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EFFECT OF STRONG ABSORPTION IN PARTICLE EXCHANGE REACTIONS

A. Dar and W. Tobocman*

Department of Nuclear Physics, The Weizmann Institute of Science, Rehovoth, Israel
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It has been suggested[1] that certain high-energy reactions are dominated by the one-particle exchange mechanism. This type of mechanism has also been applied[2] successfully to the so-called "direct nuclear reactions." In the case of direct nuclear reactions the exchanged particle is a nucleon or a cluster of nucleons, while for high-energy reactions it may be an elementary particle or a "Regge pole."[3] For both high-energy and direct nuclear reactions it appears that the distortion effects due to interactions in the incident and outgoing channels lead to a reduction in the contribution to the reaction of the partial waves of low angular momentum.

It has been suggested[4] that a dispersion-theoretic analysis can be used to describe the effect of elastic distortion on the one-particle exchange amplitude. Such an approach uses the unitarity and time-reversal symmetry of the S matrix together with certain assumptions about the analytic behavior of the transition amplitude. In this analysis it is customary to neglect certain terms in the unitarity relation. In this note we wish to point out that the neglect of these terms leads directly to very interesting consequences.

If $T_{\beta\alpha}$ is the scattering amplitude for the reaction $\alpha \to \beta$ where α and β represent equal- (positive-) energy channels, then unitarity and symmetry of the S matrix imply

$$\mathrm{Im}T_{\beta\alpha} = \tfrac{1}{2}(2\pi)^4\{\textstyle\sum_{\beta'}T_{\beta\beta'}{}^{\dagger}T_{\beta'\alpha} + \sum_{\alpha'}T_{\beta\alpha'}{}^{\dagger}T_{\alpha'\alpha} + \sum_{\nu}T_{\beta\nu}{}^{\dagger}\cdot T_{\nu\alpha}\}\delta^4(P_\alpha - P_\beta), \qquad (1)$$

where $T_{\beta\beta'}$ is the amplitude for elastic scattering. It is customary to assume that the third sum on the right of Eq. (1), which comprises all intermediate states ν which cannot be reached by elastic scattering in the initial or final channel, can be neglected. The justification of this step rests on the observation that since the nonelastic amplitudes are small compared to the elastic amplitudes, the terms in the third sum on the right of Eq. (1) are small compared to the terms in the first two sums. The fact that the various terms of these sums have different phases makes it unlikely that the sum will be of a different order of magnitude than its individual terms. Then rewriting Eq. (1) in the angular momentum representation gives

$$\mathrm{Im}T_l^{(\beta\alpha)} = u_l^{(\alpha)}T_l^{(\beta\alpha)*} + u_l^{(\beta)*}T_l^{(\beta\alpha)}, \qquad (2)$$

where

$$u_l^{(\alpha)} = \exp(i\delta_l^{\alpha})\sin\delta_l^{\alpha} \qquad (3)$$

and δ_l is the elastic phase shift.

We note that Eq. (2) is, in fact, two linear homogeneous algebraic equations for the real and imaginary parts of $T_l^{(\beta\alpha)}$. Thus we have three possibilities: (a) $T_l^{(\alpha\beta)} = 0$, (b) $u_l^{(\alpha)} = u_l^{(\beta)} = 0$ and $T_l^{(\alpha\beta)}$ is real, or (c) the determinant of the system of equations must vanish. The third alternative results in the following requirement:

$$|u_l^{(\alpha)}|^2 - \mathrm{Im}u_l^{(\alpha)} = |u_l^{(\beta)}|^2 - \mathrm{Im}u_l^{(\beta)}. \qquad (4)$$

This is equivalent to requiring that the lth partial-wave contribution to the reaction cross section be equal for the two channels.

For the high partial waves we will have $u_l=0$ and then we can take the transition amplitudes to be real. For the low partial waves we must either set the transition amplitudes equal to zero or require that the reaction cross section for the incident channel equal the reaction cross section for the exit channel.

We propose the first alternative as a basis for treating distortion effects in particle exchange reactions. That is, we claim that it is very plausible that when elastic distortion is present, the transition amplitude vanishes for low partial waves and is real for high partial waves. We will show that this model seems particularly appropriate when there is strong absorption, and we will show how this model leads to simple expressions for the transition amplitude.

When there is complete absorption in both the incident and exit channel, we have

$$u_l^{(\alpha)} = u_l^{(\beta)} = \tfrac{1}{2}i. \qquad (5)$$

When this is substituted into Eq. (2), it becomes an identity for all $T_l^{(\beta\alpha)}$. This implies that

$$\sum_{\nu\neq\beta,\alpha} (T_{\beta\nu}^{\dagger} T_{\nu\alpha})_l = 0. \qquad (6)$$

Thus in the case of complete absorption the "third-channel" contributions to the unitarity condition must vanish. This makes it plausible that when there is strong (but not complete) absorption the "third-channel" contributions will be small. According to the strong absorption model suggested by Blair,[5]

$$u_l^{(\alpha)} = \tfrac{1}{2}i, \quad l \leq kR,$$
$$= 0, \quad l > kR, \qquad (7)$$

where k is the three-momentum in the c.m. frame and R is an adjustable parameter. The elastic amplitude then becomes,[6] for $kR \gg 1$,

$$T_{\alpha\alpha'} = \frac{1}{k_\alpha}\sum(2l+1)P_l(\cos\theta)u_l^{(\alpha)}$$
$$\sim ik_\alpha R_\alpha^2 \frac{J_1(R_\alpha\sqrt{-t})}{R_\alpha\sqrt{-t}}, \qquad (8)$$

where θ is the scattering angle and $-t$ is the square of the four-momentum transfer.

In treating high-energy particle exchange reactions, one approach has been to neglect distortion effects and calculate the transition amplitude by first-order perturbation theory using plane waves. Our analysis suggests that strong absorption distortion effects may be included in such calculations by simply dropping the low angular momentum partial-wave contributions from the usual first-order result. The prescription is reminiscent of the Butler cutoff prescription[7] used for direct nuclear reactions. However, in our case the cutoff is made in angular momentum space while the Butler cutoff is carried out in configuration space.

It is of interest to note that the behavior of the partial-wave transition amplitude that appears to be appropriate in the presence of strong absorption does not satisfy the usual dispersion relation[8] for one-particle reaction amplitudes. This dispersion relation reads

$$\mathrm{Re}T_l(s) = T_l^{(0)}(s) + \frac{\mathrm{P}}{\pi}\int_{s_0}^{\infty} ds' \frac{\mathrm{Im}T_l(s')}{s'-s}. \qquad (9)$$

On the right of the above equation appear contributions from a pole and a right-hand cut. Contributions from other possible singularities are neglected. For high l, $T_l(s)$ must be real according to our prescription. Eq. (9) then requires $T_l = T_l^{(0)}$ for high l. This is in agreement with our prescription since $T^{(0)}$, the pole contribution, is real and, in fact, is usually taken to be the Born approximation amplitude. For low l, $T_l(s)$ must vanish according to our prescription. Since $T_l^{(0)}$ does not vanish, Eq. (9) cannot be satisfied. We conclude that when there is strong absorption the analytic behavior of the one-particle exchange amplitude is not well represented by a pole and a cut on the real axis of the square of the total c.m. energy.

To illustrate our suggestion we consider the amplitude for exchange of a scalar meson of mass $\sqrt{t'}$ with low Q value:

$$A = \frac{1}{t'-t} \approx t' + 2s(1-\cos\theta)$$
$$= \frac{1}{2s}\sum_{l=0}^{\infty}(2l+1)Q_l\left(1+\frac{t'}{2s}\right)P_l(\cos\theta), \qquad (10)$$

where Q_l is the Legendre function of the second kind. To include the effects of strong absorption according to our prescription, one must drop the first kR terms of the sum. For $kR \gg 1$ we can replace Q_l and P_l by their asymptotic forms in terms of Bessel functions and approximate the sum over l by an integration over impact parameter:

$$A_{\text{S. A.}} \approx \int_R^\infty db\, bJ_0(b\sqrt{-t})K_0(b\sqrt{t'})$$

$$= \frac{R}{t'-t}\{t'^{1/2}J_0(R\sqrt{-t})K_1(R\sqrt{t'}) - (-t)^{1/2}J_1(R\sqrt{-t})K_0(R\sqrt{t'})\}. \quad (11)$$

We see that the result is an additional factor in the amplitude which will lead to a cross section that is more sharply peaked in the forward direction and has an oscillatory behavior at larger angles. In particular, for $t' \gg -t$,

$$A_{\text{S. A.}} \sim [K_1(R\sqrt{t'})/\sqrt{t'}]J_0(R\sqrt{-t}). \quad (12)$$

This type of behavior is observed in direct nuclear reactions[9] and in elementary particle reactions like $\bar{p}+p \to \bar{B}+B$.[10]

Formula (11) may be derived also by substituting into the unitarity relation Eq. (1) the Fourier-Bessel representation[11] of the scattering amplitudes:

$$T_{\beta\alpha}(s,t) = \int_0^\infty db\, bJ_0(b\sqrt{-t})H(s,b). \quad (13a)$$

The form of H follows from requiring the diffraction form for the elastic amplitude:

$$ikR^2\frac{J_1(R\sqrt{-t})}{R\sqrt{-t}} = ik\int_0^R db\, bJ_0(b\sqrt{-t}). \quad (13b)$$

In place of Eq. (10) one uses

$$\frac{1}{t'-t} = \int_0^\infty db\, bJ_0(b\sqrt{-t})K_0(b\sqrt{t'}). \quad (13c)$$

The derivation involves the orthogonality relation

$$\frac{k}{4\pi}\int d\Omega_2 J_0(b_1\sqrt{-t_{12}})J_0(b_2\sqrt{-t_{23}}) \approx \frac{\delta(b_1-b_2)}{2kb_1}J_0(b_1\sqrt{-t_{13}}). \quad (14)$$

A somewhat more realistic implementation of the strong absorption effect might be to use a gradual rather than abrupt cutoff in angular momentum or impact parameter.

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*Fellow of the Alfred P. Sloan Foundation. Permanent address: Case Institute of Technology, Cleveland, Ohio.

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