UNITARY SYMMETRY AND ELECTROMAGNETIC SELF-ENERGIES OF NEUTRAL VECTOR MESONS*

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It has been recently suggested by Schwinger[1] and by Gürsey and Lee[2] that the nine observed vector mesons, ρ, ω, K^*, and φ, belong to a degenerate nonet. The secondary interaction, which violates the fundamental symmetry, induces mass splittings and the actual masses can be then related by a mass formula obtained in first-order perturbation for the secondary interaction. In this way, Gürsey and Lee[2] obtain with a definite model the remarkable relations

$$2m_\varphi + m_\omega + m_\rho = 4m_{K^*} \text{ and } m_\rho \simeq m_\omega.$$

The question naturally arises whether the observed mass difference $\Delta m = m_\omega - m_\rho \simeq 30$ MeV could be of electromagnetic origin. We show in this note, by calculating certain contributions to the self-energies, that a large amount of Δm could indeed be of this kind. To this end, we shall use unitary symmetry and information extracted from the measured electromagnetic decays of the ω meson.

The possibility that the observed Δm is an electromagnetic effect has been already mentioned generally in the articles dealing with ω-ρ mixing,[3] though often subsequently rejected on the grounds that it is too large to be of such nature.

We proceed by considering the nine vector mesons as eigenstates of the Hamiltonian[2]

$$H = H_0 + h \tag{1}$$

where H_0 is the part which is invariant under SU(3) while h is the weaker interaction which causes the mass splittings. As a result of the Hamiltonian (1), the vector mesons emerge with their heavy masses. We now switch on the electromagnetic interaction $h^{(e)}$, and we ask for the mass shifts of the neutral vector mesons due to the interaction with the photon field. In order to obtain these contributions to the self-energies, we have to solve for the W_i given by

$$\begin{vmatrix} h_{11}^{(e)} - W_i & h_{12}^{(e)} & h_{13}^{(e)} \\ h_{21}^{(e)} & h_{22}^{(e)} - W_i & h_{23}^{(e)} \\ h_{31}^{(e)} & h_{32}^{(e)} & h_{33}^{(e)} - W_i \end{vmatrix} = 0 \tag{2}$$

Here the states 1, 2, and 3 represent the eigenstates ρ^0, ω, and φ of (1). Obviously, we do not know how to compute all the possible Feynman diagrams. We choose to take (to order α) the "pole diagrams" only, i.e., diagrams of the type vector meson $\to \gamma \to$ vector meson, which might be the most important ones.

For the interaction of a neutral vector meson with the electromagnetic field we use the effective Lagrangian[4]

$$L_{\text{int}} = \frac{e}{2f_v}(V_{\mu,\nu} - V_{\nu,\mu})(A_{\mu,\nu} - A_{\nu,\mu}). \tag{3}$$

The effective couplings f_v are related by unitary symmetry which gives[5]

$$\langle\gamma|\rho^0\rangle : \langle\gamma|\chi_8\rangle : \langle\gamma|\chi_1\rangle = 1 : 1/\sqrt{3} : 0, \tag{4}$$

where χ_1 is a unitary singlet and χ_8 is the $T=0$ member of a unitary octet. These relations follow by assuming that the electromagnetic current is given by $j_e(x) = j_3(x) + (1/\sqrt{3})j_8(x)$, where $j_i(x)$ are the currents corresponding to the eight generators of SU(3). For the eigenfunctions of the ω and φ states we use the expressions[6] obtained in reference 2

$$\omega = (\sqrt{2}/\sqrt{3})\chi_1 - (1/\sqrt{3})\chi_8$$
$$\varphi = (1/\sqrt{3})\chi_1 + (\sqrt{2}/\sqrt{3})\chi_8. \tag{5}$$

We shall assume that the masses of the neutral vector mesons [due to (1)] obey $m_\rho = m_\omega = m$ and $m_\varphi \neq m$. Then, because of the large mass difference between φ and ρ^0, ω we can take the terms $h_{13}^{(e)}$ to be negligibly small. The contribution to the other terms of the pole diagrams is then of the general form

$$(\delta M)_{ij} = e^2 M / 2f_i f_j. \tag{6}$$

We denote $f_\rho = f$, and diagonalizing (2) after using (4) and (5) to calculate the various terms we obtain

$$W_1 = 0;\ W_2 = 5e^2 m/9f^2;\ W_3 = e^2 m_\varphi/9f^2. \tag{7}$$

There is no shift in the ρ^0 position, while the φ-meson shift is only $\sim\frac{1}{5}$ of that for the ω meson.

In order to estimate the magnitude of W_i we turn to the electromagnetic decay $\omega\to 2\pi$, for which we assume the same type of main diagram, i.e., $\omega\to\gamma\to\rho\to 2\pi$. Writing $f_{\rho\pi\pi}\epsilon_\mu^{(\rho)}(\rho_1-\rho_2)^\mu$ at the $\rho\to 2\pi$ vertex and using again (4) and (5) we obtain for the 2π partial decay width of ω

$$\Gamma_\omega(2\pi)=\frac{f_{\rho\pi\pi}^2}{4\pi}\left(\frac{\alpha}{f^2/4\pi}\right)^2\frac{1}{108}\frac{(m_\omega^2-4m_\pi^2)^{3/2}}{m_\omega^2}$$

$$\times\frac{m_\rho^4}{(m_\omega^2-m_\rho^2+\frac{1}{4}\Gamma_\rho^2)^2+m_\rho^2\Gamma_\rho^2},\qquad(8)$$

where we have used for the ρ a propagator with complex mass $m_\rho^*=m_\rho-\frac{1}{2}i\Gamma_\rho$.

The 2π decay of ω has been recently measured by Walker et al.,[7] who give $\Gamma_\omega(2\pi)/\Gamma_\omega(3\pi)=1.8_{-0.6}^{+1.2}\%$. Combining this result with $\Gamma_\omega(3\pi)\simeq 8.5$ MeV,[8] one has $\Gamma_\omega(2\pi)=150_{-50}^{+105}$ keV. We have used here $m_\rho=754$ MeV, $m_\omega=783$ MeV, and $\Gamma_\rho=100$ MeV. From this partial width for the 2π decay one obtains $(f^2/4\pi)\simeq 0.41$ which gives

$$W_2=7.5_{-1.4}^{+2.1}\text{ MeV}.\qquad(9)$$

The self-energy of φ turns out to be, from this calculation, approximately 2 MeV.

It is desirable at this point to have an estimate for other contributions to the self-energies. The next "lightest" possible intermediate state is $\pi\gamma$. The knowledge of the ratio $(\omega\to\pi+\gamma)/(\omega\to 3\pi)$,[9] together with the absolute value for the $\omega\to 3\pi$ partial width, makes such an estimate possible. We use for the vertex $(\omega\pi\gamma)$ the gauge-invariant expression

$$(f_{\omega\pi\gamma}/m_\pi)\epsilon_{\mu\nu\sigma\tau}\epsilon_\mu^{(\gamma)}k_\nu^{(\gamma)}\epsilon_\sigma^{(\omega)}k_\tau^{(\omega)},$$

without any form factor dependence for the off-the-mass-shell values. Then we have

$$\Gamma_\omega(\pi\gamma)=\frac{f_{\omega\pi\gamma}^2}{4\pi}\frac{(m_\omega^2-m_\pi^2)^3}{24m_\pi^2m_\omega^3}\simeq(1\text{ MeV})_{\text{exp}},$$

$$f_{\omega\pi\gamma}^2/4\pi\simeq 1.0\times10^{-3}.\qquad(10)$$

The contribution to the self-energy of the diagram $\omega\to\pi+\gamma\to\omega$ is given by

$$\delta M_{22}^{(\pi\gamma)}(p)=\frac{2i}{(2\pi)^3E(p)}\frac{f_{\omega\pi\gamma}^2}{4\pi}\int\frac{d^4k\,p_\mu k_\nu p_\sigma k_\tau(g_{\mu\sigma}g_{\nu\tau}-g_{\mu\tau}g_{\nu\sigma})}{(k^2+i\epsilon)[(p-k)^2-m_\pi^2+i\epsilon]}.\qquad(11)$$

This expression is divergent, and to estimate it we use a regularization procedure, with a convergence factor[10]

$$C(k^2)=\int_0^\infty\frac{\lambda^4}{(k^2-\lambda^2)^2}G(\lambda)d\lambda$$

and

$$\int_0^\infty\lambda^2G(\lambda)d\lambda=0.\qquad(12)$$

The resulting expression is logarithmically dependent[11] on the cutoff λ^2, and is given, if we neglect m_π compared to m, by

$$\delta m_{22}^{(\pi\gamma)}=\frac{1}{16\pi}\frac{f_{\omega\pi\gamma}^2}{4\pi}\left(\frac{m}{m_\pi}\right)^2m\left(\ln\frac{\lambda^2}{m^2}+23/18\right).\qquad(13)$$

If we choose a cutoff $\lambda^2\sim 10m^2$ one arrives at $\delta m_{22}^{(\pi\gamma)}\simeq 1.7$ MeV. As the result is not critically dependent on the cutoff value, it probably gives a reasonable estimate for this contribution. Its magnitude is slightly larger than $\delta m_{22}^{(\gamma)}$ which was used in diagonalizing (2). Nevertheless, as on the basis of unitary symmetry and the analysis of the scarce experimental evidence we expect $f_{\rho\pi\gamma}<f_{\omega\pi\gamma}$,[12] the final contribution to W_2 will probably not increase the value given in Eq. (9) by more than a factor of 1.5-2.

On the basis of the numerical estimates presented, we are inclined to conclude that at least a significant amount of the ω-ρ^0 mass difference is due to electromagnetic effects. However, we should caution that our conclusions are based on the approximate validity[13] of Eqs. (4) and (5) and the experimental result of reference 7.

A by-product of our calculation is the rate for the 2π decay of the φ meson. By using the value $(f^2/4\pi)\simeq 0.4$ and replacing the appropriate factors in (8), using again (5), one obtains $\Gamma_\varphi(\pi^+\pi^-)\simeq 16$ keV. With the experimentally measured width[8] to $K\bar{K}$ of 3.1 MeV, one has

$$(\varphi\to\pi^++\pi^-)/(\varphi\to K+\bar{K})\simeq 0.5\%.$$

Also, by using our value for f we can estimate the partial decay widths of ω and φ into lepton pairs. Taking the phase-space expressions from

Gell-Mann, Sharp, and Wagner,[14] we obtain

$$\Gamma_\omega(l^+l^-)=3.8\text{ keV and }\Gamma_\varphi(l^+l^-)=9.8\text{ keV}.$$

This gives

$$[\omega\to(e^++e^-\text{ or }\mu^++\mu^-)]/(\omega\to\pi^++\pi^-+\pi^0)\simeq0.05\%,$$

while

$$[\varphi\to(e^++e^-\text{ or }\mu^++\mu^-)]/(\varphi\to K+\bar{K})\simeq0.3\%.$$

This should make the detection of the lepton pairs more feasible in the φ decays. In any case, the measurement of the leptonic decays of the neutral vector mesons would allow a much safer estimate for the f_v's, as was already pointed out in reference 14.

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[1]J. Schwinger, Phys. Rev. Letters 12, 237 (1964).

[2]F. Gürsey and T. D. Lee, Phys. Rev. (to be published).

[3]S. L. Glashow, Phys. Rev. Letters 7, 469 (1961); J. C. Taylor, Phys. Rev. Letters 8, 219 (1962); J. Bernstein and G. Feinberg, Nuovo Cimento 25, 1343 (1962); J. Harte and R. G. Sachs, Phys. Rev. (to be published). I wish to thank Professor W. Wada for a stimulating conversation on this subject.

[4]For a detailed discussion of the photon-neutral vector meson interaction see G. Feldman and P. T. Mathews, Phys. Rev. 132, 823 (1963).

[5]Y. Ne'eman, Nucl. Phys. 26, 222 (1961); M. Gell-Mann, Phys. Rev. 125, 1067 (1962).

[6]These wave functions are practically equivalent to those obtained by using a mixing angle determined from the mass formula and the experimental masses. See, e.g., J. J. Sakurai, Phys. Rev. 132, 434 (1963).

[7]W. D. Walker et al., Phys. Letters 8, 208 (1964).

[8]N. Gelfand et al., Phys. Rev. Letters 11, 436,438 (1963).

[9]This ratio is given by different experiments as 7-18%. See reference 8 for a detailed list of references.

[10]R. P. Feynman, Phys. Rev. 76, 769 (1949).

[11]The quadratic term is eliminated by the requirement of gauge invariance and conservation of the vector meson current.

[12]See, for example, P. Singer, Phys. Rev. 130, 2441 (1963).

[13]It has been pointed out (M. Nauenberg, private communication) that if the fundamental objects of SU(3) are triplets (reference 2, for example), the electromagnetic current could have also a scalar part. This would alter the relations given in Eq. (4). For several simple assignments for the charges of these objects we found that our numerical estimates are practically not changed. I am grateful to Professor Nauenberg for valuable discussions on this question.

[14]M. Gell-Mann, D. Sharp, and W. G. Wagner, Phys. Rev. Letters 8, 261 (1962).