is to exploit hypothesis (a) and try to build a pseudoscalar, strangeness-conserving interaction among three pseudoscalar mesons in frame F'. We can build the following isoscalars:

$$(\bar{K}'K')\eta', \quad (\bar{K}'\vec{\tau}K')\vec{\pi}', \quad (\vec{\pi}'\cdot\vec{\pi}')\eta', \quad \eta'\eta'\eta'; \tag{6}$$

they all have $CP=-1$. So, we cannot build a $K\pi\pi$ interaction that transforms like Y' and does not violate CP. Clearly, we also cannot build an interaction that transforms like I_3' (and therefore belongs to an octet) and conserves CP, because this should have a unitary partner that transforms like Y'.

Since the experimental rate for $K_1^0 \to 2\pi$ is quite high, it might seem difficult to conceive of this mode as suppressed. In fact, it is very difficult to say what this rate should be. If we introduce a phenomenological coupling

$$M^3 G K_1^0(\vec{\pi}\cdot\vec{\pi}), \tag{7}$$

where G is the Fermi coupling constant and M an adjustable mass parameter, the rate depends on the sixth power of M. This makes any prediction completely unreliable. However, if we try to extract M from the experimental value for the lifetime of K_1, we get $M \sim 1.5 m_\pi$ which is a rather low value.

As has been discussed by B. W. Lee, not much can be said on hyperon decays without resorting to further restrictions, such as R invariance.[8] However, we feel that the success of the theory in explaining, at least qualitatively, the $(K^0 \to 2\pi)$ - $(K^+ \to 2\pi)$ puzzle is very encouraging.

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[1]N. Cabibbo, Phys. Rev. Letters 10, 531 (1963).

[2]Y. Ne'eman, Nucl. Phys. 26, 222 (1961).

[3]M. Gell-Mann, California Institute of Technology Report CTSL-20, 1961 (unpublished); Phys. Rev. 125, 1067 (1962).

[4]We remember that the superscripts in Eq. (1) refer to the change of strangeness. In the usual notation (that of reference 3) we have $j_\mu^{(0)} = j_\mu^1 + ij_\mu^2$, $j_\mu^{(1)} = j_\mu^4 + ij_\mu^5$.

[5]We speak of Lagrangians in a phenomenological way, and do not enter the question of possible intermediate bosons.

[6]If the Lagrangian is thought of as arising from current-current interactions, and the currents belong to the octet representation, then the possible choice of a representation is restricted to 1, 8, or 27.

[7]B. W. Lee (to be published); S. Coleman and S. L. Glashow (to be published) have also considered this possibility.

[8]In this case $\mathcal{L}_W$ is invariant under an SU_2 subgroup of SU_3 generated by the isotopic-spin operators in frame F', I'. This interesting possibility has been proposed by B. d'Espagnat and J. Prentki, Nuovo Cimento 24, 497 (1962), and by M. Baker and S. L. Glashow, Nuovo Cimento 26, 803 (1962).

[9]The CP transformation must be defined in such a way that it leaves unchanged the structure of the Lie-group algebra of SU_3. If the commutation relations among the generators, $[F^i, F^l] = if^{ilm}F^m$, are to be unchanged by CP, assumed to operate on the F as $CPF^i(CP)^{-1} = \epsilon^i F^i$, the coefficients $\epsilon^i = \pm 1$ are restricted by the relations $(\epsilon^i\epsilon^l\epsilon^m - 1)f^{ilm} = 0$. Among the many solutions we select $\epsilon^1 = \epsilon^4 = \epsilon^7 = +1$, $\epsilon^2 = \epsilon^3 = \epsilon^5 = \epsilon^6 = \epsilon^8 = -1$. For the octet of PS mesons we then have $(CP)\pi^i(CP)^{-1} = \epsilon^i \pi^i$. Note that we have selected a solution in which F^7 has $CP = 1$, so that U conserves the CP character of the Lagrangian.

[10]Perhaps the only serious evidence; see for example the report by R. H. Dalitz at the Brookhaven Conference on Weak Interactions, September 1963 (to be published).

RESONANCE POLES NEAR AN INELASTIC TWO-PARTICLE THRESHOLD*

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It is well known that an unstable particle or resonance is characterized by a pole in the scattering amplitude of its decay products. This pole lies in the complex energy plane on the Riemann sheet reached by a path passing through the physical cut on which the resonance energy is located. But, how does this pole move if the interaction between these particles is modified continuously in such a way that the resonance energy crosses an inelastic threshold? Interest in this question has been aroused recently by Oakes and Yang[1] in connection with the classification of the meson-baryon resonances by symmetry schemes which are partially violated. As an example, Oakes and Yang considered the motion of the pole associated with the Y_1^* resonance in the framework of SU_3 symmetry, when the symmetry-breaking interaction is gradually switched off.

Under the assumptions that (a) the Y_1^* resonance becomes a bound state, and (b) a single pole is associated with this resonance throughout its motion, they conclude that the pole must change Riemann sheets, moving clockwise around the $\pi\Sigma$ threshold. In particular, for some strength of the symmetry-breaking interaction, the pole appears on a Riemann sheet far removed from the physical one, and the resonance would then not be observable. This would raise serious difficulties in the classification of resonances in any scheme with broken symmetry, and would also place the resonances and stable particles on quite a different footing.

We would like to show that in a simple model, the assumption (b) above, does not hold. There exists another pole associated with the resonance pole, but on a different Riemann sheet. By suitably varying the parameters of this model, it is this second pole which becomes the resonance pole. We shall also discuss briefly the symmetry mixing of the resonance state.

We consider an extended Lee model[2] with two N particles,[3,4] N_1 and N_2, interacting with a V and a θ particle. We adjust the parameters in the model (N_i - V - θ coupling constant g_i, and N_i, V, and θ masses, m_i, $m_V{}^0$, and μ, respectively) in such a way that the V particle becomes unstable and appears as a resonance in the $N_i - \theta$ channels above the higher of the two $N_i - \theta$ thresholds, say $N_2 - \theta$. Corresponding to this resonance there is a pole in the complex energy plane of the $N_i - \theta$ scattering amplitude at $E - i\Gamma$, located in the Riemann sheet reached by passing through the physical cut above the $N_2 - \theta$ threshold.[5] E and Γ correspond approximately to the resonance energy and the half-width, respectively, and are determined by the condition,

$$d(E+i\Gamma) - 2i\sum_{i=1}^{2}\rho_i(E+i\Gamma) = 0, \tag{1}$$

where

$$d(z) = z - m_V{}^0 + \sum_{i=1}^{2}\frac{1}{\pi}\int_{m_i+\mu}^{\infty}\frac{\rho_i(E')dE'}{(E'-z)}. \tag{2}$$

In (1) and (2), $\rho_i(z) = (4\pi)^{-1}g_i{}^2u_i{}^2(z)k_i(z)$, where u_i is a real cut-off function introduced to obtain convergence in the integrals, and is assumed to be analytic in the neighborhood of the thresholds, and k_i is the momentum of the θ in the $N_i - \theta$ channel.

Moreover, there exists another pole at $\overline{E} - i\overline{\Gamma}$ in the Riemann sheet reached by continuation below the $N_2 - \theta$ threshold, where $\overline{E}$ and $\overline{\Gamma}$ are determined by the condition

$$d(\overline{E}+i\overline{\Gamma}) - 2i\rho_1(\overline{E}+i\overline{\Gamma}) = 0. \tag{3}$$

It can readily be seen from (1) and (3) that if the resonance is narrow, i.e., $\Gamma \ll m_2+\mu$, and close to the $N_2 - \theta$ threshold, these two poles must lie near each other on their respective sheets.

If we vary the parameters of the model continuously in such a way that the resonance moves below the threshold, its energy and half-width are then characterized by $\overline{E}$ and $\overline{\Gamma}$, respectively. The essential point which we want to emphasize is this: The pole associated with the resonance when it is above the threshold does not become the pole associated with the resonance when it moves below threshold. Instead, there are always two poles, one on each Riemann sheet, which move continuously without changing sheets in the neighborhood of the inelastic threshold. The resonance is determined by the pole on that Riemann sheet which is closest to the physical sheet at the resonance energy.[6] Actually, when the resonance energy is very close to the inelastic threshold, it can no longer be completely described by a single pole. For example, in the case that the inelastic channel is in an s wave, the effect of both poles appears as a cusp on the resonance.

In conclusion, we would like to comment on the question of the mixing of symmetry representations in the resonance state.[2] For $g_1 = g_2$ and $m_1 = m_2$ our model is invariant under the transformation $N_1 \leftrightarrow N_2$. In general, the components ψ_+ and ψ_- of the resonance state[7] in the symmetric and the antisymmetric representation are given by

$$\psi_\pm = g_1\rho_1 \pm g_2\rho_2, \tag{4}$$

where ρ_i is evaluated at the position of the resonance pole. We note, in particular, that if there is a mass splitting, and the resonance lies in between the two thresholds,

$$|\psi_+| \cong |\psi_-|. \tag{5}$$

This large symmetry mixing is similar to that found by Oakes and Yang. However, if as in our model, the number of resonance levels in the physical situation remains unchanged by the symmetry-breaking interaction, it may still be possible to establish the representation to which these resonances belong in the limit of exact symmetry.[8]

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†John S. Guggenheim Fellow.

‡National Science Foundation predoctoral Fellow.

[1]R. J. Oakes and C. N. Yang, Phys. Rev. Letters 11, 174 (1963).

[2]T. D. Lee, Phys. Rev. 95, 1329 (1954).

[3]This model has the property that the scattering matrix f for the processes $N_i+\theta \to N_j+\theta$ satisfies the condition determinant $f\equiv 0$, which implies that at each energy there exists a combination of $N_1\theta$ and $N_2\theta$ states which does not scatter. This condition can be relaxed by including, for example, bilinear couplings in the θ without modifying our results.

[4]Similar models have recently been discussed by P. K. Srivastava, Phys. Rev. 131, 461 (1963). We thank H. S. Mani for bringing this article to our attention.

[5]Since the $N_i-\theta$ scattering amplitude $f(E)$ satisfies the property $f^*(E)=f(E^*)$ there is also a complex conjugate pole at $E+i\Gamma$.

[6]For small Γ, we find $E\cong\bar{E}$ and $\Gamma>\bar{\Gamma}$ when the resonance is above threshold, while $E<\bar{E}$ and $\Gamma\cong\bar{\Gamma}$ when it lies below threshold.

[7]Following reference 1, we define the resonance state ψ to be the solution of $(1-i\rho R)\psi=0$, where R is the reaction matrix.

[8]The two poles also appear in a model with a pair of coupled two-particle channels, in which the resonance is generated by a fixed pole on the real axis below the thresholds. The motion of the resonance poles as a function of the residue of the fixed pole and the masses of the particles was found to be the same as in the extended Lee model. However, the symmetry mixing in the resonance state turns out to be quite small. [For a discussion of this model in the case of a single channel, see, for example, G. F. Chew, S-Matrix Theory of Strong Interactions (W. A. Benjamin, Inc., New York, 1961), p. 52.]

ERRATA

ELASTIC p-p CROSS SECTIONS AT HIGH MOMENTUM TRANSFERS. G. Cocconi, V. T. Cocconi, A. D. Krisch, J. Orear, R. Rubinstein, D. B. Scarl, W. F. Baker, E. W. Jenkins, and A. L. Read [Phys. Rev. Letters 11, 499 (1963)].

In Table I, column 1, entitled "$-t$ $(\mathrm{BeV}/c)^2$," line 2 should read 3.50 instead of 3.16. In column 4 entitled "$(d\sigma/d\omega)_{\mathrm{c.m.}}$ $(\mathrm{cm}^2/\mathrm{sr})$," line 4 should read 7.43×10^{-32} instead of 7.43×10^{-33}.

HELICON-PHONON INTERACTION IN METALS. John J. Quinn and Sergio Rodriguez [Phys. Rev. Letters 11, 552 (1963)].

The calculations displayed in Fig. 1 are valid only in the regions in which the absorption of helicon and acoustic waves is not excessive (i.e., when the real part of the frequency is much larger than its imaginary part). This is not a severe limitation and the results of Fig. 1 are correct in those regions of magnetic field for which there is no absorption.