PION-PION INTERACTION AND THE K_{e4} DECAY*

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Although the K_{e4} mode of K-meson decay is rather rare, its study can lead to valuable information of several kinds. In addition to testing the selection rules of the weak interactions (such as $\Delta S = \Delta Q$) and the structure and strength of the weak current of the strongly interacting particles, it provides an almost unique opportunity to study the pion-pion interaction at a low relative kinetic energy without the presence of other strongly interacting particles.

A number of authors have studied the K_{e4} process

$$K^+ \to \pi^+ + \pi^- + e^+ + \nu \tag{1}$$

and

$$K^+ \to \pi^0 + \pi^0 + e^+ + \nu, \tag{2}$$

some neglecting[1-4] and some including[5-7] final-state interaction of the two pions. The partial decay rate predicted for process (1) by all previous authors has been considerably larger than the recently measured experimental rate[8]; this, in itself, justifies a new attack on this problem. In addition, a study of the three-pion decay modes of η and K mesons has led Brown and Singer[9] to postulate the existence of a scalar, isospin-zero resonance (the "σ meson"), of mass m_σ about 400 MeV and width Γ_σ about 75 MeV. We shall assume that K_{e4} decay is mediated by this resonance and show that a single dimensionless weak axial vector coupling constant g_A determines the rate of both K_{e4} and $K_{\mu 2}$ decays, predicting a ratio of the rates which is in accord with experiment.

Since $m_\sigma < m_K$, a real σ could appear in the decays (1) and (2) and would constitute an appreciable final-state interaction, significantly affecting the decay rates and the momentum and angular distributions. This is the more likely because the low Q value of the decay favors the low angular momentum states. Especially for the decay (2), where p-wave pion-pion interaction is forbidden, we may expect σ dominance. We assume σ dominance also in the process (1), although here p-wave interference may affect certain distributions.

We, therefore, make the following assumptions:

(a) The lepton current is taken as $\bar{e}\gamma_\lambda(1+\gamma_5)\nu$.

(b) The current of the strongly interacting particles is defined by

$$\langle\sigma|A_\lambda|K\rangle = G(P_1+P_2)_\lambda, \tag{3}$$

where P_1 and P_2 are the pion four-momenta and G represents the K-σ-lepton vertex and has dimensions of m^{-2}. Included in G is the coupling of the lepton current to the vertex.

(c) The $\sigma \to \pi^+ + \pi^-$ coupling is related to the assumed mass m_σ and width Γ_σ of the σ meson by

$$g^2 = 32\pi m_\sigma^{\ 2}\Gamma_\sigma/3(m_\sigma^{\ 2} - 4m_\pi^{\ 2})^{1/2}. \tag{4}$$

Detailed discussion of this quantity is given in reference 9.

(d) The σ-dominant processes are assumed to take place as illustrated in Fig. 1; that is, the vertices are assumed to have the minimum structure permitted by the conservation laws. The electron mass is everywhere neglected.

In the following calculations, for various m_σ and Γ_σ, we obtain (1) the decay rates for processes (1) and (2), (2) the $K_{e4}/K_{\mu 2}$ branching ratio, (3) the invariant mass spectrum of the two pions, (4) the electron and pion momentum spectra, and (5) the pion-pion opening angle distribution.

A. Matrix element. – Using the assumptions discussed above, the invariant matrix element for process (1) becomes

$$M = Gg(P_1+P_2)_\lambda \bar{e}\gamma_\lambda(1+\gamma_5)\nu\left[s^2 - \left(m_\sigma - \frac{i\Gamma_\sigma}{2}\right)^2\right]^{-1} \tag{5}$$

where we have associated with the σ meson a

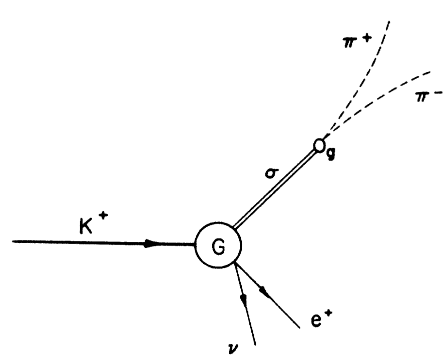

FIG. 1. Feynman diagram for the process $K^+ \to \pi^+ + \pi^- + e^+ + \nu$.

propagator $[s^2-(m_\sigma-i\Gamma_\sigma/2)^2]^{-1}$, $(s^2)^{1/2}$ being the $\pi-\pi$ invariant mass. The matrix element for process (2) will be given by $M/\sqrt{2}$. The results which follow are all for process (1) and are given in the rest frame of the K meson.

From (5) we obtain for the decay probability

$$dw=|Gg|^2 4m_K(E_eE_\nu+\vec{P}_e\cdot\vec{P}_\nu)$$
$$\times[(s^2-m_\sigma^2)^2+m_\sigma^2\Gamma_\sigma^2]^{-1}$$
$$\times\{\delta^{(4)}(P_K-P_1-P_2-P_e-P_\nu)(16\omega_1\omega_2E_eE_\nu)^{-1}$$
$$\times d^3P_1d^3P_2d^3P_ed^3P_\nu/(2\pi)^{12}\}, \tag{6}$$

the expression in the brackets being the invariant phase space, ρ_{inv}.

From (6) we can proceed to the distributions of interest. However, it is desirable to compare them with corresponding spectra when no final-state interaction is assumed. We obtain the latter from Eq. (6) by the replacement

$$|Gg|^2[(s^2-m_\sigma^2)^2+m_\sigma^2\Gamma_\sigma^2]^{-1}\to|C|^2 \tag{7}$$

where C has dimensions m^{-3} and will be considered the effective coupling.

B. Effective mass spectrum.—Starting with (6) and integrating out all lepton variables, we are left with

$$dw/ds=[|Gg|^2/12(2\pi)^5m_K^{\;3}2^5]s(1-4m_\pi^{\;2}/s^2)^{1/2}$$
$$\times[(s^2-m_\sigma^2)^2+m_\sigma^2\Gamma_\sigma^2]^{-1}$$
$$\times\{(m_K^{\;4}-s^4)(m_K^{\;4}+s^4-8m_K^{\;2}s^2)$$
$$-24m_K^{\;4}s^4\ln(s/m_K)\}. \tag{8}$$

The spectrum for the no-interaction case is then obtained by making use of (7). Expression (8) is plotted as a function of s for a number of values[10] of m_σ and Γ_σ and compared with the no-interaction case in Fig. 2. We see that the spectrum is sensitive to the position and width of the σ resonance. It differs from the effect of a "scattering length" interaction such as that considered by Ciochetti[5] which shifts the peak of the spectrum toward the pion-pion threshold. Intuitively, it is clear that if an appreciable "no-interaction" S-wave background is present in addition to the σ-meson effect we will get, at least, considerable broadening of the $\pi-\pi$ effective-mass spectrum.

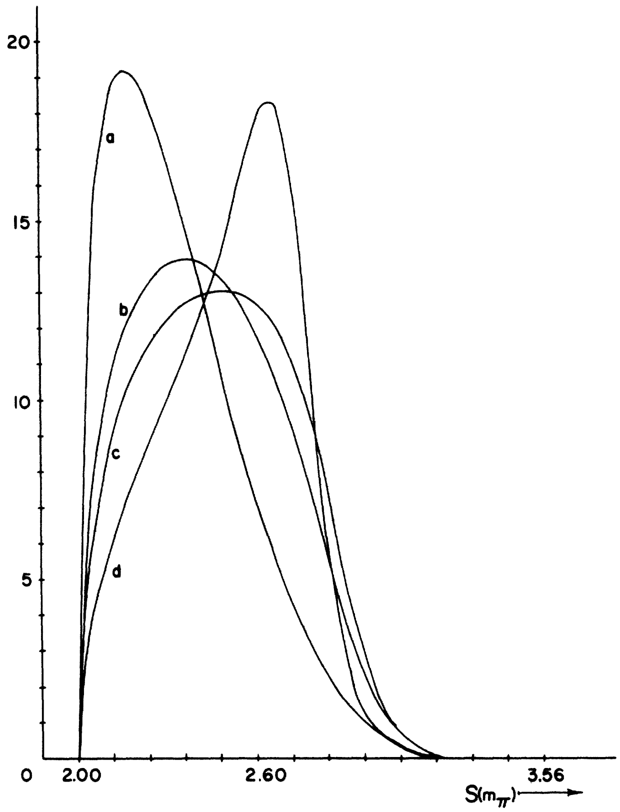

FIG. 2. Spectrum of the effective mass s of the two pions in the K_{e4} final state. Curve labels denote the following cases: (a) no final-state interaction; (b) $m_\sigma=400$ MeV, $\Gamma_\sigma=100$ MeV; (c) $m_\sigma=400$ MeV, $\Gamma_\sigma=75$ MeV; (d) $m_\sigma=381$ MeV, $\Gamma_\sigma=48$ MeV.

Effects of a relative p wave in the pion-pion system will contribute only incoherently to its invariant mass spectrum. This is also the case for the electron momentum spectrum which is shown in Fig. 3(A). On the other hand, the pion momentum spectrum given by our model [Fig. 3(B)] will be affected by the p-wave interference term that we have neglected, and should be accepted with that reservation. While this caution applies also for the pion-pion angular correlation given in Fig. 4, the striking effects of including the σ resonance (which are understandable from simple kinematical considerations) should remain essentially unaltered.

C. Decay Rate.—Writing Eq. (8) in the form

$$dw=m_KG^2g^2f(s)ds \tag{9}$$

and integrating, we obtain a rate $\Gamma_{e4}(\sigma)$, where σ refers to the mass and width assumed for the σ meson. We shall compare this rate to that of

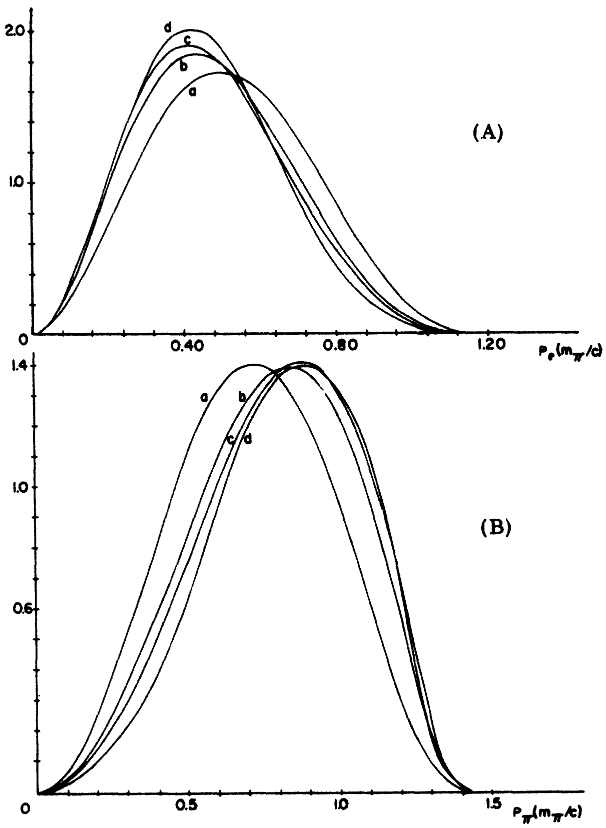

FIG. 3. (A) Electron momentum spectrum and (B) pion momentum spectrum. Curve labels denote the following cases: (a) no final-state interaction; (b) $m_\sigma = 400$ MeV, $\Gamma_\sigma = 100$ MeV; (c) $m_\sigma = 400$ MeV, $\Gamma_\sigma = 75$ MeV; (d) $m_\sigma = 381$ MeV, $\Gamma_\sigma = 48$ MeV.

the decay $K \to \mu + \nu$,

$$\Gamma_{\mu 2} = (g_A/m_k)^2 (m_K m_\mu^2/4\pi)[1 - m_\mu^2/m_K^2]^2$$
$$= (g_A/m_K)^2 N, \quad (10)$$

and for this it is convenient to write

$$G^2 = (g_A'/m_K)^2 (m_K m_\sigma)^{-1} \quad (11)$$

in order to compare the dimensionless parameters g_A and g_A'. We shall discuss below the motivation for choosing the characteristic length $(m_K m_\sigma)^{-1/2}$. We have then

$$\Gamma_{e4}(\sigma) = (g_A'/m_K)^2 N'(\sigma), \quad (12)$$

with

$$N'(\sigma) = (g^2/m_\sigma)\int_2^{3.45} f(s)ds. \quad (13)$$

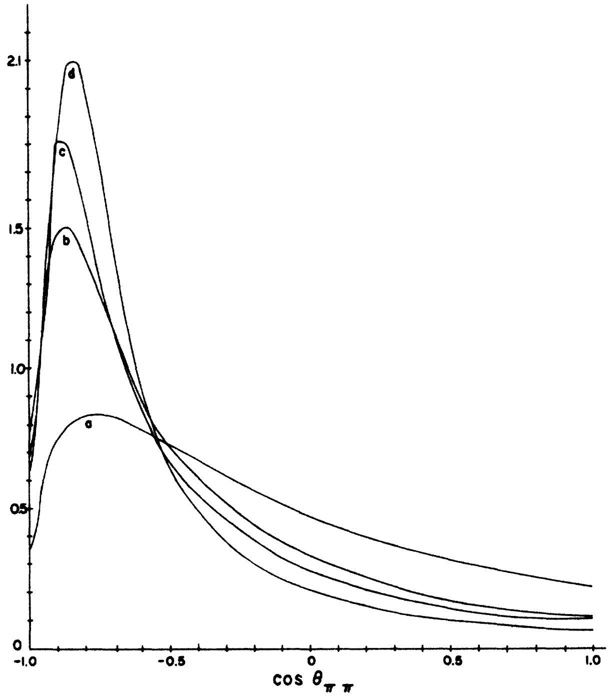

FIG. 4. Distribution of the angle between the pions in the K-meson rest frame. Curve labels denote the following cases: (a) no final-state interaction; (b) $m_\sigma = 400$ MeV, $\Gamma_\sigma = 100$ MeV; (c) $m_\sigma = 400$ MeV, $\Gamma_\sigma = 75$ MeV; (d) $m_\sigma = 381$ MeV, $\Gamma_\sigma = 48$ MeV.

The ratio of the rates is then

$$\Gamma_{e4}(\sigma)/\Gamma_{\mu 2} = (g_A'/g_A)^2 N'(\sigma)/N \quad (14)$$

and the values of $N'(\sigma)/N$ are given in Table I. It is very interesting to note that the experimental ratio[8]

$$\Gamma_{e4}/\Gamma_{\mu 2} = (4 \pm 1) \times 10^{-5} \quad (15)$$

corresponds to the choice $g_A' \approx g_A$.

We discuss now the significance of this result, which depended, of course, on our choice of the length $(m_K m_\sigma)^{-1/2}$. Consider the matrix elements characteristic of the two decays being compared, namely $\langle 0|A_\lambda|K\rangle$ for the $K_{\mu 2}$ and $\langle \sigma|A_\lambda|K\rangle$ for the

Table I. Values of the ratio $N'(\sigma)/N$.

m_σ \ Γ_σ	100 MeV	75 Mev	50 Mev
425 MeV	1.35×10^{-5}	1.21×10^{-5}	1.03×10^{-5}
400 MeV	2.275×10^{-5}	2.25×10^{-5}	2.11×10^{-5}
375 MeV	3.57×10^{-5}	4.07×10^{-5}	4.24×10^{-5}

K_{e4}, coupled respectively to the leptons. Since the dimensions of these matrix elements differ by a factor of length, we have associated the effective weak coupling constant g_A with the former and $(m_K m_\sigma)^{-1/2} g_A'$ with the latter, g_A and g_A' being dimensionless and $(m_K m_\sigma)^{-1/2}$ being regarded as a characteristic length. We find this choice intuitively appealing since the vacuum and σ meson have the same quantum numbers. Since the masses of the K and σ mesons are so close, our results will not be appreciably altered if any other reasonable alternative is adopted for this characteristic length.

It would appear, then, that the two predominantly axial vector leptonic decay modes of the K mesons, namely $K \to l + \nu$ and $K \to \pi + \pi + l + \nu$, are both describable in terms of a single dimensionless effective coupling constant $g_A \approx 1.5 \times 10^{-7}$, providing that the K_{e4} final state is dominated by the σ meson.

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COMPILATION OF RESULTS ON THE TWO-PION DECAY OF THE ω †

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The decay $\omega^0 \to \pi^+ + \pi^-$ has been the subject of some theoretical discussion.[1-3] The interest derives from the fact that although it necessarily violates isotopic spin conservation, the partial rate may nevertheless be appreciable because phase space and angular momentum barriers favor it over the normal three-pion decay. The rate for two-pion ω decay has been estimated by Sakurai[3] to be of the order

$$\Gamma_{\pi\pi} \simeq \frac{10 \text{ keV}}{f_\omega^{\,2}/4\pi}$$

where f_ω is the coupling of the ω to the nucleon. With $f_\omega^{\,2}/4\pi \sim 1$ the fraction of two-pion ω decay which might be expected is

$$R = \Gamma_{\pi\pi}/\Gamma_\omega \simeq 10^{-3},$$

using the recently reported measurement of the ω width.[4] This estimate can probably not be expected to be more than a rough guilde.

A number of experiments bearing on R have been reported, with divergent results. Those known to us are given in Table I.

In these experiments, the reactions producing the ω^0 are in general different, but the results are in every case based on the invariant mass distribution of a $\pi^+\pi^-$ pair. The following analysis therefore applies to all experiments listed in Table I.

The mass dependence of the amplitude for the production of the $\pi^+\pi^-$ pair in these reactions may be written

$$M(m, q_1 \cdots q_n) = A(q_1 \cdots q_n) + \frac{B(q_1 \cdots q_n)}{(m - m_\omega) + \frac{1}{2} i \Gamma_\omega}, \quad (1)$$

where A is the amplitude for $\pi^+\pi^-$ production not resonating as ω, and is assumed independent of