PHYSICAL REVIEW LETTERS

VOLUME 12 | 25 MAY 1964 | NUMBER 21

LIGHT-OFF-LIGHT SCATTERING IN A PLASMA

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(Received 25 March 1964)

Recently there has been considerable theoretical[1] and experimental[2] interest in the incoherent scattering of optical beams or microwaves from plasmas. The experiments in this field are difficult because the cross sections (related to the Thomson cross section $\simeq 5\times10^{-25}$ cm^2) are small. We have calculated the cross section for the elastic scattering of light by light in the presence of a plasma[3] to lowest order in the plasma parameter $r_s \equiv (n/q_d^3)^{-1}$, where n is the electron density and $q_d = 2\pi/l_d$ with $l_d^2 = KT/4\pi ne^2$. It is the purpose of this note to point out that the light-off-light scattering experiment offers certain advantages over single-beam scattering, so much so that under suitable conditions the counting rate in the light-off-light scattering experiment can exceed the rate in single-beam scattering.

Figure (1a) shows the process under consideration. Photons 1 and 2 with energies ω_1, ω_2 and momenta $\vec{k}_1, \vec{k}_2$ collide and are scattered into the final-state photons with energies ω_3, ω_4 and momenta $\vec{k}_3, \vec{k}_4$. The plasma acts as the active medium. It is polarized by the incident beams and reradiates the final-state photons.

For photons incident head on (the incident momenta point in opposite directions), there are eight independent amplitudes depending on the polarization of the final and initial photon beams [see Fig. (1b)]. The beams can be polarized perpendicular (⊥) to or parallel (∥) to the scattering plane. In order to simplify this discussion, we choose a particular polarization: photon No. 1 has ∥ polarization; photon No. 2, ⊥ polarization; photon No. 3 has ∥ polarization; and photon No. 4, ⊥ polarization. For this choice of polarization, the differential cross sec-

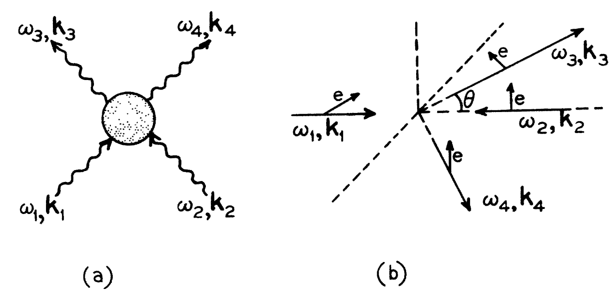

FIG. 1. (a) Fundamental diagram for the light-off-light scattering amplitude. (b) Diagram showing the kinematics and polarizations for the light-off-light scattering. The light beams are in the same horizontal plane. θ is the scattering angle measured in this plane relative to beam No. 2. Beams Nos. 1 and 3 are polarized in the scattering plane. Beams Nos. 2 and 4 are polarized perpendicular to the scattering plane.

tion is[3]

$$\frac{d\sigma}{d\Omega_3}=\tfrac{1}{2}r_0^{\,2}\left(\frac{\omega_3}{\omega_4}\right)\left(\frac{q_2}{k_1k_2}\right)\left(\frac{q\hbar}{mc}\right)^2\left|\frac{1-\epsilon(q,\omega)}{\epsilon(q,\omega)}\right|^2(\cos\theta)^2, \quad (1)$$

where $q=|\vec{k}_1-\vec{k}_3|$, $\omega=\omega_3-\omega_1$, $r_0=e^2/mc^2$ is the classical radius of the electron, $\cos\theta$ is the scattering angle [see Fig. (1b)], and $\epsilon(q,\omega)$ is the so-called causal dielectric function,

$$\frac{1-\epsilon(q,\omega)}{\epsilon(q,\omega)}=\int_{-\infty}^{+\infty}e^{-i\omega t}\langle T\{\rho_q(t)\rho_{-q}(0)\}\rangle, \quad (2)$$

where $\rho_q(t)$ is the spatial Fourier component of the electron density operator in the Heisenberg representation. $\epsilon(q,\omega)$ differs from the usual dielectric function because it has a different imaginary part:

$$\mathrm{Im}[\epsilon(q,\omega)]^{-1}=\coth(\tfrac{1}{2}\beta\hbar\omega)\,\mathrm{Im}[\epsilon_+(q,\omega)]^{-1}, \quad (3)$$

where $\epsilon_+(q,\omega)$ is the usual Lindhard dielectric function,[4] and $\beta=1/KT$ with K the Boltzmann constant and T the temperature. A similar expression may be written for any of the other choices of polarization.[3]

When $\epsilon(q,\omega)\neq 0$, the ratio inside the absolute value sign in Eq. (1) is of the order unity or smaller. Near a zero of $\epsilon(q,\omega)$, however, there is a resonance in the scattering cross section. This simply means that the intermediate state to which the many-body system was excited in our fourth-order process could have been a real state. Of course, these states have a finite lifetime, i.e., $\epsilon(q,\omega)$ has an imaginary part. The total cross sections are not infinite but just large.

For comparison, consider the incoherent, inelastic scattering of a single light beam from a plasma (see Fig. 2). Near the so-called plasma line, where $\omega_3=\omega_1\pm\omega_p$ and when $\omega/q\gg v_e$ (the electrons' thermal velocity), the differential cross section for scattering per unit volume of plasma (averaged over incoming polarization) may be written as[1]

$$\frac{d\sigma}{d\omega_3 d\Omega_3}=2\pi n r_0^{\,2}(1+\cos^2\theta)\times\left(\frac{q^2}{q_d^{\,2}}\right)\left(\frac{\omega_3}{\omega_1}\right)\frac{1}{\omega}\mathrm{Im}[\epsilon_+(q,\omega)^{-1}], \quad (4)$$

where $(\omega_1,\vec{k}_1)$, $(\omega_3,\vec{k}_3)$ are the energies and momenta of the incoming and outgoing photons, respectively, and n is the plasma density. This cross section, like that for light-off-light scattering, Eq. (1), has a resonance near the zero of $\epsilon_+(q,\omega)$. Experimentally, the important difference is that although $\epsilon_+(q,\omega)$ can get extremely small, there is still a factor $d\omega_3$ to contend with. The so-called plasma line has a high maximum intensity, but the cross section integrated over energy of the outgoing photon is small. DuBois and Gilinsky[1] have shown that the total cross section integrated over the plasma line is independent of its width and is given by

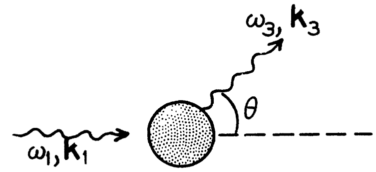

FIG. 2. Fundamental diagram for the incoherent scattering of a single beam from a plasma.

$$\frac{d\sigma_{\mathrm{total}}}{d\Omega_3}=2\pi^2 n r_0^{\,2}\left(\frac{q}{q_d}\right)^2(1+\cos^2\theta)\left(\frac{\omega_3}{\omega_1}\right). \quad (5)$$

The factor $d\omega_3$ in Eq. (4) is purely a density-of-final-states effect. The inelastic light-scattering experiment involves the available density of final "plasmon" states. In the light-off-light scattering the number of counts near the resonance is determined solely by the spectral width of the probing beams. Consequently, in the light-off-light scattering experiment near a resonance, larger counts can be obtained than with the single-beam experiment.

We now estimate the magnitude of light-off-light scattering cross section away from and at resonance. Typically, $\omega_3/\omega_4\approx 1$, $q^2/k_1k_2\approx 1$ (if we are away from the forward direction). For light sources operating in the visible at a frequency $\hbar\omega\approx 1.5$ eV, the quantity $q\hbar/mc$ is of the order of 10^{-5}. For these values of the parameters,

$$\frac{d\sigma}{d\Omega_3}\approx 10^{-36}\left|\frac{\epsilon(q,\omega)-1}{\epsilon(q,\omega)}\right|^2\ \mathrm{cm^2/sr}. \quad (6)$$

Away from a resonance where the ratio involving the dielectric function is of the order unity, the cross section is extremely small and would be difficult to measure.

To determine how large the cross section can be at a resonance, let us assume that a fully ionized nondegenerate plasma acts as the scattering medium. At resonance, $\omega\simeq\omega_p$, $\mathrm{Re}\epsilon_+(q,\omega)=0$ and $\mathrm{Im}\epsilon_+(\omega,q)\simeq\omega_p\tau$, where τ represents the lifetime of the intermediate state of the many-body system. The lifetime τ is determined either by Landau damping, or by collisions, or by the photon-induced natural lifetime,[5] whichever leads to shorter τ. We assume, of course, that $\omega_p\tau \gg 1$. In typical plasmas and for light in the visible part of the spectrum, k is of the order q_d. Therefore, unless q is made appreciably smaller than k by measuring the scattered light in the nearly forward direction, Landau damping will be very severe.[6] Therefore $q \simeq k\theta$, where θ is the small angle that the scattered beams make with incident beams. Then Eq. (1) becomes

$$\frac{d\sigma}{d\Omega_3} = 8\pi^2 r_0^{\;2}\left(\frac{\omega_3}{\omega_4}\right)(\theta^2)\left(\frac{q}{q_d}\right)^2\left(\frac{KT}{mc^2}\right)(\omega_p\tau)^2. \quad (7)$$

To compare the counting rate in the light-off-light scattering experiment with that in the single-beam scattering experiment (Eq. 5), we calculate the ratio $\mathfrak{R}$ of the total number of scattered photons N_S in the two cases:

$$\mathfrak{R} = \frac{N_{s\ \text{light-off-light}}}{N_{s\ \text{single-beam}}} \simeq 2\theta^2\left(\frac{KT}{mc^2}\right)\left(\frac{N_{\text{ph}}}{n}\right)(\omega_p\tau)^2, \quad (8)$$

where N_{ph} is the density of photons in the light beam, n is the plasma density, and T its temperature. Now, while the first two factors in Eq. (8) are much smaller than unity,[7] the last two factors can each be made larger than unity sufficiently so that $\mathfrak{R} \gtrsim 1$. Consequently, not only can counting rates be made larger at resonance, but because two scattered beams are present in the light-off-light experiment, coincidence techniques, or modulation and phase-synchronous detection techniques, can be used to discriminate against background. With present-day laser and plasma technologies, a meaningful light-off-light scattering experiment is feasible.

[1]W. E. Gordon, Proc. IRE 46, 1824 (1955); E. E. Salpeter, Phys. Rev. 120, 1528 (1960); M. N. Rostoker and N. Rosenbluth, Phys. Fluids 5, 776 (1962); J. Dawson and A. Ron, Phys. Rev. 132, 497 (1963); D. F. DuBois and V. Gilinsky, Phys. Rev. 133, A1308, A1317 (1964).

[2]K. W. Bowles, Phys. Rev. Letters 1, 454 (1958); V. C. Pineo, L. G. Craft, and H. W. Briscoe, J. Geophys. Res. 65, 2629 (1960); S. Fiocco and E. Thompson, Phys. Rev. Letters 10, 89 (1963).

[3]P. M. Platzman and N. Tzoar (to be published).

[4]A. J. Glick and R. A. Ferrell, Ann. Phys. (N.Y.) 11, 359 (1960).

[5]We wish to thank Dr. P. A. Wolff for pointing out to us the existence of this photon-induced upper limit on τ.

[6]If microwave beams are used for which $k \ll q_d$, it is no longer necessary to measure the scattered radiation in the nearly forward direction.

[7]Again, if microwave beams are utilized, the first factor in Eq. (8) need not be small.

TRIPLET CORRELATIONS IN LIQUIDS*

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(Received 24 April 1964)

The reliability of the superposition approximation (SA) for $g^{(3)}(r,t,s)$, the three-particle distribution, has been investigated in some detail, for a system of particles at liquid density, interacting through a physically reasonable potential. The superposition approximation for $g^{(3)}$ is $g_{\text{SA}}^{(3)} = g^{(2)}(r)g^{(2)}(t)g^{(2)}(s)$, where $g^{(2)}$ is the pair-distribution function. In this note we report the results for $g^{(3)}(r,r,s)$.

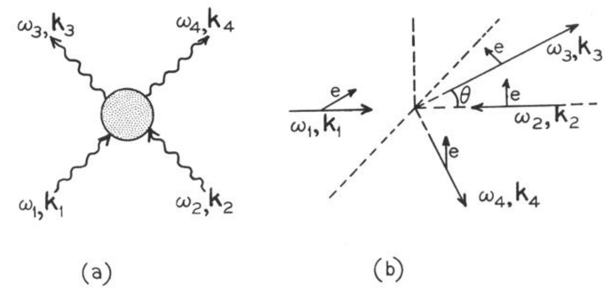
Let a denote the first-neighbor distance in $g^{(2)}$. It is found that:

(1) For $s<a$, the SA exaggerates the probability of occurrence of triplet distributions (r,r,s), and this exaggeration is greater for $r<a$.

(2) The ratio $R_r(s) = g^{(3)}(r,r,s)/[g^{(2)}(r)]^2$ shows a secondary maximum, and this is not given correctly by $g^{(2)}(s)$ which is the SA for $R_r(s)$; this maximum in $R_r(s)$ reflects the short-range order arising out of the sharp first maximum in $g^{(2)}$ which occurs at a. This maximum also seems to be present in the results of Alder[1] for a hard-sphere liquid.

(3) $g^{(2)}$ has been compared with $g_{\text{SA, BG}}^{(2)}$ which is found[2] by using $g_{\text{SA}}^{(3)}$ in the integral equation of Born and Green (BG), showing that $g_{\text{SA, BG}}^{(2)}$ is inaccurate to about the same extent as $g_{\text{SA}}^{(3)}$ itself.

The method used is the following. The motion of atoms in a liquid is simulated on a computer using classical dynamics on a system of 864 particles interacting through a Lennard-Jones potential having $\sigma = 3.4$ Å and $\epsilon/k_B = 120°$K,



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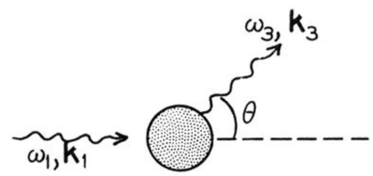

FIG. 2. Fundamental diagram for the incoherent scattering of a single beam from a plasma.