DYNAMICS OF THE π-ω RESONANCE*

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The dynamical model presented here strongly indicates a 1^+ π-ω resonance, having all the properties of the "B meson" observed by Abolins et al.,[1] at 1220 MeV, including the absence of its decay into two pseudoscalar mesons or into $\pi+\varphi$.

π-ω scattering can occur in all $I=1$, $G=+1$ states with spin and parity different from 0^+. Odd-parity, odd-J (angular-momentum) states can connect to two pseudoscalar mesons; in the remaining states, $\pi+\omega$ is the lowest known two-meson threshold. The following remarks are restricted to these states. If the B is seen to decay into $\pi+\pi$ or $K+\bar{K}$, it cannot be the resonance we shall predict here. In these states the next known threshold is $\pi+\varphi$, so that elastic π-ω unitarity is approximately valid in a substantial region.

The ρ meson is the known particle of lowest mass which can be exchanged in either of the crossed channels; therefore, we assume that the ρ-exchange force, Fig. 1, is the most important. In the partial-wave amplitudes, Fig. 1 gives rise to a short cut close to the physical region, and therefore may be expected to dominate low-energy scattering, in analogy with nucleon or (3, 3) resonance exchange in pion-nucleon scattering. The most important omitted effect is probably the exchange of the B meson itself.

If the $\pi\varphi$ channel were included, one should expect from the same arguments that ρ exchange remains the dominant force, since ω and φ have identical quantum numbers. From the experimental absence of three-pion decays of the φ, we know that the $\pi\rho\varphi$ coupling must be much smaller than the $\pi\rho\omega$ coupling; therefore, $\pi\varphi$ scattering should be negligible compared to $\pi\omega$ scattering. Thus the plausible assumption of the dominance of ρ exchange predicts that the decay of a $\pi\omega$ resonance into $\pi+\varphi$ should be greatly suppressed, as is observed for the B.

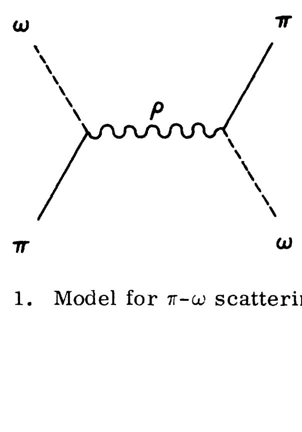

FIG. 1. Model for π-ω scattering.

The effect of the force shown in Fig. 1 can be studied by a version of the N/D method. The absence of appreciable connection to the $\pi\varphi$ channels permits the extension of approximate unitarity up to a much higher mass, for example, the $K^*(725)$-$\bar{K}$ or η-ρ thresholds. The absence of obvious competing forces (nearby singularities in the partial-wave amplitude) has already been noted. Therefore, in the spin-parity states under consideration, $\pi\omega$ scattering is an extraordinarily "clean" problem for a simple one-channel N/D calculation, and should be as good as any other calculation of this type.

The effective $\pi\rho\omega$ vertex is taken as

$$(h/\mu)\omega^{\mu}\rho^{\nu}p_{\rho}^{\lambda}p_{\omega}^{\sigma}\epsilon_{\mu\nu\rho\sigma}, \tag{1}$$

which defines the dimensionless coupling constant h. Gell-Mann, Sharp, and Wagner[2] have estimated the realtion of h to the width of the ω, assuming that the ω decays completely through a virtual ρ. For a width of 8 MeV,[3] their formula gives $h^2/4\pi=0.35$.

The partial-wave projections of Fig. 1, $B^{JP}(s)$, which are the Born terms in $T^{JP}=e^{i\delta}\sin\delta$, are easily computed in the helicity representation. Here, we give explicitly only the 0^- and 1^+ terms. Let E_π and E_ω be the energies of the π and ω, respectively, and q their common momentum; $W=E_\pi+E_\omega$, $s=W^2$, and

$$x=1+(2m_\pi^{\ 2}+2m_\omega^{\ 2}-m_\rho^{\ 2}-s)/2q^2. \tag{2}$$

Then

$$B^{0^-}=(h^2/4\pi)\tfrac{2}{3}(q/4W)(m_\omega^{\ 2}/m_\pi^{\ 2})[Q_0(x)-Q_2(x)], \tag{3}$$

which corresponds to a repulsive force. Thus, in this model a pseudoscalar resonance is impossible. The analogous formulas for $J=2$ show that the ρ-exchange force is repulsive in both the 2^+ and 2^- states.

There are two 1^+ states. In the helicity representation it is more convenient to write T as a matrix between transverse (t) and longitudinal (l) states rather than the more familiar S and D waves. The elements of the B^{1+} matrix are

$$B_{tt}^{1+} = -\frac{h^2}{4\pi}\frac{q}{4Wm_\pi^{\,2}}\{(\tfrac{1}{3}q^2 + \tfrac{4}{3}E_\pi E_\omega)Q_0(x) + (E_\pi^{\,2} + \tfrac{4}{5}E_\omega^{\,2})Q_1(x) + (\tfrac{2}{3}E_\pi E_\omega - \tfrac{1}{3}q^2)Q_2(x) + \tfrac{1}{5}E_\omega^{\,2}Q_3(x)\},$$

$$B_{tl}^{1+} = B_{lt}^{1+} = -\frac{h^2}{4\pi}\frac{qm_\omega}{2\sqrt{2}\,Wm_\pi^{\,2}}\{\tfrac{1}{3}E_\pi[Q_0(x) - Q_2(x)] + \tfrac{1}{5}E_\omega[Q_1(x) - Q_3(x)]\},$$

$$B_{ll}^{1+} = (q/4W)(h^2/4\pi)(m_\omega^{\,2}/m_\pi^{\,2})\tfrac{2}{5}[Q_1(x) - Q_3(x)]. \quad (4)$$

T was computed by the first-order, or determinantal, method, with elastic unitarity, which has been described in detail elsewhere.[4] The amplitude is written $T = BD^{-1}$, where

$$D_{ij} = \delta_{ij} - \frac{(s - s_0)}{\pi}\int_{(m_\omega + m_\pi)^2}^{\infty}\frac{B_{ij}(s')ds'}{(s' - s)(s' - s_0)}. \quad (5)$$

The mass of the resonance is the solution to $\mathrm{Re}\det D(m_r^{\,2}) = 0$, and its width may be defined as

$$\Gamma_r = \frac{\mathrm{Im}\det D(m_r^{\,2})}{m_r \mathrm{Re}(d/ds)\det D(m_r^{\,2})}. \quad (6)$$

The integral in Eq. (5) diverges, so that a cutoff is necessary. The computation was performed for a range of cutoffs. The subtraction point s_0 was taken at the left end of the short cut to insure approximate crossing symmetry as well as possible. The results do not vary appreciably as s_0 is moved over the short cut. In this method, the resulting matrix is not symmetric, and the ratio $R = T_{tl}/T_{lt}$ is a measure of the approximation. In all cases, $0.77 < R < 1.0$, a considerably better result than has been found in other determinantal calculations. The results are summarized in Table I for various values of the cutoff energy Z.

Table I. Position and width of 1^+ resonance.

$h^2/4\pi$	0.10	0.15	0.20	0.25	0.35
Position (MeV)					
$Z = 1.5$ BeV	1303	1195	1130	1099	1042
$Z = 1.75$ BeV	1171	1090	1046	1013	966
$Z = 2.00$ BeV	1088	1019	980	...	...
Width (MeV)					
$Z = 1.5$ BeV	712	270	170	119	63
$Z = 1.75$ BeV	140	73	40	21	10
$Z = 2.00$ BeV	51	19	4	...	...

For a reasonable range of cutoffs, a low-energy 1^+ resonance may be expected if $h^2/4\pi$ is of the order predicted by the ω width. To obtain a resonance at 1220 MeV, one must take $h^2/4\pi$ of the order of 0.1 or 0.15, more in agreement with the self-consistent value predicted by Zemach and Zachariasen[4] than with the value predicted from the formula of Gell-Mann, Sharp, and Wagner,[2] together with the width measurement of Gelfand et al.[3,5]

In sum, the model of ρ-exchange dominance, together with the observed rates of ω and φ decays into three pions, results in a force of the proper sign and strength to predict an axial-vector $\pi\omega$ resonance which does not decay into 2π, $K+\bar{K}$, or $\pi+\varphi$, and therefore may be the B meson.

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[5]Of course, in this type of computation, particularly in view of the necessity of a cutoff, one may not take seriously the exact numerical results.