Corporation.

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CURRENT OSCILLATIONS AND COLLECTIVE WAVES IN CdS

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This Letter describes a new type of oscillatory behavior of electron currents flowing in CdS under conditions where the electron drift velocity exceeds the velocity of sound. The frequencies of these oscillations are not determined by the transit time of sound through the sample alone, as reported by earlier investigators,[1,2] but also depend sensitively upon the voltage applied to the crystal, the electron density, and the ambient temperature of the crystal. These oscillations are believed to arise from the coupling between long-wavelength collective phonon waves[3,4] and mobile electrons.

The observations described here were made on photoconducting CdS crystals which were insulating in the dark ($\rho > 10^9$ Ω-cm). Electrical contact to the CdS was made with indium, and the current flow was in the basal plane of the crystal. Voltage pulses between 10 μsec and 100 μsec in duration were applied to samples whose lengths ranged between 4 to 16 mm. The oscillations reported here occurred only if sufficient voltage was applied to the crystal to produce sonic amplification. Under these conditions, the sample was observed to emit maxima of acoustic noise in the 10- to-100-Mc/sec frequency band simultaneously with minima in the current.

The oscillations of present interest (type I) differ from those previously described by McFee[2] (hereafter called type II) in several important respects. The period of the type-II oscillation was equal to $2(L/v_S)$ where v_S is the velocity of the amplified sound modes, and L is the length of the crystal. This period is substantially independent of any intensive property of the crystal. In contrast, the longest observed period for the type-I oscillations was about $1.7(L/v_S)$. Further, the period of type-I oscillations could be substantially changed by varying the electron density,

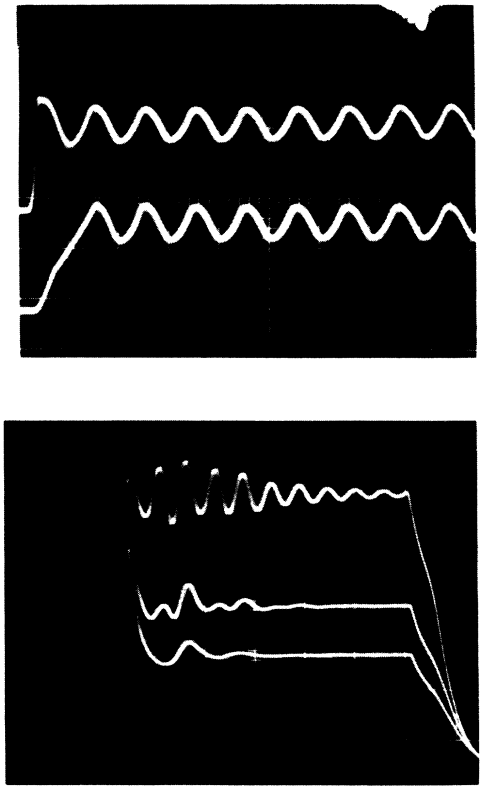

FIG. 1. (a) Current oscillations in a CdS sample. Top trace was recorded with rise time of voltage pulse of 0.2 μsec; lower trace was recorded with rise time of 4.0 μsec. Note that oscillations have same amplitude and frequency (about 250 kc/sec) in both traces. (b) Modal shift in current oscillations in a CdS sample with fixed applied voltage. Increasing the electron density from about 10^{13}/cm^3 (lowest trace) to about 2×10^{13}/cm^3 (upper trace) has caused the appearance of the second harmonic frequency (about 400 kc/sec).

n, the ambient temperature of the crystal, T, or the voltage applied to the sample, V. The shortest period observed was less than $0.1(L/v_S)$.

The dependence of the amplitude of the oscillations upon the rise time of the voltage pulse also serves to distinguish the two types of oscillation. The amplitude and frequency of the type-I oscillations were independent of the rise time of the voltage pulse for variations in rise time between 0.1 μsec and 15 μsec. In striking contrast, the type-II oscillations were only observable for rise times less than 0.5 μsec. The insensitivity of the type-I oscillations to the rise time of the voltage pulse is illustrated in Fig. 1(a).

Although the frequency of the type-I oscillations is, in general, a continuous function of n, T, and V, it was sometimes observed that very small changes in any one of these parameters could cause an abrupt change of the frequency to a harmonic or subharmonic mode of oscillation. The development of a harmonic mode with small increases in the electron density is illustrated in Fig. 1(b). The frequency dependence of the lowest frequency mode upon n and V is shown in Fig. 2.

The type-II oscillations can be explained in terms of a pulse of acoustic flux reflecting back and forth within the sample.[2] The maximum reaction on the current occurs when the pulse is at its greatest amplitude near the anode end of the crystal. In contrast, the frequencies of type-I oscillations cannot be uniquely related to the transit time of any simple acoustic mode.

The longest period of the type-I oscillations is the approximate one-way transit time across the sample of the collective phonon waves previously reported. Observations which further suggest a relation between these oscillations and collective phonon waves is their similar temperature dependence. Both the "trailing-pulse" phenomenon, which has been interpreted as a manifestation of collective phonon waves, and the type-I oscillations have been observed only when the ambient temperature of the crystal was between 250°K and 350°K.[3] In the remainder of this note we shall outline the theory of the interaction of free electrons and collective phonon waves and point out the relationship of this theory to the present observations.

The collective phonon-wave dispersion relation[4] can be extended by including the charge continuity equation and Poisson's equation. A self-consis-

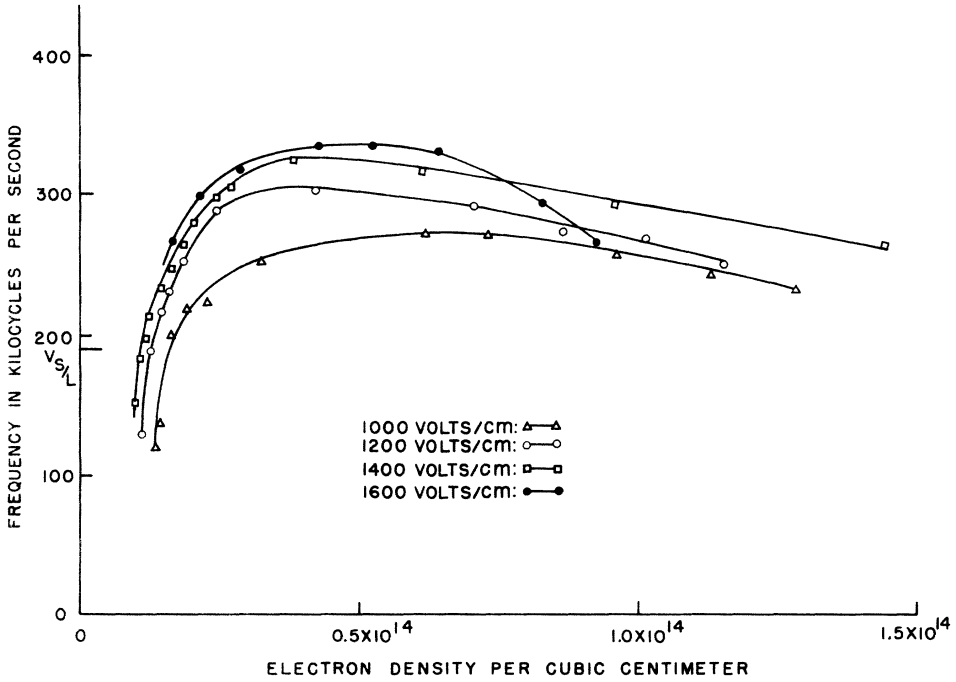

FIG. 2. Frequency of the current oscillations for the lowest frequency mode of oscillations as a function of electron density for various applied electric fields. Ambient temperature of the CdS crystal was 25 ± 2°C.

tent linearized solution of these equations yields the dispersion relation for plane waves

$$\omega^2 = \omega_0^2 + v_{II}^2 q^2, \quad (1)$$

where v_{II} is the velocity of a collective wave, as previously described, and

$$\omega_0 = \left[\beta \frac{4\pi e^2 n}{M/n}\right]^{1/2}. \quad (2)$$

Here, n is the density of charge carriers, e is the electronic charge, and β is a dimensionless factor of order $\frac{1}{10}$. The apparent phonon mass per unit volume, M, is defined as $M = \sum_k \hbar k N(k)/\lambda$, where $N(k)$ is the phonon occupation number of a mode k, and λ is the local phonon drift velocity. In the short-wavelength limit, $v_{II} q \gg \omega_0$, the solution is the collective wave previously described.

In the long-wavelength limit $v_d q < \omega_0$, the phase velocity of the wave given by Eq. (1) is larger than the electron drift velocity. The curve $\omega = v_d q$ crosses the dispersion relation of Eq. (1), and the undamped solution becomes $\omega \approx v_d q$. Ultimately, ω approaches $-\omega_0$ at $q = 0$, but the solutions in this region are damped.

The dispersion relation, Eq. (1), is similar to that of electrostatic waves in plasmas[5]: ω_0 is the plasma frequency for a system of particles whose density and charge are that of the electrons and whose mass is the apparent phonon mass per charged particle. The factor β is a correction which arises from the viscous interaction between the electrons and phonons of the active frequency band[3,4] and the rest of the specific heat of the solid.

For very small values of n, ω_0 is small and $\omega \approx v_{II} q$. In this same limit, the amplification and nonlinear effects are small and a plane-wave approximation is most valid. In the experiment described, a fixed voltage is applied across the finite crystal which imposes the boundary condition

$$\int_0^L E_1(x,t)dx = 0, \quad (3)$$

where $E_1(x,t)$ is the time-varying component of the local electric field, i.e., that associated with the wave amplitude. The values of q, which satisfy this condition, are $q = 2\pi N/L$, $N = 1, 2, \cdots$. For large values of n, $\omega \approx \omega_0$, the frequency ultimately is limited by the crossover with $\omega \approx v_d q$. The solution of ω at all but the lowest values of n is complicated by the breaking down of the linearized plane-wave solution. Further analysis is in progress on this point. This analysis shows excellent agreement at the lowest frequencies and amplitudes observed, i.e., $\omega = v_{II} 2\pi/L$, and at higher frequencies there is order-of-magnitude agreement. The estimated value of ω_0 at the highest electron densities used is $\sim 10^6 - 10^7$ sec^{-1}.

The nature of these oscillations can be qualitatively understood in terms of a forward transit through the crystal of a collective phonon wave with the accompanying variation of an electrostatic field. This electrostatic field provides a feedback mechanism across the crystal. Local variations in phonon density associated with the collective wave give rise to local variations in the carrier mobility. The nonuniform mobility will cause the accumulation of space charge which is the source of the electrostatic field. In effect, this mechanism can be thought of as a collective phonon wave with electrons bunched in the regions of maximum gradient of the phonon density. This accumulation of charge adds to the elastic stiffness of the propagating wave, causing its dispersion at long wavelengths. The constancy of the applied voltage, as implied in Eq. (3), and the fact that the collective waves can propagate only in the direction of carrier drift, select particular wavelengths of the collective wave and insures the repetitive nature of these oscillations. The type-I oscillations are then a result of a volume instability effect which is produced by phonons amplified from the thermal background. This is in contrast to the type-II oscillations which depend on the generation of a sound pulse at the crystal boundary due to a piezoelectric shock of a rapidly varying applied voltage.

The previous discussion is concerned chiefly with phenomena occurring in CdS. However, the theoretical results should apply to all direct materials with piezoelectrically active phonon bands. This mechanism may be responsible for the gigacycle oscillations observed by Gunn[6] in GaAs. The frequency ω_0 is proportional to the ratio of electron density over electron effective mass. Increases in v_d also raises ω_0 somewhat. In GaAs the value of ω_0 could be four orders of magnitude larger than in the experiments described here. If this were the case, the fundamental mode of oscillation determined by crystal length and characterized by the wave number q_0 [determined by the boundary condition, Eq. (3)] would meet the condition $v_d q_0 \ll \omega_0$ so that $\omega \approx v_d q_0$, and the damping could only be overcome when ω is not too far from ω_0. Large threshold drift velocities would be necessary to bring $v_d q_0 \approx \omega_0$.

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MAGNETORESISTANCE AND MAGNETIC BREAKDOWN*

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The galvanomagnetic properties of metals have been widely used to study the motion of the conduction electrons.[1] Generally, for high magnetic fields, two distinct types of transverse magnetoresistance $\rho_\perp(H)$ are found, either a resistance that saturates, i.e., approaches a constant value as the field is increased, or a resistance which increases without bound as H^2. These properties can be uniquely related to the presence or absence of open trajectories.[2] The appearance of quadratic behavior indicates (a) in metals with an odd number of conduction electrons per unit cell: the existence of open trajectories with an average direction in k space not perpendicular to the direction of the current; (b) in metals with an even number of conduction electrons per unit cell: either no open orbits and equal "volume" of electron and hole trajectories (the so-called compensated case), or open orbits not perpendicular to the current.

When high magnetic fields are used in metals which have relatively small energy gaps, the dynamics of the motion is such that there is a finite probability for the electrons to ignore such gaps and describe trajectories which connect two different pieces of Fermi surface (thus changing the character of the orbits). This effect is called magnetic breakdown,[3-5] and it has been found experimentally in several metals, mostly those with hexagonal close-packed structure.[6-10] The probability of breakdown, that is, the probability for an electron to make a transition between two different orbits, has been found by Blount[4] to be

$$P=\exp[-(H_0/H)], \tag{1}$$

$$H_0=K\Delta^2 mc/E_F e, \tag{2}$$

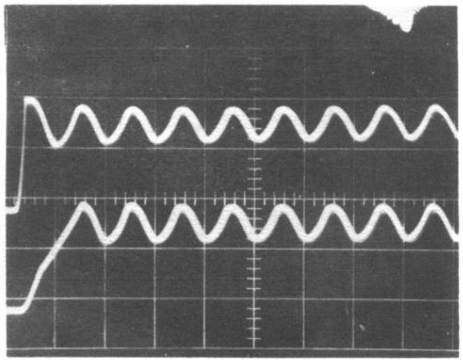
where Δ is the energy gap, E_F the Fermi energy, and K a constant of the order unity.[11]

When orbits of different character are coupled by magnetic breakdown, new types of behavior must be expected for $\rho_\perp(H)$. In particular, when the two limiting cases $P\to 0$ and $P\to 1$ correspond to nonsaturation and saturation, respectively, $\rho_\perp(H)$ is likely to exhibit a maximum at fields of the order of H_0.

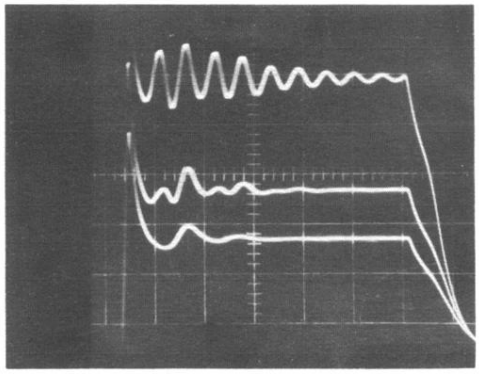
We have computed semiclassically the transverse conductivity and resistivity tensors by means of a modification of Chambers' path-integral method,[12] which includes the possibility of magnetic breakdown at a finite number of points in the orbit. This new method involves a matrix M, a function of P only, which connects in the proper way the path integrals over pieces of the Fermi surface on which the electron trajectories develop. This results in a classical network, similar to the quantum-mechanical network introduced by Pippard[5,13] in the study of the quantization of coupled orbits.

We have solved a large number of cases, all two-dimensional (i.e., neglecting longitudinal effects), which result from a spherical Fermi surface with a finite number of Bragg reflections. The intrinsic relaxation time was assumed to be constant throughout. We included cases with no axis of symmetry and with two-, four-, and sixfold symmetries. We also studied magnetic breakdown which corresponded to transitions between the following kinds of orbits: (I) open + electron → electron, (II) open + electron + hole → electron + hole (compensated), (III) hole → electron, (IV) electron + hole (compensated) → electron + electron. An example of class (I) is shown in Fig. 1, where transitions between an "undulating cylinder" and a "lens" make the final orbit equal to the original circle. Figure 1(c) shows the transverse magnetoresistance in the x direction as a function of $\omega\tau$, where $\omega=eH/mc$ is the cyclo-

(a)



(b)



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