PHYSICAL REVIEW LETTERS

VOLUME 12 4 MAY 1964 NUMBER 18

DOPPLER-SHIFTED CYCLOTRON RESONANCE WITH HELICON WAVES IN SODIUM*

M. T. Taylor
Department of Physics and Laboratory of Atomic and Solid State Physics, Cornell University, Ithaca, New York
(Received 9 April 1964)

When a transverse electromagnetic field exists inside a metal in a magnetic field and the electron mean free path is larger than the wavelength, the electrons can undergo cyclotron resonance even when the wave frequency is very much smaller than the cyclotron frequency. This occurs because an electron at the Fermi surface travels through the wave very rapidly and as a consequence of the Doppler effect it "sees" an apparent frequency very much larger than the actual frequency of the wave. This was predicted by Kjeldaas[1] for the case of transverse sound waves propagating in a metal parallel to a magnetic field and later Stern[2] pointed out that this should also occur for helicon waves. The maximum magnetic field at which Doppler-shifted cyclotron resonance is possible has been called the Kjeldaas edge. This edge is related to the Gaussian curvature of the Fermi surface at the point where the electrons have the largest component of velocity along the magnetic field. This means that for a spherical Fermi surface the position of the Kjeldaas edge is determined by the radius of the sphere in wave-vector space.

The effect has been observed in metals using sound waves[3] and helicon waves.[4] In neither of these studies was it possible to achieve a precise comparison between theory and experiment. The effect has also been observed in the damping of Alfvén waves in bismuth[5]; in this case the complexity of the band structure of bismuth makes detailed comparison difficult.

This Letter describes a method of studying the edge in simple metals which permits comparison between theory and experiment to an accuracy of a few percent. This method looks promising as an experimental technique for measuring the curvature at points on the Fermi surface.

Helicon waves are circularly polarized electromagnetic waves which can propagate through a metal when $\omega_c\tau \gg 1$ (ω_c is the cyclotron frequency and τ is the relaxation time), and they can be described as a dynamic manifestation of the Hall effect.[6] An electron at the Fermi surface with velocity v_F (~10 cm/sec) will travel through the slow helicon wave (velocity ~ 10^3 cm/sec) of frequency ω and wave vector $\vec{q}$, parallel to magnetic field $\vec{B}$, and "see" an apparent frequency ω_a given by

$$\omega_a = \omega \pm \vec{q}\cdot\vec{v}_F. \tag{1}$$

There are two situations to be considered:

(a) At small magnetic fields ($ql \geq \omega_c\tau \gg 1$ where $l = v_F\tau$) there are always some electrons on the Fermi surface which see an apparent frequency equal to their cyclotron frequency; for helicon waves $\omega \ll \omega_c$ thus

$$\omega_a = \omega_c \leq qv_F. \tag{2}$$

Therefore some electrons will undergo cyclotron resonance as long as the inequality in Eq. (2) holds and thus absorb energy from the wave.

This absorption process will cease at a particular magnetic field, the Kjeldaas edge, defined by the equality $\omega_q = \omega_c = qv_F$. At this field, energy will cease to be absorbed from the wave causing an abrupt change in the skin depth of the helicon wave.

(b) At large magnetic fields ($\omega_c\tau > ql \gg 1$) the above absorption process does not exist and the wave propagates with the dispersion relation

$$q^2 = \frac{ne\mu_0\omega}{B} f\left(\frac{ql}{\omega_c\tau}\right), \tag{3}$$

where n is the number of electrons per unit volume. The function f tends to unity when $\omega_c\tau \gg ql$; i.e., the dispersion relation goes over to the simple form derived from local theory.[7]

The field for the edge, B_{edge}, from Eqs. (2), (3), and the relation $\omega_c = eBv_F/hk_F$ assuming a spherical Fermi surface of radius k_F, is given by

$$B_{\text{edge}} = \left\{\frac{2}{3\pi}\frac{\mu_0 h}{e}k_F^{\;5}\right\}^{1/3} \nu^{1/3} f_{\text{edge}}^{\;1/3}. \tag{4}$$

The experiment measures the surface impedance of polycrystalline sodium with a magnetic field normal to its surface. At the edge, the skin depth of the helicon wave will change abruptly and this change manifests itself as a peak in the resistive component of the surface impedance.[8] Figure 1 shows a block diagram of the experimental arrangement. A slab of pure sodium (resistance ratio ~7500) is placed in a coil at 4°K. The coil is a component of a twin-T radio-frequency bridge.[9] The bridge is balanced and then unbalanced slightly so that the resistive component of the surface impedance can be studied. The output from the bridge is amplified and detected;

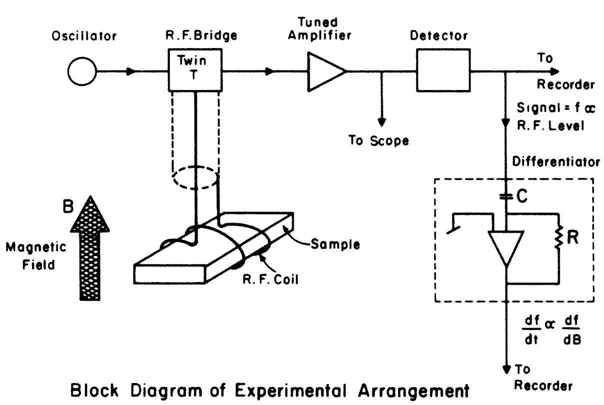

FIG. 1. Block diagram of experimental arrangement. The sample and coil are immersed in liquid helium. The magnetic field can be swept from 0 to 55 kG.

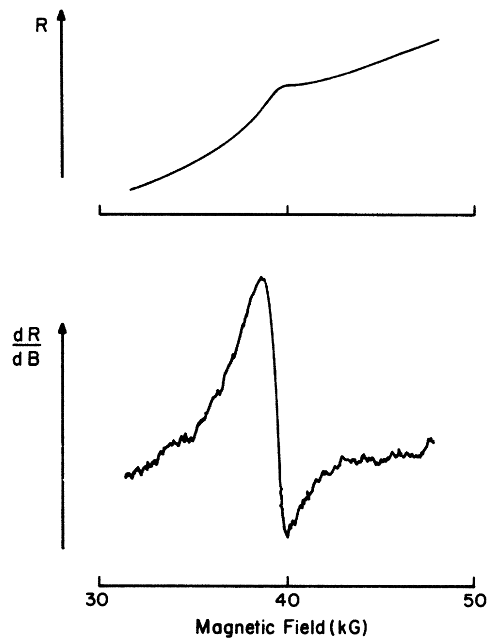

FIG. 2. Variation of the resistive component of the surface impedance, R, and its derivative as a function of magnetic field at a frequency of 35 Mc/sec.

the detector output is a measure of the unbalance of the bridge and is recorded as a function of magnetic field on an X-Y recorder. To enhance the observability of the edge, the signal from the detector is differentiated; this is a time differentiation but since the field is changed linearly in time, it is equivalent to a differentiation with respect to field. Figure 2 shows the variation of the resistive component of the surface impedence and its derivative with field at 35 Mc/sec. Measurements have been made over the frequency range 14-50 Mc/sec, so that the position of the edge, B_{edge}, varied from 30 to 45 kG. In this field range $\omega_c\tau$ and ql vary from approximately 50 to 80. A plot of the position of the edge in kG against the cube root of frequency is shown in Fig. 3. It is a straight line through the origin as predicted by Eq. (4).

In order to calculate the Fermi radius from the slope of this line a value must be assigned to f_{edge}. It has been shown[7] that as $ql \to \infty$, $f_{\text{edge}} \to 1.5$. From calculation for finite values of ql, it is found that f_{edge} is essentially a constant for $ql > 30$. Since f_{edge} enters as a fifth root, the final answer for k_F is relatively insensitive to the value chosen. The value obtained for the radius of the Fermi sphere of sodium was $(0.92 \pm 0.02) \times 10^8$ cm^{-1}; this is to be compared

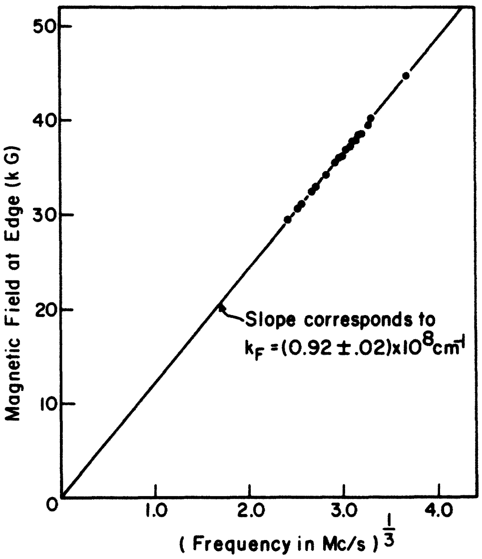

FIG. 3. Plot of the value of the magnetic field at the edge against the cube root of the frequency. Measurements were made at 18 frequencies between 14 and 50 Mc/sec.

with the theoretical value of 0.93×10^8 cm^{-1} obtained assuming one electron per atom and the lattice constant of 4.225 Å.[10] The principal error in k_F comes from the lack of precision in the calibration of the superconducting magnet, so the above accuracy can be improved.

To summarize, cyclotron resonance has been performed using helicon waves with frequencies ~10 000 times smaller than the electron cyclotron frequency and the experimental technique used promises to lead to accurate measurements of the curvature at points on the Fermi surface.

The author wishes to thank R. Bowers for his encouragement, interest, and support of this work. He also acknowledges the help of J. Houck and S. Tallman in the construction of the experiment.

---

*Work supported by the U. S. Atomic Energy Commission and the Advanced Research Projects Agency.

[1]T. Kjeldaas, Phys. Rev. 113, 1473 (1959).
[2]E. A. Stern, Phys. Rev. Letters 10, 91 (1963).
[3]A. R. Mackintosh, Phys. Rev. 131, 2420 (1963).
[4]M. T. Taylor, J. R. Merrill, and R. Bowers, Phys. Letters 6, 159 (1963).
[5]J. Kirsch, Phys. Rev. 133, A1390 (1964).
[6]See bibliography in reference 4.
[7]J. J. Quinn and S. Rodriguez, Phys. Rev. 133, A1590 (1964).
[8]P. B. Miller and R. R. Haering, Phys. Rev. 128, 126 (1962).
[9]H. L. Anderson, Phys. Rev. 76, 1460 (1949).
[10]C. S. Barrett, Acta Cryst. 9, 671 (1956).

SURFACE BARRIER IN TYPE-II SUPERCONDUCTORS*

R. W. De Blois and W. De Sorbo
General Electric Research Laboratory, Schenectady, New York
(Received 7 February 1964)

Bean and Livingston[1] have recently predicted the existence of an image-effect surface barrier to the penetration of magnetic flux into a type-II superconductor. We report here the experimental observation of a greatly enhanced superconducting to mixed-state transition field in some regions of an electropolished specimen of $Nb_{0.993}O_{0.007}$ and in another of $Nb_{0.33}Ta_{0.67}$. Our observations are consistent with the predictions of the Bean-Livingston theory that a plane surface on a type-II superconductor will produce a nucleation barrier that will increase the initial flux penetration field to near the thermodynamic critical field H_c, and that roughening the surface will permit flux penetration near the superconducting to mixed-state transition field H_{c1}. They are also shown to be inconsistent with the most plausible alternative explanation that electropolishing may have produced a chemically or structurally altered surface sheath. Joseph and Tomasch have also recently found experimental evidence for the existence of the surface barrier by a different technique than ours.[2]

The preparation of the $Nb_{0.993}O_{0.007}$ wire[3] specimen has been described earlier.[4] The $Nb_{0.33}Ta_{0.67}$ specimen was sectioned from a 0.76-mm wire annealed 291 hours at 1650°C and 10^{-7} mm Hg. Electropolishing was carried out in an acid mixture of two parts H_2SO_4 to one part HF. Several minutes near 10 V were followed by several minutes near 3 V. The voltage measurements were between the specimen and a platinized platinum cathode.

Since the existence of localized imperfections,