys. 33, 367 (1961). Extrapolation of our model to 1.12 BeV/c favors, however, a $D_{3/2}$ resonance.

ERRATA

STUDY OF THE $K_{\mu 3}$ DECAY SPECTRUM. V. Bisi, G. Borreani, R. Cester, A. Debenedetti, M. I. Ferrero, C. M. Garelli, A. Marzari-Chiesa, B. Quassiati, G. Rinaudo, M. Vigone, and A. E. Werbrouck [Phys. Rev. Letters 12, 490 (1964)].

The second sentence of reference 4 should read, "The form factors measured in this reference (f_V, g_V) are related to those in our work (f_+, f_-) by $2g_V/f_V = (f_-/f_+)-1$." In the published article, the -1 was treated as an exponent instead of as an additive term.

MESON-BARYON RESONANCES IN U(3)⊗U(3) SYMMETRY. I. S. Gerstein and K. T. Mahanthappa [Phys. Rev. Letters 12, 570 (1964)].

In the second line after Eq. (2), read $Y=(2/\sqrt{3})m + \frac{2}{3}K\nu$ instead of $Y=\frac{2}{3}m+\frac{1}{3}(K-1)\nu$. In the fourth line after Eq. (2) read $(K-1)\nu\equiv 0$ (mod 3) instead of $K\equiv 0$ (mod 3). In Eq. (6) read ψ_{000} instead of $\psi\cdots$.