MESON-BARYON RESONANCES IN U(3) $\otimes$U(3) SYMMETRY*

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In an attempt to generalize the octet model of strong interactions,[1] and in particular to explain the existence of the nine vector mesons, one is led to consider the full SU(3) symmetry. For those representations that occur in SU(3) but not in the octet model [SU(3)/Z_3], one has the well-known problem of particles with fractional charge.[2] One method to maintain integral charge[3] is to consider U(3) instead of U(1)$\otimes$SU(3), where U(1) is the baryon gauge group.[4] If ν is the eigenvalue of the generator of baryon gauge transformations then we can arrange[5]

$$2\lambda+\mu+\nu\equiv 0 \pmod 3, \qquad (1)$$

where the representation is denoted by $D_\nu(\lambda,\mu)$.[5] This leads to

$$I_3+Y/2=\text{integer}=Q. \qquad (2)$$

Y is the conventional SU(2) hypercharge and is given by $Y=\frac{2}{3}m+\frac{1}{3}(K-1)\nu$, where m is the eigenvalue of the operator M given by deSwart[6] and $K\equiv 0 \pmod 3$. However this cannot be the proper symmetry group since octets with $\nu(=B)=1$ are not permitted by (1) and we consider U(3)$\otimes$U(3) as a possibility.[7] In this note we discuss the meson-baryon resonances in this framework.

Our two basic fields are taken to be U(3) triplets ψ with baryon number 1 and V with baryon number 2. It will be clear that to arrive at multiplets consistent with those already known in SU(3) we must take both ψ and V to be the basis vectors of $D(1,0)$ which is possible if we identify

$$B=-\nu \qquad (3)$$

in the V space. We must still specify the hypercharge of the triplets which we do as follows: For the ψ field we take $Y=(2/\sqrt{3})m+2\nu/3$ and for the V field $Y=(2/\sqrt{3})m-\nu/3$. Hence we have

$$I=0,\ Y=0;\ I=\tfrac{1}{2},\ Y=1. \qquad (4)$$

This assignment of hypercharge is opposite to that of Schwinger[7] and is necessary to insure a proper determination of the meson-baryon resonances (see below).

We construct the baryons as

$$\bar\psi\otimes V=(3,3),$$

the pseudoscalar and vector meson as

$$\psi\otimes\bar\psi=(3\otimes\bar 3,1)=(8,1)\oplus(1,1),$$

and the meson-baryon resonances as

$$\psi\otimes\bar\psi\otimes\bar\psi\otimes V=(3\otimes\bar 3\otimes\bar 3,3)$$
$$=(\overline{15},3)\oplus(6,3)\oplus(\bar 3,3)\oplus(\bar 3,3).$$

With our assignments (3) and (4) we find the isotopic spin and hypercharge assignments of the 18-dimensional representation (6, 3) to be

$$I=\tfrac{3}{2},\tfrac{1}{2},\tfrac{1}{2},\tfrac{1}{2},1,1,0,0,$$
$$Y=1,1,-1,-1,0,0,0,-2, \qquad (5)$$

and so the resonances can be assigned to this representation. If we had used the hypercharge assignment of Schwinger[7] as do Sawyer and Johnson[8] then all hypercharges in (5) would be reversed and we would be forced either to use the 45-dimensional representation or to build the resonances from the product $\bar\psi\otimes V\otimes V\otimes\bar V$ which is not a meson-baryon state. This will make a difference since the symmetry-breaking term of Schwinger is not symmetric in ψ and V.

We would also like to point out that if we introduce no fields other than ψ and V, then since U(1)$\otimes$SU(3) is not a subgroup of U(3) we cannot find a symmetry-breaking interaction which gives SU(3) symmetry and the success of the octet model must arise in a more sophisticated manner. In this we also differ with the methods of reference 8.

The symmetry-breaking term in the Lagrangian is given by Schwinger as

$$\mathfrak{L}_{\text{int}}=\sum_\nu\{\psi\cdots,\psi_\nu\}\bar V_{-\nu}\begin{pmatrix}3 & \bar 3 & 1\\ \nu & -\nu & 0\end{pmatrix}+\text{H.c.}, \qquad (6)$$

where we use the notation and phase conventions of deSwart,[6] and $\nu=I,I_z,Y$. Although we use the SU(3) Clebsch-Gordan coefficients in the following, this is to be regarded as a choice which will yield apparent SU(3) symmetry in the results rather than an imposition of SU(3) symmetry at this level. In fact (6) is only one of many choices to obtain U(2) symmetry. Constructing the lowest order mass operator yields

$$M = T_{000}{}^1 \mathcal{T}_{000}{}^1 + T_{000}{}^8 \mathcal{T}_{000}{}^1 + T_{000}{}^1 \mathcal{T}_{000}{}^8 - \tfrac{7}{6} T_{000}{}^8 \mathcal{T}_{000}{}^8$$
$$+ \tfrac{1}{2}(T_{110}{}^8 \mathcal{T}_{1-10}{}^8 - T_{100}{}^8 \mathcal{T}_{100}{}^8 + T_{1-10}{}^8 \mathcal{T}_{110}{}^8)$$
$$+ (T_{1/2-1/2-1}{}^8 \mathcal{T}_{1/2,1/2,1}{}^8 - T_{1/2,1/2-1}{}^8 \mathcal{T}_{1/2-1/2,1}{}^8$$
$$+ T_{1/2,1/2,1}{}^8 \mathcal{T}_{1/2-1/2-1}{}^8 - T_{1/2-1/2,1}{}^8 \mathcal{T}_{1/2,1/2-1}{}^8)$$
$$-(T_{1/2-1/2-1}{}^{27} \mathcal{T}_{1/2,1/2,1}{}^8 - T_{1/2,1/2-1}{}^{27} \mathcal{T}_{1/2-1/2,1}{}^8$$
$$+ T_{1/2,1/2,1}{}^{27} \mathcal{T}_{1/2-1/2-1}{}^8 - T_{1/2-1/2,1}{}^{27} \mathcal{T}_{1/2,1/2-1}{}^8)$$
$$+ (3/\sqrt{2}) T_{000}{}^{27} \mathcal{T}'_{000}{}^8 + (\sqrt{2}/\sqrt{3})$$
$$\times (T_{100}{}^{27} \mathcal{T}_{100}{}^8 - T_{110}{}^{27} \mathcal{T}_{1-10}{}^8 - T_{1-10}{}^{27} \mathcal{T}_{110}{}^8), \quad (7)$$

where T's are operators in the ψ space and $\mathcal{T}$ in the V space.

The mass formula for the baryons and mesons has been given by Schwinger,[7] and we turn our attention to the baryon-meson resonances. We construct 18 orthonormal states from the set (6, 3) using the SU(3) Clebsch-Gordan coefficients[9]; the states thus have SU(3) isotopic spin and hypercharge content and may be grouped into a decuplet $(N^*, Y_1{}^*, \Xi^*, \Omega)$ and an octet $(\tilde{N}, \tilde{\Lambda}, \tilde{\Sigma}, \tilde{\Xi})$. Using (7) and the Wigner-Eckart theorem, and redefining the reduced matrix element appropriately, we find

$$M(N^*) = a_0 + 16a_1 - a_2 - 2a_3 + 5a_4,$$
$$M(Y_1{}^*) = a_0 + 20a_1 - (16/3)a_4,$$
$$M(\Xi^*) = a_0 + 24a_1 + a_2 + 2a_3 + 7a_4,$$
$$M(\Omega) = a_0 + 28a_1 + 2a_2 + 4a_3 - 18a_4, \quad (8)$$

$$M(\tilde{N}) = a_0 - 11a_1 - a_2 - 2a_3 - 20a_4,$$
$$M(\tilde{\Sigma}) = a_0 - 33a_1 + a_2 - a_3 - (41/3)a_4,$$
$$M(\tilde{\Xi}) = a_0 - 31a_1 + 3a_3 + 20a_4,$$
$$M(\tilde{\Lambda}) = a_0 - 17a_1 - a_2 + a_3 + 3a_4, \quad (9)$$

$$\langle \tilde{\Sigma} | Y_1{}^* \rangle = \sqrt{2}\,[a_1 + a_2 - a_3 + (11/3)a_4],$$
$$\langle \tilde{\Xi} | \Xi^* \rangle = \sqrt{2}\,[a_1 + a_2 - a_3 + 5a_4]. \quad (10)$$

The constants a_0, a_1, a_2, a_3, and a_4 come from (1,1), (8,8), (1,8), (8,1), and (27,8), respectively. If we ignore (i) the mixing (10) and (ii) the coefficient a_4, then (8) satisfies the usual decuplet equal-spacing relation and (9) the G-M-O octet formula.[10] In fact, (i) and (ii) are not independent; if we require that the mixing is zero then (10) shows that $a_4 = 0$. Thus we obtain apparent SU(3) symmetry (and, in fact, the more specialized result that the mass operator transforms like the neutral member of an octet) in this model, by the dynamical assertion of no mixing, as in the baryon formula of Schwinger.

Requiring $\langle \tilde{\Sigma} | Y_1{}^* \rangle = \langle \tilde{\Xi} | \Xi^* \rangle = 0$ leaves three unknowns in (8) and (9). Fixing the position of the N^* and the level spacing, b, of the decuplet determines two of them and hence the position of the octet is undetermined. If, for example, the $I = \frac{1}{2}$, $J = \frac{3}{2}$ (parity not known) resonance[11] at $M = 1515$ MeV is identified as $\tilde{N}$, then the remaining members of the octet lie just above the highest energies presently studied; in fact, $M(\tilde{\Lambda}) = 2390$ MeV. It should be emphasized that the model does predict 18 baryon-meson resonances all with $J = \frac{3}{2}^+$, even though they split into a set of 10 and a set of 8, each satisfying the appropriate G-M-O formulas separately.

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