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UNITARITY AND FORM FACTORS IN THE PRODUCTION PROCESS $\pi + N \to \rho + N$

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The peripheral model has provided a relatively successful interpretation of many high-energy reactions. In this paper we consider one of the most thoroughly examined reactions,

$$\pi + N \to \rho + N, \tag{1}$$

in order to better understand the peripheral model and to elucidate the nature of the ρ resonance.

There are three important related problems that are raised in connection with the peripheral model for this and similar reactions:

(i) The lowest partial waves exceed the unitarity limit[1] of

$$\sigma_j \leq \pi(j+\tfrac{1}{2})\lambda\!\!\!^{-2}.$$

(ii) Quite aside from this, elastic πN diffraction scattering reveals that ρ production (1) must be suppressed, particularly in the low partial waves because of competing absorption processes[2] (i.e., the initial- and final-state absorption must be taken into account in a manner analogous to distorted-wave Born approximation calculations of low-energy physics).

(iii) The "form factor"

$$F^2(t) = [(m_\pi^{\ 2} - \Lambda^2)/(t - \Lambda^2)]^2 \tag{2}$$

now introduced phenomenologically[3] into the cross section requires a quite unphysically small Λ to fit the data.[4] The form factor at either vertex or associated with the propagator in pion exchange will be governed by the lowest mass intermediate state to which a pion can couple. Up to energies over one BeV, no significant intermediate state of the correct properties is known. Thus qualitatively we expect $\Lambda^2 \gtrsim 50 m_\pi^{\ 2}$, whereas the phenomenological value which has been used is $\approx 6 m_\pi^{\ 2}$.[4] Using the form

$$F^2 = [(m_\pi^{\ 2} - \Lambda^2)/(t - \Lambda^2)]^n \tag{3}$$

with $n = 4$ or 6 instead of (2) yields a higher Λ^2 but does not change the qualitative situation.[5]

We will calculate the effects (ii) of absorption on the peripheral process (1). We find below that these absorption effects are quite sizable at all production angles, although, of course, largest away from the forward direction. This immediately suggests an entirely different attitude toward the peripheral process as observed in the physical region from that suggested by a "form factor." Qualitatively, no ρ remains unscathed, even in the forward direction, as it leaves the nucleon. This means that the width and position of the ρ will vary significantly with momentum transfer[6] and also with total energy. As others have suggested, we strongly emphasize that one should observe that the ρ is broader at large momentum transfer and that it approaches the free ρ for high total energy.[7] The polarization of the ρ may also be modified from the predictions of the peripheral model, increasingly with increasing momentum transfer. (We are talking about the decay angular distribution in $\cos\theta_{\pi\pi}$ and the Treiman-Yang angle.) In order to discover the true properties of the ρ, one needs first to examine experimentally the momentum transfer dependence of these quantities.[6]

The cross section for

$$\pi^- + p \to \rho^0 + n \tag{4}$$

in the peripheral or one-pion exchange model can be written in terms of the usual scalar variables s and t as

$$\frac{d\sigma_{\pi N\to\rho N}^{\text{per}}}{d\Omega}=\left|T_{\pi N\to\rho N}^{\text{per}}\right|^2=\frac{1}{4}\left(\frac{g_{NN\pi}^2}{4\pi}\right)\left(\frac{g_{\rho\pi\pi}^2}{4\pi}\right)K\frac{-t[t-(m_\rho+m_\pi)^2][t-(m_\rho-m_\pi)^2]}{(t-m_\pi^2)^2}F^2(t)\chi, \quad (5)$$

where

$$K=\frac{1}{sm_\rho^2}\left\{\frac{[s-(m_N+m_\pi)^2][s-(m_N-m_\pi)^2]}{[s-(m_N+m_\rho)^2][s-(m_N-m_\rho)^2]}\right\}^{1/2},$$

$g_{\bar{N}N\pi}^2/4\pi\approx 15$ (the $\bar{N}N\pi$ coupling constant), $g_{\rho\pi\pi}^2/4\pi=\frac{3}{2}\Gamma_\rho m_\rho^2/(\frac{1}{4}m_\rho^2-m_\pi^2)^{3/2}\approx 2$ (the $\rho\pi\pi$ coupling constant), and $\chi\approx(2/\pi)\tan^{-1}(\Delta E_{\text{obs}}/\Gamma_\rho)$ (the fraction of the ρ peak experimentally observed). The amplitude in (5) can be written as

$$T_{\pi N\to\rho N}^{\text{per}}=(\varphi_{\rho N},H_{\pi N\to\rho N}^{\text{per}}\varphi_{\pi N}), \quad (6)$$

where the φ's are free-state vectors. Now taking into account the initial- and final-state interactions,[8] we have[9]

$$T_{\pi N\to\rho N}=(\chi_{\rho N}^{(-)},H_{\pi N\to\rho N}^{\text{per}}\chi_{\pi N}^{(+)}), \quad (7)$$

where, for example, the lth partial wave of the πN state projected onto the πN channel has the radial form

$$\langle r|\chi\rangle_{\pi N,l}=j_l(kr)\tfrac{1}{2}[\eta_{\pi l}\exp(2i\delta_{\pi l})+1]+g_l(kr)\left(\frac{\eta_{\pi l}\exp(2i\delta_{\pi l})-1}{2i}\right), \quad (8)$$

where $g_l(kr)\to n_l(kr)$ outside the interaction region. Here $\eta_{\pi l}=\exp(-2\delta_{\pi l}{}^I)$, with $\delta_{\pi l}{}^I$ the imaginary part of the πN phase shift. The real part of the phase shift is $\delta_{\pi l}$. In the following, we neglect the spins of the nucleon and rho. Then we make the partial-wave analysis of (6):

$$T_{\pi N\to\rho N}^{\text{per}}=\sum_l(2l+1)b^{\text{per}}P_l(\cos\theta). \quad (9)$$

We consider only the "mass-shell" correction to the Born approximation. We further assume that $\eta_{\pi l}=\eta_{\rho l}$. Then we obtain for (7), using (8) and (9),

$$T_{\pi N\to\rho N}^{U}=\sum(2l+1)b^{\text{per}}[\tfrac{1}{2}(\eta_l+1)]^2P_l, \quad (10)$$

where the superscript U reminds us that this is the unitarized amplitude. For the calculation we assumed

$$d\sigma_{\pi N\to\pi N}/d\Omega=(k^2\sigma_{\text{total}}^2/16\pi^2)\times\exp[-\tfrac{1}{4}R^2 2k^2(1-\cos\theta)], \quad (11)$$

in order to determine the η_l. Using a more complicated form in order to get a better fit to the elastic-scattering data did not lead to significant changes in (11). For $l<30$, the numerical calculations consisted of projecting out the partial-wave amplitudes from (5) and (12) to find b_l^{per} and η_l, modifying each b_l^{per} as in (11) and summing the series back. For the higher partial waves ($l>30$), the full Born approximation $T_{\pi N\to\rho N}^{\text{per}}$ was used.

The calculated ρ-production angular distribution is shown in Fig. 1 for a pion laboratory momentum, $k_{\pi\text{lab}}$, of 3.3 BeV/c.[10] In addition to the unitarized cross section (c), we show (a) the phenomenological formula [i.e., the peripheral cross section (5) with $F^2(t)$ given by (2)] which fits the data rather well,[4] and (b) the peripheral formula (5) with $F^2(t)$ given by (3) and $\Lambda^2=65m_\pi^2$. The elastic-scattering range parameter R in (12) was taken as[11] 1.07 F. It is seen that there is a significant difference between the curves (a) and (c) at extreme forward angles. This difference is, however, a detailed effect and difficult to check experimentally. The interesting result is that with an appropriate form factor [Eq. (3) with $\Lambda^2=65m_\pi^2$] absorption corrections lead to agreement with experiment, and these corrections are very large even at forward angles, though, of course, largest at the largest angles.

This result suggests that the data need reinterpretation and that the actual ρ may be significantly different in mass and width from the currently accepted values. From curves (b) and (c) in Fig. 1 we see that the suppression of the peripheral cross section at forward directions is about $\frac{1}{3}$ and at modest momentum transfer ($t\approx-15m_\pi^2$) is about $\frac{1}{7}$. Thus it is suggested that the ρ-mass plot be made in momentum transfer bins. We stress that the variation of the observed ρ with t is probably significant in the region of very small t.[6] In Fig. 2, data from a

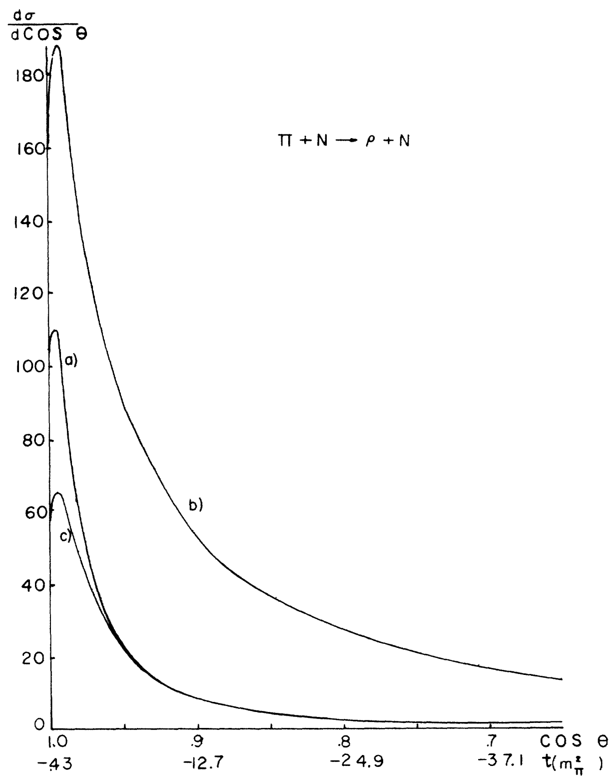

FIG. 1. Plots of ρ-production cross section (5) versus production angle of the ρ for $k_{\pi\text{lab}} = 3.3$ BeV/c. Curve (a) corresponds to a theoretical fit to Guiragossián's data using the phenomenological $F^2(t)$ given by (2) with $\Lambda^2 = 6m_\pi^2$; (b) is calculated using $F^2(t)$ given by (3) with $\Lambda^2 = 65m_\pi^2$; (c) is our unitarized result starting from curve (b). A similar curve is obtained using $F^2(t)$ as given by (2) with $\Lambda^2 = 35m_\pi^2$.

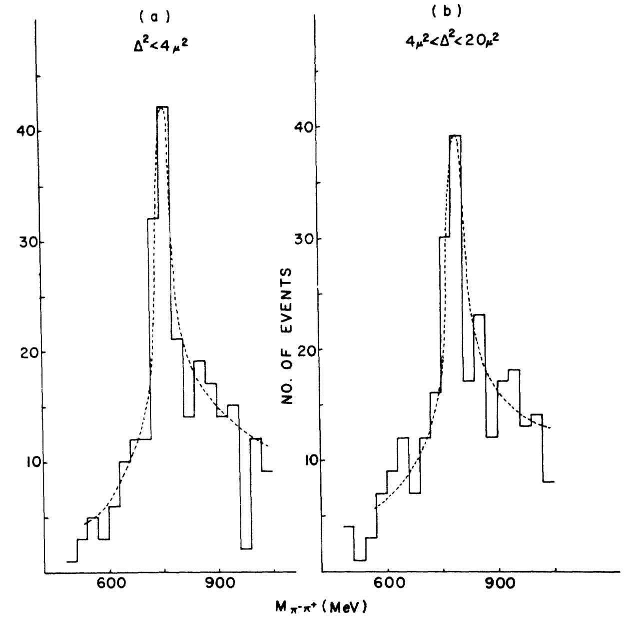

FIG. 2. The ρ^0 peak in two momentum transfer intervals as determined by Lee et al. ($\Delta^2 = -t$, $\mu = m_\pi$). In (a) the width and position are $\Gamma_\rho = 80$ MeV and $m_\rho = 765$ MeV; in (b), $\Gamma_\rho = 100$,eV and $m_\rho = 795$ MeV. In this experiment $k_{\pi\text{lab}} = 3.63$ BeV/c.

recent experiment of the Michigan bubble chamber group[12] are shown which suggest the effect under discussion and indicate that the ρ width is much smaller than currently accepted.[13] The experimental result is not definitive and some theoretical understanding is needed of the dependence of the width and position on momentum transfer to enable accurate determination of the free ρ position and width.

Another aspect is the interpretation of experimental effects which previously could not be explained in terms of rho production. In the light of the above discussion we suggest that much of the "background" observed at larger momentum transfer may be ρ's and many of the isotropically decaying pion pairs may come from depolarized ρ's. (This is in addition to the observed[12] production of the two-pion s wave.) In the rest frame of the ρ, the peripherally produced ρ should decay as $\cos^2\theta_{\pi\pi}$ with respect to the incident pion, but spin-orbit coupling should be a significant effect if the total energy is not too high.[14] So at nonforward production angles, some $\sin^2\theta_{\pi\pi}$ decay should occur. Several experimental results can be explained by this depolarization (other reasons can also be thought of): (1) Isotropic decay in the two-pion rest frame grows relatively by an order of magnitude as t goes from m_π^2 (the extrapolation point) to $t \approx -10m_\pi^2$.[12] (2) The isotropic decay term in the cross section in a plot against the two-pion mass is peaked at the ρ mass.[12] (3) Interference of $\omega \to \pi^+ + \pi^-$ decay with ρ^0 decay is said to be observed.[15] Such interference cannot occur in the simple peripheral model, if the ρ is produced by π exchange and the ω by ρ exchange, because the polarizations are orthogonal.

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[3]This arbitrary, but simple, form has been used to fit a number of different experiments in addition to (1). See, for example, G. Goldhaber et al., Phys. Letters 6, 62 (1963); E. Ferrari and F. Selleri, Nuovo Cimento 27, 1450 (1963).

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[5]F. Selleri (unpublished) has, however, obtained much larger Λ^2 based on the form $F(t) \to 0.28$ at large negative t. We find this form unsatisfactory as the experiments of Goldhaber and Guiragossián (references 3 and 4) require much smaller form factor at very large $|t|$.

[6]The calculations shown in Fig. 1 for $k_{\pi \mathrm{lab}} = 3.3$ BeV/c indicate that this variation should be significant at small production angles, i.e., $-t < 20 m_\pi^2$.

[7]We are unable to state the direction of the shift in position although it seems reasonable that at high energy the ρ mass will be higher at high-momentum transfer. Small deflections of the pions as they leave the vicinity of the nucleon will, on the basis of phase space, favor higher ρ mass.

[8]Initial- and final-state interactions are mainly absorptive, however, not (we assume) primarily due to iteration of $H_{\pi N \to \rho N}^{\mathrm{per}}$.

[9]The results that follow can also be obtained from the S-matrix approach by the methods of Omnes and Jackson [R. Omnes, Nuovo Cimento 8, 316 (1958); J. Jackson, Nuovo Cimento 25, 1038 (1962)]. The development is more complicated, however, as the channels into which the absorption goes must be treated explicitly.

[10]A more detailed treatment involving a complex potential shows that this approximation underestimates the suppression of the very low partial-wave amplitudes. The suppression of any amplitude is seen to be at most a factor of $\frac{1}{4}$ [similar calculations were performed at higher energy. There occurs an energy-dependent difference between the pure form-factor curve (a) and the unitarized curve (c) (see reference 2)], which is not small enough for the first one or two partial waves. However, this is sufficient to satisfy the unitarity limit and give a reasonable result for the total amplitude. For orientation, some sample η_l's [that were obtained from (12) using (10)] which we used for our calculation at 3.3 BeV/c are $\eta_0 = 0.20$, $\eta_1 = 0.29$, $\eta_2 = 0.44$, $\eta_3 = 0.61$, $\eta_4 = 0.76$, $\eta_5 = 0.87$, etc.

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[13]There is also a shift in position and width depending on the momentum-transfer bin due to the variation of the kinematical minimum of the momentum transfer. For the example shown, this is a completely negligible effect.

[14]Qualitatively, we would expect large spin-orbit coupling in the final state if in strongly absorbed partial waves, the absorption varies significantly from the lth to the $(l+2)$nd wave. There will also be some coupling to the nuclear spin leading to depolarization. At high energies these spin-dependent effects would become small.

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$\Delta T = \frac{3}{2}$ NONLEPTONIC DECAY

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The relative rates for the decays $K^+ \to \pi^+ + \pi^0$ ($\Delta T = \frac{3}{2}$) and $K_1^0 \to \pi^+ + \pi^-$ ($\Delta T = \frac{1}{2}$) give an amplitude ratio of 1/23. It is difficult to understand this as a consequence of electromagnetic violation of a purely $\Delta T = \frac{1}{2}$ weak interaction. [As an illustration, consider the model[1] in which the decay proceeds through successive strong and weak couplings: $K\pi \to K^* \to \pi$. On taking into account the electromagnetic mass difference between $K^{*+} - K^{*0}$, with the aid of the theoretical (mass)^2 formula $K^{*+} - K^{*0} = K^+ - K^0$, one obtains the amplitude ratio 1/400.] It has been suggested recently[2] that the $\Delta T = \frac{1}{2}$ decay may be approximately forbidden, while the electromagnetic process is not correspondingly inhibited. We should like to point out that, contrary to this proposal, the observed rate for the $\Delta T = \frac{3}{2}$ nonleptonic decay of K^+ can be obtained from known leptonic decay rates, without invoking electromagnetic effects.

The relevant leptonic decays are $\pi^+ \to \mu^+ + \bar{\nu}$ and $K^+ \to \pi^0 + e^+ + \nu$, $\pi^0 + \mu^+ + \bar{\nu}$. They are described by the phenomenological couplings

$$(g_{\pi l}/m_\pi)\pi_\lambda \bar{\psi}_\mu \gamma^\lambda (1 + i\gamma_5)\psi_\nu$$

and, for example,

$$(g_{K\pi l}/m_K m_\pi)[K_\lambda \pi^0 - K\pi_\lambda^0 + \xi(K_\lambda \pi^0 + K\pi_\lambda^0)] \times \bar{\psi}_\mu \gamma^\lambda (1 + i\gamma_5)\psi_\nu,$$