by Landau damping, or by collisions, or by the photon-induced natural lifetime,[5] whichever leads to shorter τ. We assume, of course, that $\omega_p\tau \gg 1$. In typical plasmas and for light in the visible part of the spectrum, k is of the order q_d. Therefore, unless q is made appreciably smaller than k by measuring the scattered light in the nearly forward direction, Landau damping will be very severe.[6] Therefore $q \simeq k\theta$, where θ is the small angle that the scattered beams make with incident beams. Then Eq. (1) becomes

$$\frac{d\sigma}{d\Omega_3} = 8\pi^2 r_0^{\,2}\left(\frac{\omega_3}{\omega_4}\right)(\theta^2)\left(\frac{q}{q_d}\right)^2\left(\frac{KT}{mc^2}\right)(\omega_p\tau)^2. \qquad (7)$$

To compare the counting rate in the light-off-light scattering experiment with that in the single-beam scattering experiment (Eq. 5), we calculate the ratio $\mathfrak{R}$ of the total number of scattered photons N_S in the two cases:

$$\mathfrak{R} = \frac{N_s \text{ light-off-light}}{N_s \text{ single-beam}} \simeq 2\theta^2\left(\frac{KT}{mc^2}\right)\left(\frac{N_{\text{ph}}}{n}\right)(\omega_p\tau)^2, \quad (8)$$

where N_{ph} is the density of photons in the light beam, n is the plasma density, and T its temperature. Now, while the first two factors in Eq. (8) are much smaller than unity,[7] the last two factors can each be made larger than unity sufficiently so that $\mathfrak{R} \gtrsim 1$. Consequently, not only can counting rates be made larger at resonance, but because two scattered beams are present in the light-off-light experiment, coincidence techniques, or modulation and phase-synchronous detection techniques, can be used to discriminate against background. With present-day laser and plasma technologies, a meaningful light-off-light scattering experiment is feasible.

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[5]We wish to thank Dr. P. A. Wolff for pointing out to us the existence of this photon-induced upper limit on τ.

[6]If microwave beams are used for which $k \ll q_d$, it is no longer necessary to measure the scattered radiation in the nearly forward direction.

[7]Again, if microwave beams are utilized, the first factor in Eq. (8) need not be small.

TRIPLET CORRELATIONS IN LIQUIDS*

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The reliability of the superposition approximation (SA) for $g^{(3)}(r,t,s)$, the three-particle distribution, has been investigated in some detail, for a system of particles at liquid density, interacting through a physically reasonable potential. The superposition approximation for $g^{(3)}$ is $g_{\text{SA}}^{(3)} = g^{(2)}(r)g^{(2)}(t)g^{(2)}(s)$, where $g^{(2)}$ is the pair-distribution function. In this note we report the results for $g^{(3)}(r,r,s)$.

Let a denote the first-neighbor distance in $g^{(2)}$. It is found that:

(1) For $s<a$, the SA exaggerates the probability of occurrence of triplet distributions (r,r,s), and this exaggeration is greater for $r<a$.

(2) The ratio $R_r(s) = g^{(3)}(r,r,s)/[g^{(2)}(r)]^2$ shows a secondary maximum, and this is not given correctly by $g^{(2)}(s)$ which is the SA for $R_r(s)$; this maximum in $R_r(s)$ reflects the short-range order arising out of the sharp first maximum in $g^{(2)}$ which occurs at a. This maximum also seems to be present in the results of Alder[1] for a hard-sphere liquid.

(3) $g^{(2)}$ has been compared with $g_{\text{SA, BG}}^{(2)}$ which is found[2] by using $g_{\text{SA}}^{(3)}$ in the integral equation of Born and Green (BG), showing that $g_{\text{SA, BG}}^{(2)}$ is inaccurate to about the same extent as $g_{\text{SA}}^{(3)}$ itself.

The method used is the following. The motion of atoms in a liquid is simulated on a computer using classical dynamics on a system of 864 particles interacting through a Lennard-Jones potential having $\sigma = 3.4$ Å and $\epsilon/k_{\text{B}} = 120°$K, which are the parameters for the interaction of two argon atoms. The particles are enclosed in a box of side 10.23σ, so that the density is $1.374\ \text{g cm}^{-3}$, which is the density of liquid argon at 91.8°K; periodic boundary conditions conserve the density in the box. The potential is truncated at 2.25σ. With a time interval of 10^{-14} sec, the system of coupled differential equations is solved as a set of difference equations using a predictor-corrector formula. This gives the time evolution of the configurations from which various correlations have been calculated. Starting with a random distribution of positions and velocities, the system was allowed to settle down to equilibrium which was observed by monitoring the mean square velocity of the system expressed in °K. The velocities were then stepped down to bring the temperature into the region of 90°K, where the system was to be studied in detail. When this region was finally reached, and equilibrium re-established, it was found that the temperature was fluctuating around a mean value of 94.4°K with a relative rms deviation of 1.6%. The following results are therefore for a system at 94.4°K and with density $1.374\ \text{g cm}^{-3}$.

The pair distribution $g^{(2)}(r)$ was found by counting the number of particles lying at a distance between r and $r+\Delta r$ from a given particle; the triplet distribution $g^{(3)}(r,t,s)$ was found by counting the number of triplets in a similar way. The $g^{(3)}$ was compared with $g_{\text{SA}}^{(3)}$ and $g^{(2)}$ with $g_{\text{SA, BG}}^{(2)}$ from reference 2.

Figure 1 shows[3] $g^{(2)}(r)$ and eight representative points of $g_{\text{SA, BG}}^{(2)}$. The discrepancy is of the order of 10% or less. Figure 1 also shows $[g^{(3)}(r,r,r)]^{1/3}/g^{(2)}(r)$, which is unity in the SA; the SA for symmetric triplets is therefore about as bad as $g_{\text{SA, K}}^{(2)}$. However, for $r<a$, the SA systematically overestimates the occurrence of such triplets. A region of r is not shown in

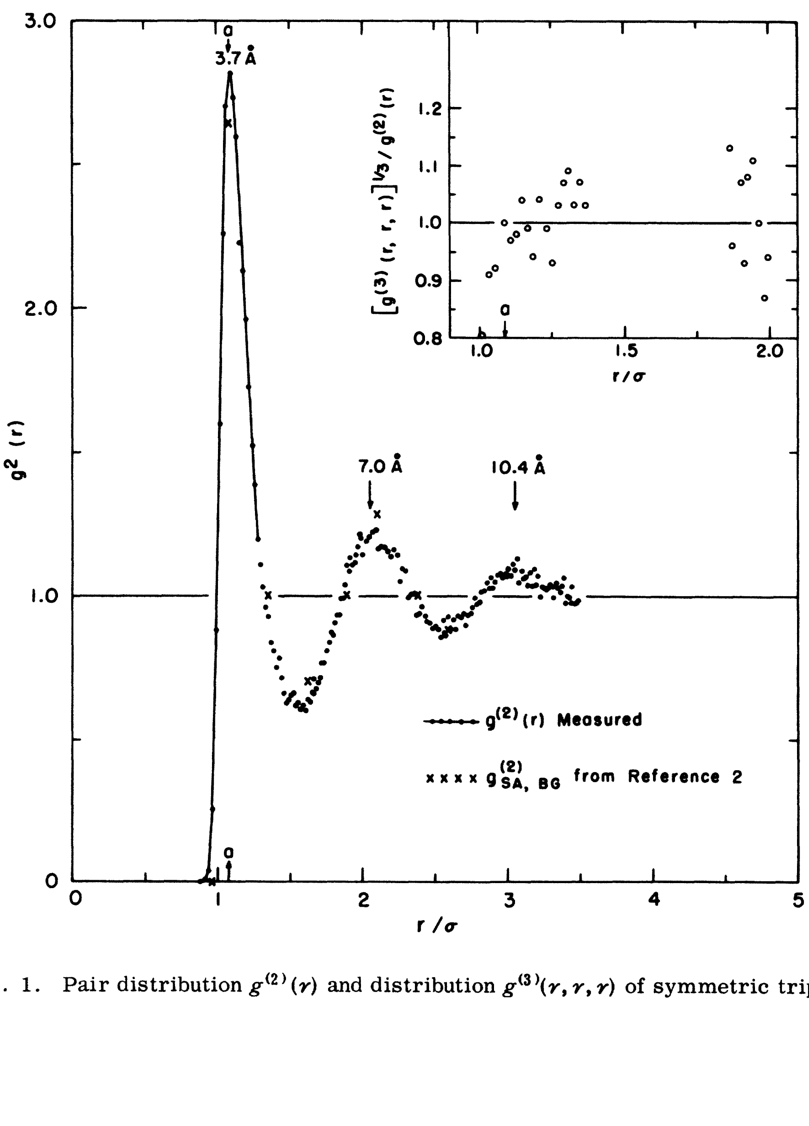

FIG. 1. Pair distribution $g^{(2)}(r)$ and distribution $g^{(3)}(r,r,r)$ of symmetric triplets.

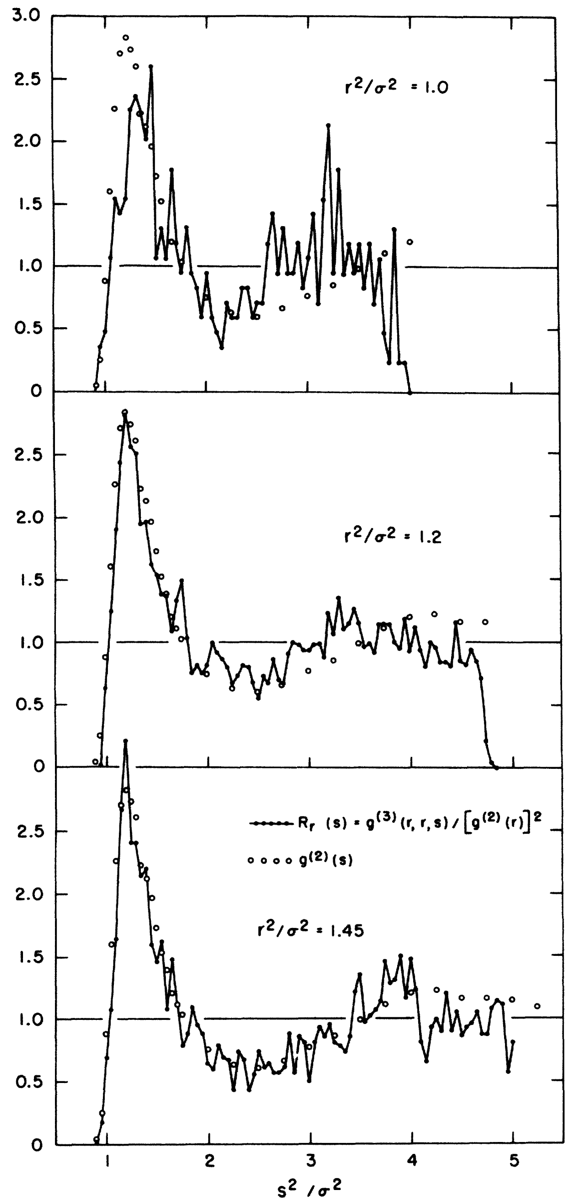

FIG. 2. Distribution $g^{(3)}(r,r,s)$ of asymmetric triplets for three values of r.

Fig. 1 because of insufficient statistics to evaluate $g^{(3)}(r,r,r)$ in that region; this is the region of the first minimum in $g^{(2)}$, where $g^{(3)}(r,r,r)$ is 100 times smaller than its peak value at $r=a$.

Figure 2 shows $R_r(s)$ for three values of r; $g^{(2)}(s)$ is also shown for comparison. For $r=\sigma$, the SA exaggerates the probability in the region below the first peak of $g^{(2)}$; for this value of r, $R_r(s)$ shows a first peak at about $s^2=1.35\sigma^2$. For the two larger r shown in Fig. 2, the SA gives the first peak in $R_r(s)$ correctly. The second maximum in $R_r(s)$ can be understood as follows: If a is the position of the sharp peak in $g^{(2)}(s)$, it is easy to show by analogy with the situation in the fcc lattice that a maximum in $R_r(s)$ is to be expected at $s=\lambda$ given by

$$\lambda^2=a^2(4-a^2/r^2).$$

With $a^2=1.2$ and $r^2=1.0$, 1.2, and 1.45, one gets $\lambda^2=3.36$, 3.6, and 3.83, respectively, in agreement with the positions of the broad maxima seen in Fig. 2.

A liquid of hard spheres has been studied on a computer by Alder and Wainwright,[4] and Alder[1] has reported results on triplet configurations; he finds that, as an approximation for $g^{(2)}(r)$, $[g^{(3)}(r,r,r)]^{1/3}$ is more accurate than $g_{\mathrm{SA}}^{(2)}(r)$ calculated from the Born-Green integral equation. In the more realistic model considered here, $g_{\mathrm{SA,\,BG}}^{(2)}(r)$ does not seem to be worse than $[g^{(3)}(r,r,r)]^{1/3}$. It is interesting to see, from Table IV of Alder,[1] that a broad maximum in $R_r(s)$ is present in his results also.

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[3]The Fourier transform of this $g^{(2)}(r)$ has peaks at $\kappa\sigma=6.8$, 12.5, 18.5, and 24.8, whereas the x-ray spectrum of Eisenstein and Gingrich [A. Eisenstein and N. S. Gingrich, Phys. Rev. 62, 261 (1942)] shows peaks at 6.8, 12.3, 18.4, and 24.4.

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