NOTES ON W_3 AND RELATED SYMMETRIES

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Schwinger has recently proposed[1] a strong-interaction symmetry group $W_3 = U^{(1)}(3) \otimes U^{(2)}(3)$. W_3 predicts a ninth $(\frac{1}{2})^+$ baryon and accommodates a ninth 0^{-+} meson. In this note we examine some other aspects of a possible W_3 group on the baryon-meson level, that is, independently of the possible existence of particle triplets of a more fundamental character than baryons and mesons.[1] As a presumed W_3 invariance must be at least as approximate as SU(3), the following remarks are at best some guide to distinguish consequences of a general symmetry from those of specific dynamical models. In what follows we shall confine ourselves to the subgroup SW_3 of W_3, $SW_3 = SU^{(1)}(3) \otimes SU^{(2)}(3)$ with respective generators $F_J^{(1)}$, $F_J^{(2)}$, $J = 1, \cdots, 8$. SW_3 is related to SU(3) of the eightfold way (generators F_J) by

$$F_J = F_J^{(1)} + F_J^{(2)}. \tag{1}$$

The restriction to SW_3 is inessential for the subsequent discussion of strong and weak effects.

(a) Strong interactions.—Consider first the usual eight baryons and eight pseudoscalar mesons and one possible effective coupling between them[2]:

$$[\bar{b}mb] = \bar{b}_{\alpha\beta} m_{\beta\gamma} b_{\gamma\alpha}, \tag{2}$$

$$b_{\alpha\beta} = \begin{pmatrix} 2^{-1/2}\Sigma^0 + 6^{-1/2}\Lambda, & \Sigma^+, & p \\ \Sigma^-, & -2^{-1/2}\Sigma^0 + 6^{-1/2}\Lambda, & n \\ \Xi^-, & -\Xi^0, & -2\times 6^{-1/2}\Lambda \end{pmatrix}. \tag{3}$$

$m_{\alpha\beta}$ is like $b_{\alpha\beta}$ with $\Sigma \to \pi, \Lambda \to \eta, N \to K, \Xi^- \to K^-$, $-\Xi^0 \to \bar{K}^0$. Suppose now there exists a ninth $(\frac{1}{2})^+$ baryon, here called H. Define (λ is a number)

$$B_{\alpha i} = b_{\alpha i} + \delta_{\alpha i}\lambda H; \quad \text{coupling: } [\bar{B}mB]. \tag{4}$$

For $\lambda = 0$ we have SU(3), for $\lambda \neq 0$, in general, U(3). But for the special value

$$\lambda = 3^{-1/2} \tag{5}$$

we have more. Put

$$X^0 = 3^{-1/2}(-2^{1/2}\Lambda + H);$$
$$\left.\begin{matrix} Y^0 \\ Z^0 \end{matrix}\right\} = 6^{-1/2}(\Lambda + 2^{1/2}H) \mp 2^{-1/2}\Sigma^0. \tag{6}$$

Then $\bar{X}^0X^0 + \bar{Y}^0Y^0 + \bar{Z}^0Z^0 = \bar{\Lambda}\Lambda + \bar{H}H + \bar{\Sigma}^0\Sigma^0$. Thus the mixtures X^0, Y^0, Z^0 behave like particles, and $\bar{B}_{i\alpha} m_{\alpha\beta} B_{\beta i}$ is separately invariant for Greek and Latin SU(3) transformations. This is SW_3. We have quantum numbers as in Table I, where $F_3^{(i)}$ is the third component of "ith isospin," $Y^{(i)}$ is[3] the "ith hypercharge," $i = 1, 2$. B is the representation[4] $\{\bar{3}, 3\}$ of SW_3. m is $\{8, 1\}$, so for m, $F^{(2)} = 0$.

It is well known[5] that (2) or (4) corresponds to a D/F ratio $= 1$ which maximally violates R invariance. Can we have a different ratio? Define

$$B_{i\alpha}(R) = B_{\alpha i}, \quad m_{\alpha\beta}(R) = m_{\beta\alpha}.$$

Then instead of (4) we could have taken $[\bar{B}(R)m(R) \times B(R)]$. If W_3 is good, this is an alternative theory; we cannot mix both couplings. To see this take the coupling $[\bar{B}Bm']$. Can $m' \equiv m$? No, as $[\bar{B}Bm'] = \bar{B}_{i\alpha} B_{\alpha k} m_{ki}'$. Thus m' is a Latin matrix (it is $\{1, 8\}$), whereas m is Greek. It follows that SW_3 does not allow R invariance unless one introduces a second distinct pseudoscalar octet. Likewise, an R-invariant SW_3 requires two R-conjugate vector-meson octets. We shall not speculate further here on this interesting possibility, but note that, regardless, the subspace of Bm phenomena is not R invariant as soon as m' splits

Table I. Assignments for the nine baryon states $B_{\alpha i}$.

$Y^{(2)}$	$F_3^{(2)}$ \ $Y^{(1)}$, $F_3^{(1)}$	$\frac{1}{3}$, $\frac{1}{2}$	$\frac{1}{3}$, $-\frac{1}{2}$	$-\frac{2}{3}$, 0
$-\frac{1}{3}$	$-\frac{1}{2}$	Z^0	Σ^-	Ξ^-
$-\frac{1}{3}$	$\frac{1}{2}$	Σ^+	Y^0	$-\Xi^0$
$\frac{2}{3}$	0	p	n	X^0

from m. This may be of interest, as R invariance shows many signs of not being good.[6] If an H does exist, one may try to "develop around $\lambda=(\frac{1}{3})^{1/2}$." The foregoing remarks on R invariance can easily be stated for arbitrary representations.[7]

The factorized character of SW_3 leads to new and drastic selection rules, however. For example, we have the amplitude relations $(\pi^- p\,|\,\Lambda K^0, R) = -(\pi^- p\,|\,HK^0, R)\sqrt{2}$, $(K^- p\,|\,\Lambda, R) = -(K^- p\,|\,H, R)\sqrt{2}$, etc., where R is any $(\pi, K, \bar{K}, \eta)$ mixture with zero charge and hypercharge. Due to separate conservation of $Y^{(1)}$ and $Y^{(2)}$, all amplitudes $(\pi N\,|\,\Sigma K, R)$ and $(\bar{K}N\,|\,\Xi K, R)$ vanish. Thus (inelastic) associated ΣK production should be suppressed compared to ΛK production. There seems to be no evidence for this.[8] Thus, to say the least, W_3 does not make itself manifest in this respect. SW_3 does not suppress nonhyperonic reactions[8] like $(\pi^- p\,|\,nK\bar{K})$.

Suppose there exists a ninth pseudoscalar meson; call it ψ. Define (λ' is a number)

$$M_{\alpha\beta} = m_{\alpha\beta} + \delta_{\alpha\beta}\lambda'\psi. \tag{7}$$

A coupling $[\bar{B}MB]$ is reducible under SW_3. For $\lambda'=(\frac{1}{3})^{1/2}$ we have $SU^{(1)}(3) \to SU_{(1)}{}^{(1)}(3) \otimes SU_{(2)}{}^{(1)}(3)$ for mesons only. The corresponding decomposition for vector mesons has been discussed by Gürsey, Lee, and Nauenberg.[9] Their vector-meson mass formula follows by assuming that the breakdown of the factorization of $SU^{(1)}(3)$ goes in such a way that $Y_{(1)}{}^{(1)}$ remains conserved. While a general invariance under $SU_{(1)}{}^{(1)}(3) \otimes SU_{(2)}{}^{(1)}(3) \otimes SU^{(2)}(3)$ does great violence[9] to the conventional connection between particle and antiparticle, it is nevertheless found[9] that some qualitative consequences of this invariance resemble some observed phenomena. If there exists a ψ, the factorization of $SU^{(1)}(3)$ would inhibit pure π decays of ψ but not (for sufficiently heavy ψ) $\psi \to \eta + 2\pi$. It is interesting that there is some evidence that points to an effect of this kind.[10] As $SU^{(1)}(3) = SU(3)$ for mesons, it follows that the factorization of $SU^{(1)}(3)$ for mesons may have a different domain of validity than SW_3.

The success of the Gell-Mann–Okubo mass formula indicates that if there is a W_3 group, its breakdown should have SU(3) as a stage, at least for baryons. $SW_3 \to SU(3)$ by an effective spin-spin coupling $F_J{}^{(1)}F_J{}^{(2)}$ which for $\{\bar{3}, 3\}$ equals $\frac{1}{2}(F^2 - \frac{8}{3})$. As an orientation, let us add such a term to the mass formula: $M = M_0 + aY + b[I(I+1) - \frac{1}{4}Y^2 - \frac{1}{3}F^2] + cF_J{}^{(1)}F_J{}^{(2)}$. For $\{\bar{3}, 3\}$, $a \simeq -190$ MeV, $b \simeq 35$ MeV, $c \simeq -\frac{2}{3}[M(H) - 1150]$ MeV. If[1] $H = Y_0{}^*(1405)$, $c \simeq -160$ MeV. Thus for such mass values of a possible H, breakdown of SW_3 and SU(3) are of comparable magnitude.[11] (Of course, the two physical $T=0$ baryons are linear combinations of Λ and H.)

(b) Electromagnetism.—For SW_3 we take $Q = F_3{}^{(1)} + F_3{}^{(2)} + \frac{1}{2}(Y^{(1)} + Y^{(2)})$. One can go to the U-spin representation[12] for both factors of SW_3 by making for each a rotation around the fifth direction. The nine baryons now form multiplets in $U^{(1)}$, $U^{(2)}$, $Y_U{}^{(1)}$, $Y_U{}^{(2)}$, $Q = -Y_U{}^{(1)} - Y_U{}^{(2)}$. Thus electromagnetism becomes scalar with respect to both $U^{(1)}$, $U^{(2)}$. The relabeling of the baryons in the U language is given in Table II. From the separate $U^{(1)}$, $U^{(2)}$ invariance, we get the 10 relations[13]

$$\langle n\rangle = \langle \Xi^0\rangle = \langle X^0\rangle = -\tfrac{1}{2}\langle Z^0\rangle;$$
$$\langle \Sigma^+\rangle = \langle p\rangle, \quad \langle \Sigma^-\rangle = \langle \Xi^-\rangle,$$
$$\langle X^0\,|\,Y^0\rangle = \langle X^0\,|\,Z^0\rangle = \langle Y^0\,|\,Z^0\rangle = 0.$$

Using Eq. (6) these are seen to be equivalent to the seven electromagnetic relations of SU(3), and in addition, for magnetic moments for example,

$$\mu(H) = 0, \quad \mu(\Lambda\,|\,H) = -\mu(\Lambda)\sqrt{2}, \quad \mu(\Sigma^0\,|\,H) = -\mu(\Lambda)\sqrt{6}.$$

From Eqs. (1) and the definition of Q it follows that these relations are stable under a breakdown $SW_3 \to SU(3)$. Thus, like SU(3), SW_3 yields $\mu(\Lambda) = \frac{1}{2}\mu(n)$, etc. Generally, electromagnetism does not distinguish between SW_3 and SU(3) as far as magnetic moments and related form factors are concerned. However, further amplitude relations for photoproduction exist which are easily read off from Table II, noting that all mesons have $Y_U{}^{(2)} = 0$. Note: a more general definition of Q is possible[14] with physically different implications for form factors.

(c) Nonleptonic baryon decays.—It has been argued[15] that a possible clue to strong interaction symmetries may be found in the fact that the modes $\Sigma_+{}^+$ and $\Sigma_-{}^-$ are very nearly P conserving, and this (on the basis of $|\Delta T| = \frac{1}{2}$) in opposite P channels. Of course, one cannot prove that this fact is necessarily tied to symmetry, but it

Table II. Relabeling of $B_{\alpha i}$ in the U language.

$Y_U{}^{(2)}$	$U_3{}^{(2)}$ \ $Y_U{}^{(1)}$, $U_3{}^{(1)}$	$\frac{1}{3}$, $\frac{1}{2}$	$\frac{1}{3}$, $-\frac{1}{2}$	$-\frac{2}{3}$, 0
$-\frac{1}{3}$	$-\frac{1}{2}$	$-X^0$	n	p
$-\frac{1}{3}$	$\frac{1}{2}$	$-\Xi^0$	Y^0	$-\Sigma^+$
$\frac{2}{3}$	0	$-\Xi^-$	$-\Sigma^-$	Z^0

seems reasonable to try to find such a connection whenever a new symmetry is discussed. It is perhaps of interest to note that SW_3 provides a possible tie of this kind. From Table I we see that Σ^+ and Σ^- belong to distinct $F^{(2)}$ triplets which blocks all transitions $\Sigma^+ \to \Sigma^- +$ mesons. It has been noted earlier[15] that under this condition one can attempt to find stronger rules than $|\Delta T| = \frac{1}{2}$ which accommodate the parity situation.

Corresponding to Eq. (1) we have $\vec{T} = \vec{T}^{(1)} + \vec{T}^{(2)}$ which relates the total isospin to the "isospins of first and second kind." According to Table I we have $\Delta T_3^{(1)} = 1$, $\Delta T_3^{(2)} = \frac{1}{2}$ for Σ_-^-, $\Delta T_3^{(1)} = 0$, $\Delta T_3^{(2)} = \frac{1}{2}$ for the Σ^+ modes. We are led to refine the $|\Delta T| = \frac{1}{2}$ interactions responsible for Σ decay into two parts H_1, H_0 with the following properties:

(a) H_1 has $|\Delta T^{(1)}| = 1$, H_0 has $|\Delta T^{(0)}| = 0$; both have $|\Delta T^{(2)}| = \frac{1}{2}$. H_1 and H_0 are separately P-conserving and relatively P-opposite interactions. These properties are respected by the strong interactions as we have a shared symmetry. Σ_-^- goes via H_1 only and conserves parity.

(b) The strong interactions are invariant for $B \to iT_2^{(1)}\theta B$, where θ interchanges the first and third rows in Table I, if also $\pi \to -\pi$, $\eta \to \eta$, $K^+ \to K^0$, $K^0 \to -K^+$. Let this symmetry be shared with the $\Delta T_3^{(1)} = 0$ part of H_1. Such H_1 exist. Then by an argument given previously,[16] Σ_+^+ cannot go via H_1 while Σ_0^+ can. Both Σ^+ modes go via H_0. Hence Σ_+^+ and Σ_-^- have opposite parity and Σ_0^+ is parity violating.

We shall not go into further details at this place, except for noting the qualitative fact that P violation in Λ and Ξ decay should be strong, as Λ occurs in all three triplets.[17] Rather, one should ask what meaning it can have to seek for a shared SW_3 symmetry in view of the stringent restrictions of SW_3 within the strong interactions. This question can perhaps not be discarded out of hand. It is a special example of the dynamical question where symmetries could at all "shine through." In this connection it may be relevant that the weak interactions may well probe higher frequency regimes than do some of the strong ones, due to the more singular nature of these couplings and the possibility that these interactions are mediated by vector bosons with a relatively high mass. At any rate, if the Σ decays are a clue to symmetry, a possible group factorization could be of help, and it seemed worth noting that SW_3 provides at least a simple example of this kind.[18] Finally, if there are no underlying triplets,[1] one never needs W_3; SW_3 suffices.

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[1] J. Schwinger, Phys. Rev. Letters 12, 237 (1964).

[2] [ ] denotes the trace operation. A common factor $ig\gamma_5$ (or an equivalent thereof) is suppressed.

[3] $Y^{(i)}$ is defined as $3^{-1/2}F_8^{(i)}$. This definition is not at variance with an integral charge for possible underlying triplets. It is at this point that the use of the full W_3 rather than SW_3 is essential.

[4] $\{m^{(1)}, m^{(2)}\}$ denotes the direct product of representations of dimension $m^{(i)}$ with respect to $SU^{(i)}(3)$. $\{\bar{m}\}$ is the contragredient of $\{m\}$.

[5] M. Gell-Mann, California Institute of Technology Report No. CTSL 20, 1961 (unpublished); Phys. Rev. 125, 1067 (1962).

[6] See, e.g., A. Martin and K. Wali, Phys. Rev. 130, 2455 (1963).

[7] A general representation of W_3 is provided by a tensor with two independent sets of upper indices $(u_1^{(1)}, \cdots, u_N^{(1)})$, $(u_1^{(2)}, \cdots, u_M^{(2)})$, totally symmetric in each set separately. Similarly for two independent sets of lower indices $(d_1^{(1)}, \cdots, d_P^{(1)})$, $(d_1^{(2)}, \cdots, d_Q^{(2)})$. The tensor is separately traceless with respect to the (1) and (2) indices. $B_{\alpha i}$ is of type $(u^{(1)}, d^{(2)})$, m is $(u^{(1)}, d^{(1)})$. According to Eq. (6), R invariance is the operation of interchanging u's with d's and interchanging the superscripts (1) and (2). Thus a $(u^{(1)}, d^{(1)})$ turns into a $(u^{(2)}, d^{(2)})$ which is m'. Note: m' has a vanishing "π'-nucleon" vertex!

[8] For relevant 16-BeV (π^-, p) data, see J. Bartke et al., Nuovo Cimento 24, 876 (1962).

[9] F. Gürsey, T. D. Lee, and M. Nauenberg, Phys. Rev. (to be published). Their two vector-meson mass relations have also been noted by S. Okubo, Phys. Letters 5, 165 (1963). In the present language, the mass formula is $m = a + b[Y_{(1)}^{(1)2} + Y_{(2)}^{(1)2}]$.

[10] N. Samios (private communication).

[11] For higher multiplets there are other distinct spin-spin couplings such as $d_{IJK}F_I^{(1)}F_J^{(2)}F_K^{(2)}$.

[12] S. Meshkov, C. Levinson, and H. Lipkin, Phys. Rev. Letters 10, 361 (1963).

[13] $\langle n \rangle$ denotes the expectation value of an electromagnetic quantity for a neutron state, etc. $\langle X^0 | Y^0 \rangle$, etc. are the corresponding transition elements.

[14] I want to thank T. D. Lee for pointing this out to me. This question will be discussed in detail in a forthcoming paper by M. Nauenberg.

[15] A. Pais, Rev. Mod. Phys. 33, 493 (1961).

[16] A. Pais, Phys. Rev. 122, 317 (1961).

[17] Using H_0 and H_1, one obtains $\alpha(\Lambda) \simeq -\alpha(\Sigma_0^+)$ by a further shared symmetry argument. Unlike the result for Σ_+^+ and Σ_-^- decays, this last relation is generally modified by the inclusion of the coupling responsible for nonleptonic Ξ and K decays.

[18] In the foregoing we have used a strong Bm coupling which is not R invariant. Thus the present considerations have no direct correspondence with the proposals of S. P. Rosen, Phys. Rev. Letters 12, 408 (1964). The nonleptonic baryon-decay relation discussed by M. Gell-Mann, Phys. Rev. Letters 12, 155 (1964), is compatible with the present considerations, however. It should further be noted that SU(3) cannot lead to a shared symmetry which has P conservation for both Σ_-^- and Σ_+^+ as a consequence.