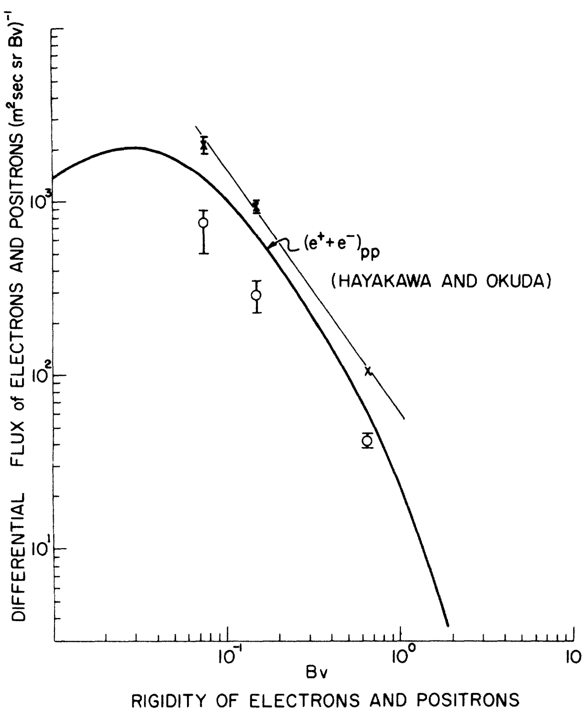

FIG. 3. The calculated energy spectrum of positrons and electrons from galactic p-p collisions (Hayakawa and Okuda, reference 5). Circles: the energy spectrum of negative excess electrons, based on reference 5 and the measured fraction $e^+/(e^+ + e^-)$. Crosses: the energy spectrum of all electrons and positrons based on reference 5 and the measured fraction $e^+/(e^+ + e^-)$.

and maintenance of the electronic system; of J. Upton and S. Lucero in the assembly and testing of the mechanical and optical system; of N. Metropolis and R. Dornberger in presenting the computer program for analysis of the events; of D. Tsui for calculations of the atmospheric corrections; and of J. Dill in scanning and measuring.

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CRITERION FOR STABILITY AGAINST RESISTIVE INTERCHANGE MODES*

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Considering the stability of hydromagnetic systems in which a small resistivity is introduced, the fastest growth rates of the new modes have been found to be proportional to the cube root of the resistivity.[1,2] In particular, when the analysis is restricted to a plane incompressible sheet-pinch with a finite gravitational field[1] or to very low β systems[3,4] such as the stellarator configuration, with a negative pressure gradient along the radius of magnetic curvature, unstable interchange modes have been shown to exist always. These modes become topologically possible when resistivity is introduced because the mass flow is no longer tied to the magnetic lines of force.

Here we consider a cylindrical pinch configuration and show that if $\beta \equiv 2p/B^2$ is of the same order of magnitude as $p'R/B^2$ and small, but not negligible, a criterion for stability against resistive interchange modes can be given. p is the pressure, p' its gradient, B the total magnetic field, and R the radius of the plasma cylinder. The condition for stability is

$$-\frac{d\ln p}{d\ln r} < \frac{\{(d\ln\iota/d\ln r)^2 + (2kr\iota)^2\}}{(1+k^2r^2\iota^2) + 0(\beta)}, \tag{1}$$

where $\iota \equiv B_\theta/krB_z$ represents the rotational transform over the length of the cylinder, k is the wave number associated with this length, r the radial space variable, and B_θ and B_z are the azimuthal and axial components of the magnetic field. Gaussian units with $c=1$ are used. It is assumed that $\chi \equiv \eta/(RV_H) \ll 1$, where η is the electrical resistivity and V_H is the hydromagnetic velocity of the system.

The given criterion is valid for interchange modes localized and nonlocalized around the singular surface[5] when $\chi^{2/5} \ll \beta \ll 1$ and only for localized modes when $0<\beta \lesssim \chi^{2/5}$. When $\beta d\ln p/d\ln r$ is finite, Eq. (1) reduces to $-d\ln p/d\ln r<0$, as previously found.[3,4] The corresponding criterion for purely hydromagnetic (nonresistive) modes is[5]

$$-\frac{d\ln p}{d\ln r} < \frac{1}{4\beta(1+k^2r^2\iota^2)}\left(\frac{d\ln\iota}{d\ln r}\right)^2$$

and is less restrictive than the condition Eq. (1).

We use a closed system of macroscopic hydromagnetic equations in which the effects of collisions are taken into account through electrical resistivity and thermal conductivity. All the terms representing finite Larmor radius effects and electron inertia are neglected. We then define as an equilibrium configuration the one in which all the physical quantities have a negligible variation on any time scale shorter than the diffusion time associated with η.[6] Since the heat flow is relatively faster than the diffusion across the lines of force,[6] we assume constant temperature. Therefore, the only equilibrium condition is

$$-p' = B_\theta(1/r)(d/dr)(rB_\theta) + B_z(d/dr)B_z.$$

Then we study the stability, looking for normal mode solutions of the form $\vec{\xi}_p(\vec{r},t) = \vec{\xi}_p(\vec{r})\exp(st + im\theta - iknz)$, where $\vec{\xi}_p$ represents a small displacement from the equilibrium. Instabilities take place in a small region of width ϵR around the singular surface $r=r_0$ where the perturbation follows the magnetic lines of force, so that $r_0 \equiv mB_\theta/(nkB_z)$. After considering the linearized perturbed hydromagnetic equations, we introduce a system of local coordinates around the singular surface, in which $\vec{e}_r$, $\vec{e}_B \equiv \vec{B}/B$, and $\vec{e}_\perp \equiv \vec{e}_r \times \vec{B}/B$ are the unit vectors, and derive the lowest order (in ϵ) set of equations for interchange modes valid inside the region ϵR:

$$q^2 d^2\xi_r/dt^2 = M^2H\tilde{B}_B + iMFt(d^2/dt^2)\tilde{B}_r, \quad (2)$$

$$\tilde{B}_r = iMFt\xi_r + (\chi/q)(d^2/dt^2)\tilde{B}_r, \quad (3)$$

$$\tilde{B}_B = -\tfrac{1}{2}\beta\tilde{P}, \quad (4)$$

$$\tilde{P} = -\vec{\nabla}\cdot\vec{\xi}_p + G\xi_r, \quad (5)$$

$$\tilde{B}_B = iMFT\xi_B + (\chi/q)(d^2/dt^2)\tilde{B}_B - (\vec{\nabla}\cdot\vec{\xi}_p) + (H - \tfrac{1}{2}\beta G)\xi_r, \quad (6)$$

$$q^2\xi_B = \tfrac{1}{2}\beta G\tilde{B}_r + iMFt\tilde{B}_B. \quad (7)$$

Here $\vec{B}_p$ and p_p represent the perturbation of the magnetic field and of the pressure. We use dimensionless units so that

$$q \equiv sR/V_H, \quad G \equiv -p'R/p, \quad t \equiv (r-r_0)/R, \quad \tilde{P} \equiv p_p/p;$$

and

$$\tilde{B}_r \equiv \vec{B}_p\cdot\vec{e}_r/B, \quad \tilde{B}_B \equiv \vec{B}_p\cdot\vec{B}/B^2, \quad \vec{\xi}_r \equiv \vec{\xi}_p\cdot\vec{e}_r/R,$$

$$\xi_B \equiv \vec{\xi}_p\cdot\vec{B}/(BR), \quad M \equiv mRB/(r_0B_z),$$

$$H \equiv 2R(B_\theta/B)^2/r,$$

and

$$F \equiv Rr(B_z/B)^2(d/dr)[B_\theta/(rB_z)].$$

The ordering leading to Eqs. (2)-(7) implies, for β and M finite, $\chi \approx \epsilon^3$, $q \approx t \approx \epsilon$, $\tilde{B}_r/\xi_r \approx \epsilon$, $\tilde{P}/\xi_r \approx (\vec{\nabla}\cdot\vec{\xi}_p)/\xi \approx \tilde{B}_B/\xi_r = O(1)$, $\xi_B/\xi_r \approx \epsilon^{-1}$, and $(d^2\xi_r/dt^2)/\xi_r \approx (d^2\tilde{B}_r/dt^2)/\tilde{B}_r \approx (d^2\tilde{B}_B/dt^2)/\tilde{B}_B \approx \epsilon^{-2}$.

By following a procedure shown in reference 2, a quadratic form can be derived from the perturbed hydromagnetic equations, indicating that for this class of modes s is real. Moreover, if we take the Fourier transform of Eqs. (2)-(7) as in references 1, 3, and 4, we can see that two modes, one with ξ_r even ($\tilde{B}_r$ odd) and one with ξ_r odd ($\tilde{B}_r$ even), have the same eigenvalue coming from the same dispersion relation. The solution of Eqs. (2)-(7) for $t/\epsilon \to \pm\infty$ overlaps the solution of the corresponding hydromagnetic equations in which all the inertial and resistive terms are neglected for $t \to \pm 0$. This solution is considered in detail in reference 7, and in the limit $t \to \pm 0$ is given by the equations

$$\tilde{B}_{re}'' + \beta GH/(2F^2t^2)\tilde{B}_{re} = 0, \quad (8)$$

$$\xi_{re} = -i\tilde{B}_{re}/(MFt), \quad (9)$$

$$\tilde{B}_{Be} = -i\tilde{B}_{re}G\beta/(2MFt); \quad (10)$$

and studied in references 1, 3, and 4. If β is small so that $\beta \approx \epsilon^\alpha \ll 1$, where $\alpha > 0$, the mode with ξ_r even becomes localized to the region ϵR in the sense that for $t/\epsilon \to \pm\infty$ it becomes small beyond any order, and the mode with ξ_r odd remains nonlocalized in the sense that for $t/\epsilon \to \pm\infty$ it connects with the nonzero solution of Eqs. (8) or (12), and (9).

The ordering leading to these modes is $q^2 \approx \epsilon^{2+\alpha}$, $\chi/q \approx \epsilon^{2-\alpha}$, $\tilde{B}_B/\xi_r \approx \epsilon^{\alpha}$, and in particular for the localized mode $\tilde{B}_r/\xi \approx \epsilon^{1+\alpha}$ and $(d^2\tilde{B}_r/dt^2)/\tilde{B}_r \approx (d^2\xi_r/dt^2)/\xi_r \approx (d^2\tilde{B}_B/dt^2)/\tilde{B}_B \approx \epsilon^{-2}$, while for the nonlocalized mode $\tilde{B}_r/\xi_r \approx \epsilon$, $(d^2\xi_r/dt^2)/\xi_r \approx (d^2\tilde{B}_B/dt^2)/\tilde{B}_B \approx \epsilon^{-2}$, and $(d^2\tilde{B}_r/dt^2)/\tilde{B}_r \approx \epsilon^{-2+\alpha}$. Clearly, the ordering for the nonlocalized mode implies that $\tilde{B}_r$ is not negligible compared to $Ft\xi_r$ and has a very slow variation as a function of t/ϵ. We see that $q \approx \chi^{1/3}\beta^{2/3}$ and $\epsilon \approx \chi^{1/3}\beta^{1/6}$; in particular, $\beta \approx \epsilon$ when $\beta \approx \chi^{2/5}$ and then $q \approx \chi^{3/5}$ (i.e., $s \propto \eta^{3/5}$).

As long as $\alpha < 1$ ($\beta \gg \epsilon$), both modes are solutions of Eqs. (2)-(7) in lowest order, and the eigenvalue is given by the asymptotic form of a unique dispersion relation in the limit $\beta \to 0$.

When $\alpha = 1$ ($\beta \approx \epsilon$), the lowest order set of equations having the nonlocalized mode as a solution is Eqs. (3)-(7), together with

$$q^2(d^2/dt^2)\xi_r = M^2H\tilde{B}_B + MFt(d^2/dt^2)\tilde{B}_r - iMT\tilde{B}_r, \quad (11)$$

where $T \equiv R^2/B_r^{\,2} d/dr(\vec{J}\cdot\vec{B})$, replacing Eq. (2). Correspondingly, Eq. (8) has to be replaced by

$$\tilde{B}_{re}'' - [T/(Ft) - \beta GH/(2F^2t^2)]\tilde{B}_{re} = 0, \quad (12)$$

and the asymptotic form of the solution at the edges of the region ϵR is changed. Then the eigenvalue depends on the equilibrium quantities outside ϵR through the parameter $\Delta \equiv \tilde{B}_{re}'/\tilde{B}_{re}(t \to +0) - \tilde{B}_{re}'/\tilde{B}_{re}(t \to -0)$ and the driving mechanism is not the interchange alone. All this is shown in detail in reference 3, where the case $D \equiv -\beta p'H/(2pF^2) \approx \epsilon$ is considered, and the complete procedure for computing s in these conditions is given in reference 1.

If $\alpha > 1$ ($\beta \ll \chi^{2/5}$), the nonlocalized mode goes into the "tearing" mode (reference 1), with $q \approx \chi^{3/5}$, which is faster than the localized interchange mode for finite wavelengths.

Therefore, the dispersion relation we obtain applies only to the latter mode when $\alpha \geq 1$. The lowest order set of equations for the localized mode is then

$$q^2\xi_r'' = M^2F^2t^2q\xi_r/\chi - \tfrac{1}{2}\beta M^2H\tilde{P}, \quad (13)$$

$$(\chi/q)\tilde{P}'' = (\tfrac{1}{2}M^2F^2t^2 + 2/\beta)\tilde{P} + 2(H-G)\xi_r/\beta. \quad (14)$$

A set of eigensolutions can be obtained; the one having the largest eigenvalue is $\xi_r = e^{-\sigma t^2/2}$, where we require $\sigma > 0$. In general, we have

$$q^{3/2} = \chi^{1/2}MF[(GH/F^2 - H^2F^2)/(1+2n) - (1+2n)]\tfrac{1}{2}\beta, \quad (15)$$

where $n = 0, 1, \cdots$. In particular, for $n = 0$, we obtain

$$\sigma^{-3} = \chi^2(GH/F^2 - H^2/F^2 - I)\beta/(2M^2F^2).$$

The requirement $\sigma > 0$ gives the condition for instability and leads to Eq. (1). Logically, $\epsilon \equiv \sigma^{-1/2}$, so that

$$\epsilon R = \chi^{1/3}[(GH/F^2 - H^2/F^2 - 1)\tfrac{1}{2}\beta]^{1/6}R/(MF)^{1/3}. \quad (16)$$

We see that in conditions of marginal stability we have $\epsilon R = 0$. We also notice that while for the models previously considered,[1,3,4] only the r components of $\vec{\xi}_p$ and $\vec{B}_p$ were involved in the stability problem, in the present case the perturbations in the $\vec{B}$ directions also play a role, so that inertial and dissipative effects are enhanced and stabilization can occur.

The localized mode is the only one existing for very short wavelengths, i.e., for $\epsilon^{-1+\frac{1}{2}\alpha} < m \leq \epsilon^{-1}$ and has then the faster growth rates. In particular, when the wavelength of the perturbation is of the same order of magnitude as ϵR, so that $m\epsilon \approx 1$, a dispersion relation, containing Eq. (15) as a special case, can be derived and the condition Eq. (1) can be shown to remain valid. If we replace Eq. (5) by the equation $\vec{\nabla}\cdot\vec{\xi}_p = 0$, corresponding to the assumption of incompressibility, we do not obtain unstable modes.

When β and $p'R/p$ are to be considered of zeroth order in ϵ, then it is possible to obtain from Eqs. (2)-(7) a properly ordered set of equations and define an eigenvalue problem leading to a condition for marginal stability. This is in general of the form

$$-d\ln p/d\ln r = (2F^2r/\beta HR)f(\beta; H^2/F^2),$$

where f is a function of two variables. A numerical solution of this problem has been obtained by Greene and Johnson for several values of β. Their results have been found in agreement with Eq. (1) for small values of β.

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d BAND OF COPPER*

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A number of calculations relating to the energy band structure of Cu have been made over the last three decades.[1,2] The density of states in the d band has been of particular interest because of the role d-band electrons play in the magnetic metals. Previously, no direct experimental method for determining the density of states was available.[2] It is the purpose of this Letter to report direct measurements of the d-band density of states and other characteristics of the energy band structure of Cu which have been made by means of photoemission studies.

The use of photoemission to obtain information about band structure has been demonstrated recently by work on Si,[3,4] the alkali antimonides,[5] and Cs_3Bi.[5] The experimental techniques have been descussed previously.[6] In the present work, the work function of Cu was lowered to a minimum value by placing approximately a monolayer of Cs on the surface. The vacuum tubes were provided with LiF windows[7] having a high-energy cutoff at 11.5 eV. A McPherson monochromator was used for measurements in the vacuum ultraviolet.

The d-band density of states will be discussed first since it is of greatest interest. Because of the large d-band density of states, the quantum yield should rise sharply and a large number of slow electrons should appear abruptly when the $h\nu = h\nu_0 = \varphi + E_d$. E_d is the separation between the d-band edge and the Fermi surface, and φ is the work function. Since φ was found to be 1.6 eV and $E_d = 2.1$ eV[8] (see Fig. 1), $h\nu_0 = 3.7$ eV. As seen in Fig. 1 the yield begins to rise sharply at $h\nu = 3.7$ eV. The effect on the energy distribution curves is even more striking [see Figs. 2(a) and 2(b)]. For $h\nu = 3.7$ eV, only a small fraction of the electrons has energies less than 0.4 eV; however, for $h\nu = 3.9$ eV the majority has less than this energy. Thus the onset of transitions from the d band is clearly evident.

As long as there is no large variation in the density of final states, the energy loss processes do not significantly distort the curves, and conservation of $\vec{k}$ is not an important selection rule,[6] the energy distribution of the escaping electrons excited initially from the d band will be determined by the density of states in that band. Insofar as the relative distribution in energy is independent of $h\nu$ when given in terms of $E-h\nu$ (where E is the kinetic energy of the electron in vacuum), these conditions must be satisfied.

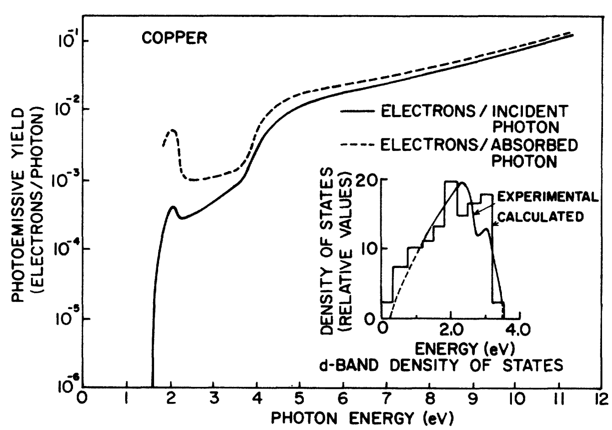

FIG. 1. The spectral distribution of the quantum yield from Cu. The reflectivity data of Ehrenreich and Philipp (reference 8) were used to obtain the yield per absorbed photon. The insert gives the density of states obtained from this work (labeled experimental) and the density of states calculated by Burdick (reference 2).