MEASUREMENT OF THE nn SINGLET S-STATE SCATTERING LENGTH*†

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A measurement is reported of the spectrum of γ rays from the reaction

$$\pi^- + d \rightarrow n + n + \gamma \qquad (1)$$

for π^- mesons stopped in liquid deuterium. A 180-deg-focusing γ-ray pair spectrometer[1] with an energy resolution of 1% was used as the detector. The measured spectrum was compared with theoretical spectra yielding a value for the neutron-neutron singlet S-state scattering length a.

It is well known from existing experimental results that for the nucleon-nucleon interaction the np and pp singlet state scattering lengths differ considerably. The apparent violation of charge independence has stimulated considerable work in attempts at a reasonable explanation.[2-6] One of the desirable pieces of information to help clarify the situation is the nn scattering length.

One of the cleanest ways of measuring a is by means of Reaction (1), since in the final state only the two neutrons interact strongly. Phillips and Crowe[7] have measured the γ-ray spectrum from this reaction and obtained a value for a of -15.9 F with probable errors of $-\infty$ F and +7.4 F. A recent measurement by Cerineo et al.[8] of the proton spectrum from the $d(n,p)2n$ reaction yields a value for a of -21.7 ± 1 F. Although the statistical accuracy of this latter experiment is quite good, there seems to be considerable uncertainty in the assumptions used in the theoretical analysis.

A detailed description of the experimental setup for the present measurement is given in reference 1. π^- mesons from the Berkeley 184-in. synchrocyclotron were stopped in liquid hydrogen and also in liquid deuterium. The spectrum of γ rays from the radiative reaction in hydrogen,

$$\pi^- + p \rightarrow n + \gamma, \qquad (2)$$

and Reaction (1) in deuterium were both measured. Identical spectrometer settings were employed for the two measurements.

The measured deuterium spectrum was compared with theory in a manner identical to that employed by Phillips and Crowe.[7] In this analysis the energy resolution function for the spectrometer is folded into theoretical spectra evaluated for different values of the nn scattering length. The scattering length is then determined by the best fit of the measured spectrum with the folded theoretical spectra.

A theoretical calculation of the spectra has been made by Watson and Stuart[9] who employ the impulse approximation and consider the two final-state neutrons to interact only in the singlet S state. The nn wave function is approximated by a square well with the correct scattering length and effective range r_0. For the deuteron S state the Hulthén wave function $(e^{-\alpha r} - e^{-\beta r})/r$ is employed. The validity of the approximations made in the calculation are discussed by Watson and Stuart and also by McVoy.[10] Calculations made by McVoy of the integrals which occur in the formula for the spectra were used by us in the numerical calculation. An IBM 7074 computer was employed. For this calculation the following constants were used: $r_0 = 2.65$ F, $\mu = 139.58$ MeV, $M_n/\mu = 6.7310$, $\alpha/\mu = 0.3274$, $\beta/\mu = 1.54$, $[\mu - (M_n - M_p) - B_0]/M_n = 0.9747\ \mu$. Here, μ is the π^--meson mass, M_n the neutron mass, M_p the proton mass, and B_0 the deuteron binding energy.

The instrument resolution is determined by the measured spectrum of the monoenergetic γ ray from Reaction (2). To be used in the folding operation it must be energy independent over the range of energy values of the theoretical spectra for which spectrum comparison is made. Of the various factors which cause energy dependence of the resolution,[7] the most important for this spectrometer is a second-order effect due to multiple scattering. The effect of this on the final results has been determined to be insignificant compared to the statistical error, and no correction was deemed necessary.

It is imperative that the measured spectra accurately describe the spectrum of γ rays from the reactions involved. Since the pair spectrometer detects a variable fraction of the electron-positron pairs for a given γ-ray energy, an energy-dependent correction to the measured data must be applied. This is easily calculated from the geometry of the pair spectrometer.[1] In addition, a correction was made to the deuterium spectrum for the variation of the pair production cross section with energy.

The folded theoretical spectra for several values of the scattering length a are shown in Fig. 1.

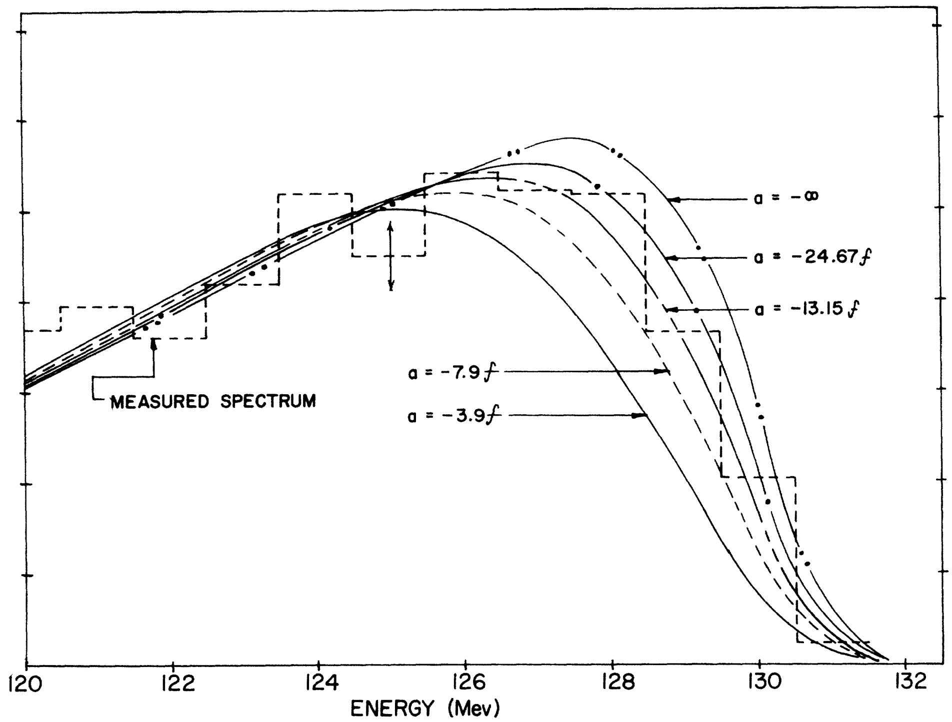

FIG. 1. The folded theoretical γ-ray spectra for various values of the neutron-neutron singlet S-state scattering length a. The measured spectrum is also shown.

Included in the figure in histogram form is the measured spectrum corrected for the energy-dependent detection efficiency and normalized to the same area as the theoretical spectrum for $a = -20$ F.

Comparison of the spectra was made by evaluating an average weighted spectrum energy (the weighted first moment[7]) over a given energy interval at the high-energy end of the spectra. The weighting factors were taken equal to the reciprocal of the square of the relative error for each data point of the measured spectrum. This method of weighting was chosen to reduce the effect of the low-energy tail of the spectrum, which is relatively insensitive to the nn interaction. Identical weighting factors were used for the theoretical spectra. The weighted first moment is thus defined as

$$M = \sum W_i N_i E_i / \sum W_i N_i,$$

where N_i is the spectrum contribution in the energy interval of mean energy E_i and $W_i = (N_i'/\sigma_i)^2$. Here, N_i' is the value of N_i for the measured spectrum corrected for the energy-dependent detection efficiency and σ_i is the standard error in N_i'.

The error in M for the folded theoretical spectra resulting from using the measured spectrum from Reaction (2) as the instrument resolution is negligible. The error on M for the measured deuterium spectrum was calculated by the standard method of error propagation and is given in Table I as the probable error in ΔM.

The values of M for the folded theoretical spectra relative to the value of M for the spectrum with $a = -\infty$ are plotted in Fig. 2 as a function of a^{-1}. (ΔM is defined as $M_a - M_{a=-\infty}$.) Smooth curves have been drawn through the calculated points. Curves I, II, III, and IV correspond to the four different energy ranges (specified in Table I) over which M as a function of a has been evaluated. The corresponding values of ΔM for the measured spectra for each of these four en-

Table I. Measured values of the scattering length for the four energy ranges considered.

Energy range (MeV)		$M_{\text{meas}} - M_{a=-\infty}$	a (F)
I	120-131.5	-0.46 ± 0.08	$-15.1^{-3.3}_{+2.5}$
II	122-131.5	-0.34 ± 0.07	$-19.1^{-5.9}_{+3.8}$
III	124-131.5	-0.29 ± 0.07	$-19.0^{-6.6}_{+4.0}$
IV	126-131.5	-0.23 ± 0.07	$-17.0^{-7.7}_{+4.3}$

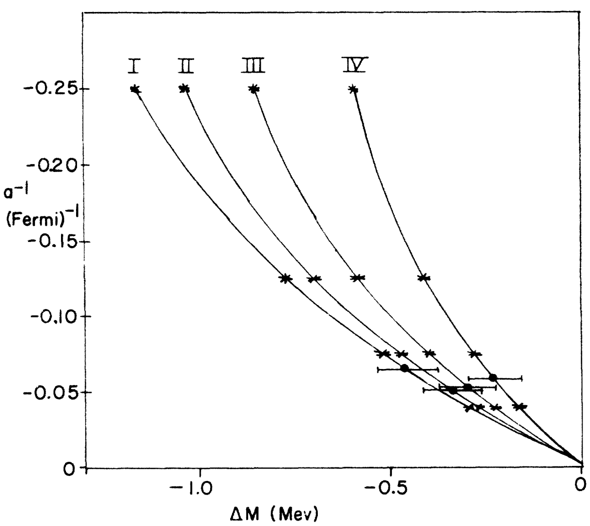

FIG. 2. Each curve represents the value of M as a function of a relative to the value of M for $a=-\infty$ plotted against a^{-1}. ΔM is equal to $M_a - M_{a=-\infty}$. Curves I, II, III, and IV correspond to the four energy ranges specified in Table I. The measured values of ΔM are indicated together with the probable errors.

ergy ranges are also given in the figure with the respective probable errors. The corresponding values for the scattering length a are specified in the table.

It is emphasized that the quoted errors on a are purely statistical and do not contain any estimate of error resulting from uncertainty in the theoretical spectra. The shapes of the theoretical spectra are expected to be quite accurate near the high-energy end where the relative momentum of the two neutrons is small. However, some of the approximations made in the theoretical treatment become less reliable as this momentum increases. The analysis by McVoy[10] indicates that uncertainties in the spectra may become significant at energies of more than 1 MeV below the high-energy end. It is pointed out that although the energy ranges over which the analysis has been made seem quite substantial compared with 1 MeV, the contribution in the folded spectra arising from the accurately known high-energy end of the unfolded spectra is relatively quite large. For example, in the folded spectrum for $a=-24.67$ F and for the energy interval 126-131.5 MeV, 50% of the contribution arises from the first 1 MeV in the high-energy peak and 75% comes from the first 2 MeV.

Cerineo et al.[8] have compared their measured value for a with that predicted by two different theoretical approaches. When compared with the predictions made by Lin,[5] a 2 to 3% difference in the coupling constants between the π^0 meson and the proton and neutron, respectively, is required to achieve agreement between the calculation and the measurement. From a comparison with the calculations of Wong and Noyes,[6] a breakdown of charge symmetry is indicated. The results of our measurement, if compared with the calculations of Wong and Noyes, also indicate that charge symmetry in the neutron-neutron interaction is violated.

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