that the vortex-wave spectrum of a type II superconductor in a magnetic field larger than the lower critical field H_{c1} has two frequency branches. For small k values the spectrum is parabolic in k, and for large k values it is linear for both branches. At $k=0$ the energy of the low-frequency branch is zero, and the high-frequency branch has a gap of magnitude $\hbar\omega_0$, where $\omega_0 = eB/cm^*$. The vortex system is excited in four elliptically polarized modes, two of which are energetically degenerate. Therefore, one should be able to observe a resonance in a type II superconductor at the cyclotron frequency. From the line width and the line shape of the resonance, information regarding the internal field gradient of the periodic vortex lattice could be obtained. An experiment of this kind would be a direct test of Abrikosov's theory.

The specific heat contribution of the vortex wave spectrum has been calculated. Near absolute zero ($T \ll T_0 = \hbar\omega_0/k_B$), it is proportional to $T^{1/2}$. At "high temperatures" when $kT \gg \hbar\omega_0$, it is independent of temperature. This "high-temperature" contribution is proportional to a postulated cutoff temperature θ_m which limits the vortex-wave spectrum to wave vectors below a value k_m. The vortex-wave specific-heat contribution is, of course, in addition to the ordinary electronic specific heat which is associated with depairing of the electrons in the superconducting state.

The author is indebted to T. G. Berlincourt for introducing him to type II superconductivity and for his valuable advice, to P. G. de Gennes for a copy of his lecture notes,[3] and to J. A. Cape, R. R. Hake, and T. Wolfram for discussions.

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[4]Interactions between the nearest neighbor vortex lines will permit the propagation of transverse waves in the vortex lattice. If one introduces a phenomenological constant α (energy/unit volume) which is proportional to the curvature of the potential energy of the vortex lattice at the equilibrium site of a vortex line, then the modes of propagation along the x and y directions in the square lattice of spacing d will give rise to an additional term C underneath the square root in Eq. (5) which is of the form

$$C = \frac{32\alpha}{n_s \omega_0 h}\left[\sin^2\left(\tfrac{1}{2}dk_x\right) + \sin^2\left(\tfrac{1}{2}dk_y\right)\right].$$

In the following discussion we shall assume that C may be neglected with respect to Ak^2 in Eq. (5).

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PHOTON-PHONON INTERACTION IN THE NEAR INFRARED

G. Rupprecht
Tyco Laboratories, Inc., Waltham, Massachusetts
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The connection of the complex index of refraction with the optically active lattice vibrations of a solid by dispersion theory is generally accepted. Reflectivity measurements have been used extensively, together with Kramers-Kronig analysis, and subsequent fitting to dispersion formulas, to characterize normal mode spectra at the center of the Brillouin zone. The parameters are resonance frequencies ω_i, oscillator strengths f_i, and damping constants γ_i. Since damping constants cannot represent the details of the interaction between photons and optical and acoustical phonons, some investigators have proposed making the damping constants frequency dependent. It is the purpose of this paper to present data on a number of cases where the classical dispersion treatment breaks down completely, as has been observed on some alkali halides.[1-3] Only by replacing the "constant" damping term by an exponentially varying, frequency-dependent term can reasonable agreement be obtained with dispersion theory. It was observed that in the

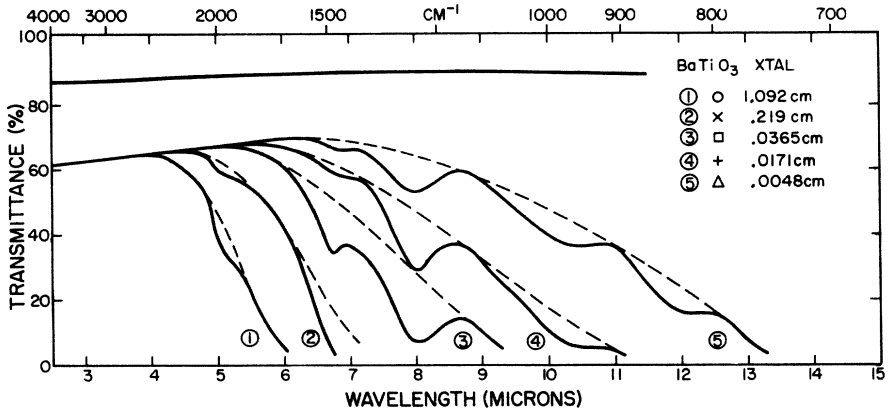

FIG. 1. Transmittance of five $BaTiO_3$ single-crystalline samples of different thicknesses as a function of wavelength. The dashed lines indicate the background absorption.

near infrared in materials such as $BaTiO_3$, $SrTiO_3$, TiO_2, SiO_2, Al_2O_3, MgO, MgF_2, and CaF_2, the absorption index, k (the imaginary part of the complex index of refraction), varies exponentially as a function of wave number, while standard dispersion theory predicts a frequency dependence as ω^{-3}.

Single-crystalline samples of thickness ranging from about 1 cm to 10^{-3} cm were prepared with parallel and highly polished surfaces and measured in transmission with the "Infracord" at room temperature. In the cases of MgF_2, CaF_2, and MgO, commercially available hot-pressed polycrystalline material was used. The measurements on SiO_2 were carried out with fused quartz. The transmittance data were evaluated according to the approximate formula $T = (1-R)^2 \times \exp(-Kd)$, where T is the transmittance, R is the reflectivity, K is the absorption coefficient, and d is the thickness of the sample. Since a sufficient number of samples of different thicknesses were measured, the ranges of validity for this formula overlap.

In Fig. 1, the "Infracord" traces for $BaTiO_3$ are shown as an example. The evaluation is made in terms of the absorption index, k, which is related to the absorption coefficient, K, by $k = K\lambda/4\pi$. In the course of this evaluation, the absorption was divided into a "background absorption," k_b, and a "peak absorption," Δk, with $k = k_b + \Delta k$. The background absorption is indicated in Fig. 1 by dashed lines. In Fig. 2, the data of Fig. 1 are shown as a function of wave number in terms of k_b and Δk_j, the maxima of Δk (dashed circles). The definition of a background absorption may appear arbitrary. It should be noted, however, that for most of the broad absorption peaks, Δk_j is usually smaller than k_b and exceeds the background absorption only slightly on a few occasions. This is true

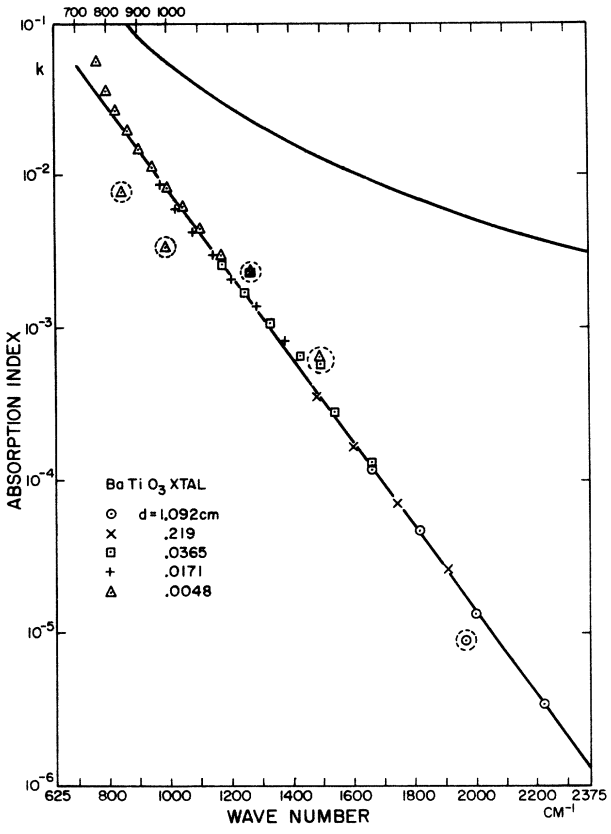

FIG. 2. Absorption index k of $BaTiO_3$ as a function of wave number calculated from data shown in Fig. 1. The points surrounded by dashed circles are the absorption peaks $\Delta k_j = k$(maximum) $- k$(background). Their wave numbers are 840 cm^{-1}, 980 cm^{-1}, 1255 cm^{-1}, 1485 cm^{-1}, and 1960 cm^{-1}. The curved solid line is $k(\omega)$ calculated from dispersion theory (see reference 1).

for all the materials under investigation with the exception of quartz, where some Δk_j exceeds k_b appreciably at several frequencies. From an experimental standpoint, k_b can be represented accurately in this frequency range by

$$k_b = k_0 \exp(-\omega/\omega_0). \quad (1)$$

The dispersion data of Spitzer et al.[4] were used to calculate $k(\omega)$ in this range. This result is indicated by the curved solid line in Fig. 2. An estimate of the frequency dependence of k in classical dispersion theory may easily be obtained from the dispersion formulas, as

$$k(\omega) = (\Sigma_i f_i \gamma_i)/2n\omega^3, \quad (2)$$

where n is the real part of the refractive index. n is only slightly frequency dependent in the range in question.

The same frequency dependence as in $BaTiO_3$ was found with other materials, which are listed in Table I together with their values for k_0 and ω_0 according to Eq. (1). In view of the overwhelmingly strong contribution of the temperature-dependent "soft mode" in $SrTiO_3$ to the optical properties, the transmittance was measured with one sample between liquid nitrogen and 200°C. No change of the absorption index, k, was observed. Equation (1) holds, therefore, independent of temperature.

In view of the presented evidence, the notion of a damping constant as used in the dispersion theory seems questionable. The experimental results suggest a re-evaluation of the absorption process. In this context, it may be significant to note that the additional absorption peaks, Δk_j, are closely related to the background absorption (Fig. 2). In the case of $BaTiO_3$, some of the Δk_j (Fig. 1) may be identified tentatively as harmonics of the fundamental frequency $\omega_1 = 492$ cm^{-1} For even modes ($2\omega_1 \approx 980$ cm^{-1} and $4\omega_1 \approx 1960$), Δk_j is smaller than k_b by a factor of 2.5; for the odd order ($3\omega_1 \approx 1485$ cm^{-1}), it is larger than k_b by approximately 50%. No convincing explanation has yet been found to the two other absorption peaks (at 840 cm^{-1} and 1255 cm^{-1}).

Table I. The frequency dependence of the absorption index in the near infrared of different solids according to the relation $k(\omega) = k_0 \exp(-\omega/\omega_0)$.

Material	Form	k_0	ω_0 (cm^{-1})
$BaTiO_3$	Single crystal	4.6	158
$SrTiO_3$	Single crystal	17	153
TiO_2	Single crystal	20.9	149
SiO_2	Polycrystal	8.3	205
MgO	Polycrystal	1.3	139
MgF_2	Polycrystal	30.9	103.9
CaF_2	Polycrystal	26	83.5

As an explanation of the " anomalous" behavior of $k(\omega)$, it is tentatively proposed that the exponential, temperature-insensitive wavelength dependence is the result of a multiphonon process in which a photon is dissipated simultaneously into n phonons such that

$$\omega = \sum_{i=1}^{n} \omega_i,$$

and the sum of their momenta,

$$\sum_{i=1}^{n} \vec{q}_i = 0.$$

The absorption index, k, may be assumed proprotional to the probability $W_n(\omega_i, \vec{q}_i)$ for this event, which will depend upon the nonlinear coupling between the different phonon branches and their density of states. If one assumes, for the sake of simplicity, that only one acoustical branch couples strongly to photons of energy $\hbar\omega_0$ and momenta $\vec{q}_i$ close to the edge of the Brillouin zone, where their density of states is largest, $W_n(\omega_0', \vec{q}_i)$ is subject to the boundary conditions $\omega = n\omega_0'$,

$$\sum_{i=1}^{n} \vec{q}_i = 0,$$

and $|\vec{q}_i| \approx \pi/a$. If $W_{(n+m)} = W_n W_m$, then

$$W_n = (p)^n = \exp[-n \ln(p^{-1})]$$

$$= \exp[(-\omega/\omega_0') \ln(p^{-1})]$$

with $\omega_0' \ln(p^{-1}) = \omega_0$.

The assumption of a multiple-phonon process leads therefore, at least in principle, to a frequency dependence of the absorption index which agrees with the experimental observation. It is expected, however, that a more detailed theory may give more meaning to the parameters k_0 and ω_0 and provide new insight into the nonlinear coupling of normal modes in solds.

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DISPERSION OF LONG-WAVELENGTH SPIN WAVES FROM PULSE-ECHO EXPERIMENTS

R. W. Damon and H. van de Vaart
Sperry Rand Research Center, Sudbury, Massachusetts
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We report here the observation of a strong dispersion in the spin-wave spectrum of yttrium iron garnet (YIG). This was observed by microwave pulse-echo measurements of the dependence of the group velocity on magnetic field or frequency. The echo delay time changes by a factor of five when the field strength is varied by less than 1 Oe, and decreases with increasing field. This occurs under conditions for which the propagation mode is purely magnetic, with no significant elastic interaction. The experiments were performed on disks of YIG at about 9300 Mc/sec, from 1.3°K to room temperature. The dispersion is attributed to the variation of the effective demagnetizing factor with spin wavelength when the wavelength becomes comparable to a sample dimension. These measurements can provide information on the shape of the spin-wave spectrum in a previously unattainable regime. The spin-wave vector at which this dispersion occurs in our experiments is approximately 2000 cm^{-1}, so the exchange contribution to the effective field is only a fraction of 1 Oe.

The observed dependence of the spin-wave pulse-echo delay time on applied field is shown in Fig. 1 for a YIG disk at 0.38 cm in diameter by 0.031 cm thick, cut in a (100) plane. The disk was in a TE_{103} rectangular cavity, excited with less than 1 mW of incident power at 9255 Mc/sec. The magnetic field was perpendicular to the disk, and the sample temperature was 1.5°K in this case. The excitation and reradiation of these spin waves is believed to occur in the nonuniform internal field near the edges of the disk.[1,2] Similar results were obtained on other samples of slightly different dimensions and for various locations within several types of microwave cavities. At room temperature, the maximum observable delay time for these echoes is only a few microseconds.

The propagation of spin waves beyond the region of generation had been reported earlier[1] for operation near the acoustic crossover of the spin-wave spectrum in a YIG disk magnetized normal to its plane. There, the propagating wave vector was near the crossover value, $k_c \approx 10^5$ cm^{-1}, where the exchange field is about 100 Oe. Under these conditions, the group velocity varied as the mode of propagation within the sample changed from elastic wave to spin wave with increasing magnetic field strength. We observe

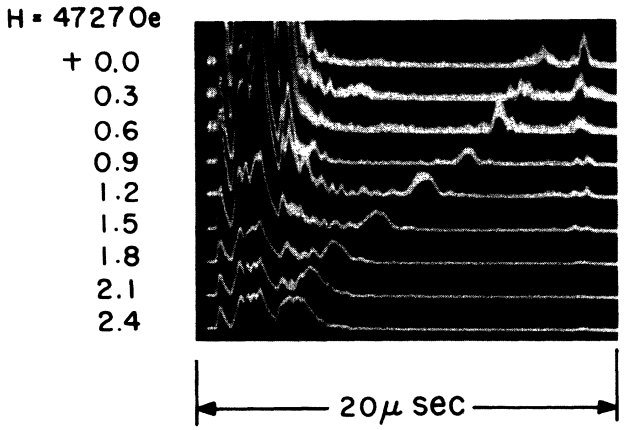

FIG. 1. Field dependence of pulse echoes observed in a (100) YIG disk, 0.380 cm in diameter, 0.031 cm thick, with perpendicular field. Time scale is 2 μsec/div, frequency 9255 Mc/sec, temperature 1.4°K. The receiver gain is adjusted at each field for approximately constant echo amplitude.