first two the $\pi\pi$ or $K\bar{K}$ must still combine with isotopic spin 1 and even G-parity. Again we can invoke the dominance of the ρ for these states and argue that their contribution should be unimportant below the $\rho\eta$ threshold (1410 MeV). The third state, $K\bar{K}\pi$ is a three-body state whose threshold is about $\omega = 2.0\ \mu_\pi$; phase space should keep this contribution small in the region we are considering. The empirical observation[1] that the resonance does not occur in 4π states other than those with the $\pi\omega$ configuration completes the argument for treating the reaction as purely elastic. It should be emphasized that the qualitative results would not be greatly changed if a small amount of inelasticity is present: In fact, as indicated above, the agreement would probably improve.

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NUCLEON-NUCLEON SCATTERING AND THE MESON RESONANCES*

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If the nucleon-nucleon scattering amplitude has a Mandelstam representation,[1] then the partial-wave amplitudes have certain analyticity properties. Application of the Cauchy theorem then leads to dispersion relations.[2] In the Cini-Fubini approximation when the left-hand discontinuity is assumed to be equal to some known function, $\mathrm{Im}f^{(B)}$, these take the form of Eq. (1). (Since we shall deal only with singlet amplitudes we omit the deuteron pole.)

$$f_J(p^2) = \frac{1}{\pi}\int_0^\infty \frac{\mathrm{Im}f_J(\nu')d\nu'}{\nu' - p^2} + \frac{1}{\pi}\int_{-\infty}^{-\frac{1}{4}m_\pi^2} \frac{\mathrm{Im}f_J^{(B)}(\nu')d\nu'}{\nu' - p^2}; \quad (1)$$

$\vec{p}$ = center-of-mass momentum. We have denoted all the quantum numbers by a single symbol, J. The "driving term" $\mathrm{Im}f_J^{(B)}$ is to be computed from a dynamical model. The model we use is to neglect all but lowest order terms corresponding to the exchange of a π meson and the η, ρ, ω, and f resonances.[3,4]

Although this specifies $f_J^{(B)}$ completely, the resulting integral equation cannot, as it stands, be solved for the physical partial-wave amplitude, f_J. Even the assumption of elastic unitarity, i.e., $\mathrm{Im}f_J(p^2) = |f_J(p^2)|^2$, which reduces Eq. (1) to an N/D problem, leads to divergent integrals because of the vector and tensor resonances.

The purpose of efforts[4] to solve Eq. (1) is twofold: (i) to determine whether the resonance approximation, analyticity, and unitarity can together provide a suitable computational scheme, and (ii) if so, to determine the appropriate values of the resonance-nucleon coupling constants. Thus, at present, it is not as important to solve Eq. (1) as to compare its solutions with experiment.

This comparison can be accomplished by the following indirect procedure.[5] We first replace $f_J(p^2)$ by $f_J(p^2)/p^{2j}$, where j is the total angular momentum of the state with quantum numbers J. The dispersion relation (1) is still valid,[2] and the second integral can be evaluated immediately by Cauchy's theorem. Taking the real part of the resulting equation, we obtain

$$f_J^{(B)}(p^2) = \mathrm{Re}f_J(p^2) - p^{2j}\frac{\mathrm{P}}{\pi}\int_0^\infty \frac{\mathrm{Im}f_J(\nu')d\nu'}{(\nu')^j(\nu' - p^2)} \equiv \frac{1}{M}h_J(p^2). \quad (2)$$

The j-fold subtraction insures the validity of this equation at $p^2 = 0$, and the nucleon mass M is introduced to make the functions h_J dimensionless.

For energies below $T_{\mathrm{lab}} = 400$ MeV the first term of h_J is known from experiment.[6] The sec-

ond term, which we call the reactive correction, is, in principle, also determined by experiment, but, in practice, the integrand must be approximated for energies greater than 400 MeV. Following Hamilton and collaborators[5] we assume that for energies greater than 2 BeV inelastic processes are as strong as they can be—i.e., $e^{2i\delta}=0$. This implies, for uncoupled waves,

$$\mathrm{Im}f_J(p^2)=1/2p, \quad T_{\mathrm{lab}}\geq 2 \text{ BeV}. \tag{3}$$

In the region between 400 MeV and 2 BeV a smooth interpolating curve is used for $\mathrm{Im}f_J(p^2)/p^{2j}$. The resulting integrals have been evaluated numerically for the singlet amplitudes with $j\leq 4$ using the YLAM and YLAN3M phase shifts,[6] and the h functions are shown in Fig. 2.

The "driving terms," $f_J^{(B)}$, have also been computed for these partial waves and have the form

$$f_J^{(B)}(p^2)=\frac{1}{M}\left\{g_{\pi N}{}^2 G_J{}^1(p^2)+\sum_{\alpha=2}^{11} Q(\alpha)G_J{}^\alpha(p^2)\right\},$$

where the $Q(\alpha)$ are quadratic combinations of the coupling constants which are to be determined.

The functions $G_J{}^\alpha$ are determined from the approximate Feynman amplitude for the scattering process shown in Fig. 1:

$$F^{(B)}(p_1p_2\to p_1'p_2')$$

$$=\sum_{r=\pi,\eta,\rho,\omega,f}\frac{V(r;p_1p_1')\cdot V(r;p_2p_2')}{m_r{}^2-t}+\text{exchange}.$$

The masses used are (in MeV)

$$m_\pi=140, \quad m_\eta=548, \quad m_\rho=750,$$

$$m_\omega=788, \quad m_f=1250.$$

The dot $(\cdot)$ represents contraction over all indices; e.g.,

$$V(\rho;p_1p_1')\cdot V(\rho;p_2p_2')=V_{\alpha\mu}(\rho;p_1p_1')V_{\alpha\mu}(\rho;p_2p_2').$$

The vertices, V, are related to the coupling constants as follows:

$$V_\alpha(\pi;pp')=ig_{\pi N}\bar{u}(p')\gamma_5\tau_\alpha u(p),$$

$$V(\eta;pp')=ig_{\eta N}\bar{u}(p')\gamma_5 u(p),$$

$$V_\mu(\omega;pp')=i\bar{u}(p')\{g_1{}^\omega\gamma_\mu+(g_2{}^\omega/4M)\sigma_{\mu\nu}k_\nu\}u(p),$$

$$V_{\alpha\mu}(\rho;pp')=i\bar{u}(p')\{g_1{}^\rho\gamma_\mu+(g_2{}^\rho/4M)\sigma_{\mu\nu}k_\nu\}\tau_\alpha u(p),$$

$$V_{\mu\nu}(f;pp')=\bar{u}(p')\{(g_1{}^f/M^2)[P_\mu P_\nu-\tfrac{1}{3}P^2I_{\mu\nu}]$$

$$+i(g_2{}^f/M)[\gamma_\mu P_\nu+\gamma_\nu P_\mu-\tfrac{2}{3}\gamma\cdot PI_{\mu\nu}]\}u(p),$$

$$k=p-p', \quad P=p+p', \quad a\cdot b=\vec{a}\cdot\vec{b}-a_0b_0,$$

$$I_{\mu\nu}=\delta_{\mu\nu}-k_\mu k_\nu/k^2.$$

Our normalization is such that $g_{\pi N}{}^2/4\pi=14.2$.

A gradient search was conducted to find values of the coupling constants which minimize the discrepancy between $f^{(B)}$ and h. The starting points were the corners of a hypercube of side 200 in the seven-dimensional parameter space. The following values give a minimum:

$$g_{\eta N}=33.9, \quad g_1{}^\omega=-0.23, \quad g_2{}^\omega=2.26,$$

$$g_1{}^\rho=0.95; \quad g_2{}^\rho=34.7, \quad g_1{}^f=2.39,$$

$$g_2{}^f=-2.31.$$

The quality of the agreement with experiment is shown in Fig. 2.

The use of the reactive correction, rather than attempting to solve Eq. (1), is fully consistent with the requirement of unitarity and analyticity and has three advantages: (i) The coupled partial waves present no more difficulty than the uncoupled ones; (ii) the physical significance of the approximations made is readily understood, and they are subject to experimental study; (iii) the difficulties and ambiguities of an N/D calculation are avoided.

The fits shown in Fig. 2 suggest that this approach to the nucleon-nucleon interaction, without auxiliary assumptions, is a promising one, and the work is now being extended to the triplet amplitudes. We wish to emphasize that the validity of the h function is in no way dependent upon the resonance approximation, and that they permit us to extend our study to the low partial waves, since unitarity is taken into account. Detailed tables will be published elsewhere, together with estimates of their expected error, which will permit the use of statistical measures of

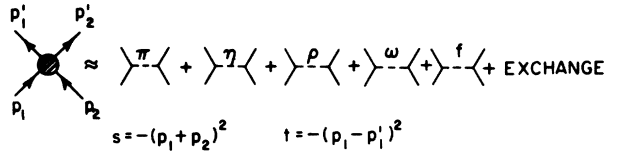

FIG. 1. The approximate scattering amplitude $F^{(B)}(p_1p_2\to p_1'p_2')$.

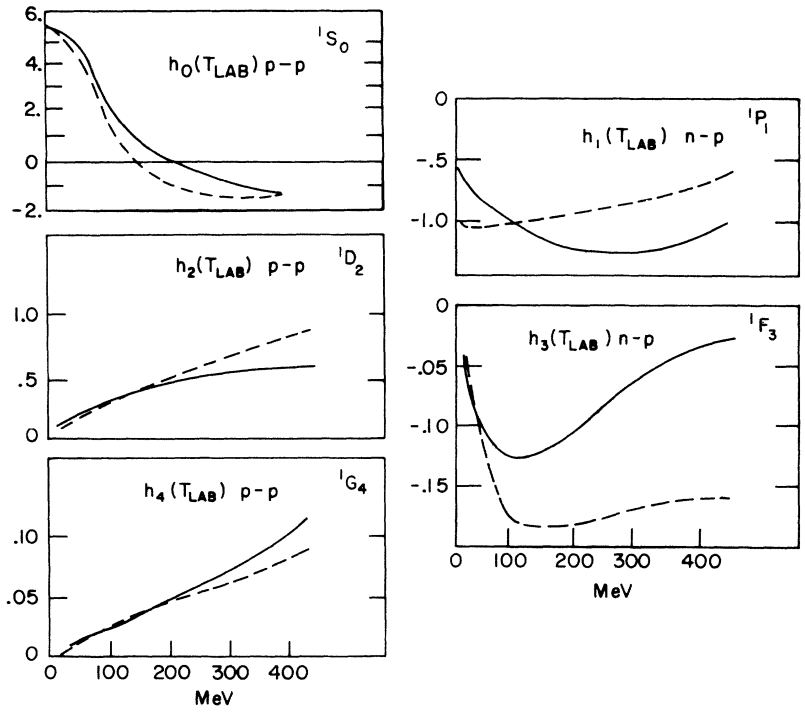

FIG. 2. Comparison between the functions h_J (solid line) determined from experimental data, and the functions $f_J^{(B)}$ (dashed line) calculated using the coupling constants listed in the text. In view of the uncertainties in the n-p phase shifts, h_1 and h_3 are not very reliable, the uncertainty being comparable to the discrepancies observed here.

"goodness of fit."

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