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QUANTUM THEORY OF INTERFERENCE OF LIGHT FROM TWO LASERS

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The interference fringes[1] or the closely related phenomena of beats[2] observed in the superposition of two independent laser beams can be explained quite simply by describing each of the beams by a classical field and taking the observed intensity pattern to be proportional to the square of the sum of the two field functions.[1,2] It has been suggested[3] that the correct quantum mechanical explanation is fundamentally different in that the average intensity has no interference term so that the interference effects are due only to a periodic variation in the correlation of the intensity at different points in space or time; this is based on the assumption that the state of the radiation field for a laser is the same as for a thermal source to the extent that the phase of the field is uniformly distributed so that the average of the field is zero. There is another quantum mechanical explanation which is possible and which seems to us quite likely to be correct. We suggest that the interference phenomena could be due mainly to an interference term in the average intensity– in close analogy to the explanation in terms of classical fields; this is based on the hypothesis that the average of the field is not zero for induced laser radiation as it is for spontaneous thermal radiation.[4]

We expand the positive-frequency part of the field operator as

$$E^{(+)}(\vec{x}, t) = \sum_n a_n f^{(n)}(\vec{x}, t), \tag{1}$$

where the functions $f^{(n)}(\vec{x}, t)$ are complex-valued positive-frequency solutions of the field equations which are orthonormalized to represent independent modes of the radiation field, and the operators a_n are the harmonic oscillator annihilation operators for these modes. The mean number of photons counted at the point $\vec{x}$ in a very short-time interval at the time t is taken to be proportional to the expectation value[5-7]

$$\langle E^{(-)}(\vec{x}, t)E^{(+)}(\vec{x}, t)\rangle = \sum_n \langle a_n^+ a_n\rangle |f^{(n)}(\vec{x}, t)|^2 + 2\,\mathrm{Re} \sum_{n<m} \langle a_n^+ a_m\rangle f^{(n)*}(\vec{x}, t) f^{(m)}(\vec{x}, t). \tag{2}$$

The quantum mechanical state or density matrix of the radiation field appears here in the assignment of expectation values to the operators $a_n^+ a_m$. Light from two independent sources is described most conveniently by modes each of which represents light from just one of the sources. The independence of the two sources is characterized by taking

$$\langle a_n^+ a_m\rangle = \langle a_n^+\rangle\langle a_m\rangle \tag{3}$$

whenever n and m label modes for different sources. We want to emphasize that nothing more is needed to describe the independence of the sources.

For light from thermal sources the density matrix is typically a mixture of eigenstates which have a definite number of photons in each of a set of suitably chosen modes; the expectation value of each of the annihilation operators a_n is zero.[8-10] The average intensity (2) then has no term representing an interference between two independent thermal sources. Interference effects such as are produced by the illumination of two slits by a single thermal source are explained by the fact that each mode generally represents light going through both slits.

For laser light the situation may be strikingly different. Consider two independent lasers each oscillating in a single mode. The average intensity (2) then is

$$\begin{aligned}\langle E^{(-)}(\vec{x},t)E^{(+)}(\vec{x},t)\rangle &= \langle a_1{}^+a_1\rangle|f^{(1)}(\vec{x},t)|^2+\langle a_2{}^+a_2\rangle|f^{(2)}(\vec{x},t)|^2\\ &\quad+2\,\mathrm{Re}\langle a_1\rangle^*\langle a_2\rangle f^{(1)*}(\vec{x},t)f^{(2)}(\vec{x},t),\end{aligned}\qquad(4)$$

where 1 and 2 label the modes of the two lasers. Here we have already done the factoring (3) which characterizes the independence of the two sources. The general properties of a quantum mechanical state require only that

$$|\langle a_n\rangle|^2\le\langle a_n{}^+a_n\rangle\qquad(5)$$

for $n=1,2$. The average intensity (4) has a maximum interference term when both of the complex numbers $\langle a_n\rangle$ have the maximum magnitudes allowed by the inequalities (5). This is the case if and only if the state of the radiation field is a pure state which is an eigenstate of the positive-frequency part of the field operator (1), a "coherent state,"[11,5,8] with eigenvalue

$$\langle a_1\rangle f^{(1)}(\vec{x},t)+\langle a_2\rangle f^{(2)}(\vec{x},t).\qquad(6)$$

The average intensity (4) then is[12]

$$\langle E^{(-)}(\vec{x},t)E^{(+)}(\vec{x},t)\rangle=|\langle a_1\rangle f^{(1)}(\vec{x},t)+\langle a_2\rangle f^{(2)}(\vec{x},t)|^2,\qquad(7)$$

which is the same as the intensity computed from a classical field whose positive-frequency part is the complex function (6). In general, for mixed states of the radiation field, the complex numbers $\langle a_n\rangle$ can have magnitudes anywhere between zero and the maximum values allowed by the inequalities (5); the interference term in the average intensity (4) will be the corresponding fraction of the maximum interference shown by the average intensity (7) for the ideal "coherent state." Qualitatively, similar results can be gotten with more than one mode for each laser.

We suggest that a laser beam might be represented correctly by a density matrix for which the magnitudes of the expectation values $\langle a_n\rangle$ are considerable fractions of the maxima allowed by the inequalities (5) and that the interference effects produced by two independent laser sources could be due mainly to a significant interference term in the average intensity (2). For the example of two independent lasers each oscillating in a single mode, the average intensity (4) would show a two-slit interference pattern when $f^{(1)}(\vec{x},t)$ represents light coming through one slit from one laser and $f^{(2)}(\vec{x},t)$ represents light coming through another slit from the other laser. If the expectation values $\langle a_n\rangle$ have nearly the maximum magnitudes allowed by the inequalities (5), this pattern will be nearly identical to that produced by a single source which illuminates both slits simultaneously with a field whose positive-frequency part is the complex function (6). In general, the interference pattern would be qualitatively the same with the fringe visibility reduced according to the reduction of the magnitudes of the complex numbers $\langle a_n\rangle$ from the maximum values allowed by the inequalities (5). The light from the two independent lasers would be coherent in exactly the same way that the light from a single source is coherent. This coherence implies no relation between the density matrices for the two beams. It is a property possessed separately by each of the beams; it depends only on the magnitudes of the expectation values $\langle a_n\rangle$. The same thing could happen for two modes of a single laser. If two modes are excited independently, and if for each the magnitude of the expectation value $\langle a_n\rangle$ is near to the maximum allowed by the inequality (5), the light from the two modes will be nearly as coherent as the light from a single mode. This is a coherence property which is not realized by light from thermal sources. We suggest that it might be realized by laser light and that it could be the basis of the interference effects observed in the superposition of two independent laser beams.

For a state which is stationary with respect to the radiation field Hamiltonian, the expectation values $\langle a_n\rangle$ vanish and the coherence property described above cannot be realized. But even for a steady state of a continuously running stable laser, we can say for sure only that we have a stationary state with respect to the Hamiltonian of the entire interacting field and laser system.

If the expectation values $\langle a_n \rangle$ vanish for laser light, as they do for light from thermal sources, interference effects between two independent laser beams could be explained only in terms of the correlation of the intensity at different points in space and time.[3] The interference phenomena would be transient. Both the position and the visibility of the interference fringes would vary randomly. This is observed in the experiments with ruby lasers.[1] But these observed random variations of the position and visibility of the interference fringes are consistent also with the explanation based on the average intensity and the hypothesis that the expectation values $\langle a_n \rangle$ are significantly different from zero. The random spiking of ruby lasers allows only one picture of the interference fringes to be taken for each pair of simultaneous spikes from the two lasers.[1] A random variation of the position of the interference fringes from picture to picture can be explained by a random variation of the phases of the expectation values $\langle a_n \rangle$ from spike to spike. A variation in the visibility of the interference fringes can be explained by a variation in the magnitude of the expectation values $\langle a_n \rangle$ for different spikes and by the possibility of a variable number of modes per spike and a variable frequency difference between the modes of the two lasers as different modes are excited for each different spike.[1] These complications can be eliminated with gas lasers operating continously at a single well-stabilized frequency. The observation of stable interference fringes produced by the superposition of two such laser beams could be explained only according to our suggestions in terms of the average intensity with expectation values $\langle a_n \rangle$ significantly different from zero. The possibility of such an experimental test is suggested by the fact that beats between two gas lasers have already been observed.[13] The sensitivity of such a test would be enhanced by the fact that the explanation based on intensity correlation depends critically on the number of photons per mode in the radiation field.[3] According to that explanation, the fringe visibility of a semistable interference pattern should decrease as the intensity is cut. Our explanation based on the average intensity predicts no such decrease of the fringe visibility.

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ANOMALOUS ENERGY SPECTRUM OF PROTONS IN THE EARTH'S RADIATION BELT*

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The spatial distribution and energy spectrum of trapped protons have been measured over a broad region of space from 2500 to 7500 km in altitude. The early results confirm and extend certain important earlier measurements, and make dilemmas for theories offered to explain them.

The communications satellite Relay I carries the instrumentation for this survey. Still operating, Relay I was launched 14 December 1962, into a medium inclination orbit extending from 1.2 to 2.2 earth radii. Four detectors produce the eight data channels used in this paper. Their characteristics are listed in Table I. Data re-