the path length is several times longer, even around r_1. With the correction, we obtain $E_{rE}/E_{rS0} = (V_{rE}/V_{rS0})(r_2/r_1)$. Figure 3 shows this ratio plotted against μB with μ again taken as 700 cm^2/V-sec. In the range $\mu B > 1$ the points approach what should be expected from Eq. (1). Whether it is strictly legitimate to use the radii ratio correction to get the critical field ratio is a question that awaits further study with samples of different size. Nevertheless, we feel that the interpretation is at least qualitatively correct and that the results lend support to the theory of acoustic-wave interaction in electric and magnetic fields.

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POSSIBLE INTERPRETATION OF THE $\pi\omega$ RESONANCE AT 1220 MeV†

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A resonance has recently been reported[1] in the interaction of π and ω mesons, its total energy being 1220 MeV and width about 100 MeV. The purpose of the present note is to suggest that this can be interpreted as the direct analog for the $\pi\omega$ system of the 33 isobar observed in π-nucleon interactions and that its spin and parity should be 2^-.

Consider the reaction

$$\pi + \omega \rightarrow \pi + \omega. \tag{1}$$

Let us assume that the basic mechanism of Reaction (1) is a coupling between π, ρ, and ω mesons. The emission of a π meson of four-momentum q by an ω meson of four-momentum p, polarization μ together with a ρ meson of four-momentum $p' = p - q$, and polarization κ is described by a vertex

$$f\epsilon_{\lambda\mu\nu\kappa} p_\lambda p_\nu'. \tag{2}$$

This is the interaction assumed, for example, by Gell-Mann, Sharp, and Wagner[2] in discussing the ω decay: Using their Eqs. (1) and (2) and taking the recent value of 9 MeV[3] for the ω width and 100 MeV[4] for the ρ width, we find $(f^2/4\pi) = 0.45$ in units where $\hbar = c = \mu_\pi = 1$.

With this interaction model, the only single-particle singularities contributing to Reaction (1) are the direct and crossed single-ρ terms illustrated in Fig. 1. We shall argue below that we can neglect coupling to all inelastic channels; then these two terms together with elastic unitarity should determine Reaction (1) at low energies. Bearing in mind the fact that both ρ and ω mesons are 1^- states and that their masses are similar and large compared with the π meson mass, we see the dynamical similarity to low-energy π-nucleon scattering.

It is well known that for the latter case the static limit[5] gives a good qualitative understanding of the importance of the 33 state and an approximate connection between the position and width of the resonance. We shall try the same approximation here, though its validity is even more suspect. In the rest frame of the ω meson, the interaction (2) becomes $Mf\epsilon_{ijk}q_k$, where M is the mass of the ω meson and ij are the polarizations of ω and ρ mesons. The three

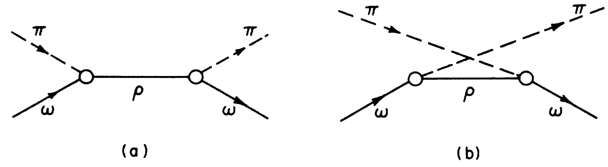

FIG. 1. Contributions to $\pi\omega$ scattering.

matrices $(S^k)_{ij} = -i\epsilon_{ijk}$ form a representation of the vector-meson spin operator and thus the interaction has the simple static form $(\vec{S}\cdot\vec{q})$. In the static limit only p states are allowed; using the index α to represent the three states 0^-, 1^-, and 2^-, respectively, we can easily construct projection operators and find the static limit of the Born approximation:

$$\left.\begin{array}{l} h_\alpha{}^B(\omega) = \lambda_\alpha/4\pi\omega \\ \lambda_\alpha = \frac{1}{4}(-2, -1, 1)f^2 \end{array}\right\}, \qquad (4)$$

where ω is the π-meson energy and h_α is the partial-wave amplitude $\exp(i\delta_\alpha)\sin\delta_\alpha/q^3$. The crossing matrix in the static limit is

$$A_{\alpha\beta} = \frac{1}{6}\begin{bmatrix} 2 & -6 & 10 \\ -2 & 3 & 5 \\ 2 & 3 & 1 \end{bmatrix}. \qquad (5)$$

It is evident that the 2^- state is the analog of the 33 state for πN scattering, being attractive in Born approximation and having attractive contributions from all three crossed states. These qualitative statements should not be too sensitive to the static approximation and we therefore expect a resonance in the 2^- state.

We cannot predict the position of the resonance, even granted the validity of the approximation. However, we can compare with the πN case. The value of λ_α for the 2^- state is 0.11, while the corresponding number for the 33 state is 0.12. This is consistent with the fact that the $\pi\omega$ resonance is somewhat higher ($\omega = 2.62\ \mu_\pi$) than the 33 resonance ($\omega = 2.0\ \mu_\pi$) but of a similar order of magnitude, though there is no reason for the cutoff to be exactly the same in the two cases. If we use an effective range formula for the phase shift

$$\cot\delta_\alpha = (\omega/\lambda_\alpha q^3)(1 - \omega/\omega_0), \qquad (6)$$

we can calculate the expected $\pi\omega$ cross section, choosing ω_0 to give a maximum at the correct energy.

The results are shown in Fig. 2. The width of the peak obtained is considerably greater than the width of the experimentally observed resonance.[1] However, the agreement seems quite adequate in terms of the model used, since the shape is sensitive to the coupling strength assumed. (The effect of reducing f^2 by a factor 2 is shown in Fig. 2.) The discrepancy is in the right direction, since the assumptions involved in the static approximation and those of pure elastic scattering all tend to exaggerate the effective strength of the Born term in the 2^- state. An order-of-magnitude increase in the assumed value of f^2 would make it impossible to obtain a resonance as high as 2.62 μ_π, while an order-of-magnitude decrease would correspond to an extremely narrow resonance and to an attraction in the 2^- state so weak that it would be unlikely to dominate the reaction. The consistency of the model is thus not completely trivial.

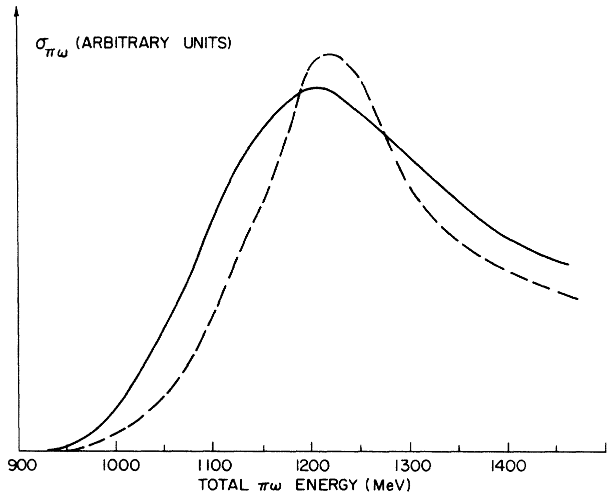

FIG. 2. Total cross section for elastic $\pi\omega$ scattering in the static limit, adjusted to give a maximum at 1220 MeV. Solid curve for $\lambda = 0.11$; dashed curve for $\lambda = 0.05$.

It remains to justify the rather serious assumption that Reaction (1) is mainly elastic. The $\pi\omega$ system has isotopic spin 1 and even G-parity. If we consider all s, p, and d states, the spin and parity assignments are 0^-, $1^\pm$, $2^\pm$, and 3^+. Up to the energy region we are considering the only two-body states with any of these assignments are the 1^- states of 2π and $K\bar{K}$. The most important contribution from this assignment is that due to the ρ meson, and this has already been taken into account in the present model. This approximation is used qualitatively in assessing the sign of the force in the various p states and quantitatively only in calculating the contribution of Fig. 1(b) to the 2^- state: Within the accuracy of the model this should be reasonable. There is no two-body channel coupled to the 2^- state of the $\pi\omega$ system. As far as three-body channels are concerned, the only ones are $\pi\pi\eta$, $K\bar{K}\eta$, and $K\bar{K}\pi$. In the first two the $\pi\pi$ or $K\bar{K}$ must still combine with isotopic spin 1 and even G-parity. Again we can invoke the dominance of the ρ for these states and argue that their contribution should be unimportant below the $\rho\eta$ threshold (1410 MeV). The third state, $K\bar{K}\pi$ is a three-body state whose threshold is about $\omega = 2.0\ \mu_\pi$; phase space should keep this contribution small in the region we are considering. The empirical observation[1] that the resonance does not occur in 4π states other than those with the $\pi\omega$ configuration completes the argument for treating the reaction as purely elastic. It should be emphasized that the qualitative results would not be greatly changed if a small amount of inelasticity is present: In fact, as indicated above, the agreement would probably improve.

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NUCLEON-NUCLEON SCATTERING AND THE MESON RESONANCES*

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If the nucleon-nucleon scattering amplitude has a Mandelstam representation,[1] then the partial-wave amplitudes have certain analyticity properties. Application of the Cauchy theorem then leads to dispersion relations.[2] In the Cini-Fubini approximation when the left-hand discontinuity is assumed to be equal to some known function, $\mathrm{Im}f^{(B)}$, these take the form of Eq. (1). (Since we shall deal only with singlet amplitudes we omit the deuteron pole.)

$$f_J(p^2) = \frac{1}{\pi}\int_0^\infty \frac{\mathrm{Im}f_J(\nu')d\nu'}{\nu'-p^2} + \frac{1}{\pi}\int_{-\infty}^{-\frac{1}{4}m_\pi^2} \frac{\mathrm{Im}f_J^{(B)}(\nu')d\nu'}{\nu'-p^2}; \tag{1}$$

$\vec{p}$ = center-of-mass momentum. We have denoted all the quantum numbers by a single symbol, J. The "driving term" $\mathrm{Im}f_J^{(B)}$ is to be computed from a dynamical model. The model we use is to neglect all but lowest order terms corresponding to the exchange of a π meson and the η, ρ, ω, and f resonances.[3,4]

Although this specifies $f_J^{(B)}$ completely, the resulting integral equation cannot, as it stands, be solved for the physical partial-wave amplitude, f_J. Even the assumption of elastic unitarity, i.e., $\mathrm{Im}f_J(p^2) = |f_J(p^2)|^2$, which reduces Eq. (1) to an N/D problem, leads to divergent integrals because of the vector and tensor resonances.

The purpose of efforts[4] to solve Eq. (1) is twofold: (i) to determine whether the resonance approximation, analyticity, and unitarity can together provide a suitable computational scheme, and (ii) if so, to determine the appropriate values of the resonance-nucleon coupling constants. Thus, at present, it is not as important to solve Eq. (1) as to compare its solutions with experiment.

This comparison can be accomplished by the following indirect procedure.[5] We first replace $f_J(p^2)$ by $f_J(p^2)/p^{2j}$, where j is the total angular momentum of the state with quantum numbers J. The dispersion relation (1) is still valid,[2] and the second integral can be evaluated immediately by Cauchy's theorem. Taking the real part of the resulting equation, we obtain

$$f_J^{(B)}(p^2) = \mathrm{Re}f_J(p^2) - p^{2j}\frac{P}{\pi}\int_0^\infty \frac{\mathrm{Im}f_J(\nu')d\nu'}{(\nu')^j(\nu'-p^2)} \equiv \frac{1}{M}h_J(p^2). \tag{2}$$

The j-fold subtraction insures the validity of this equation at $p^2 = 0$, and the nucleon mass M is introduced to make the functions h_J dimensionless.

For energies below $T_{\mathrm{lab}} = 400$ MeV the first term of h_J is known from experiment.[6] The sec-