

sublattice magnetizations and vary rapidly in the vicinity of the compensation temperature.

In conclusion, it should be emphasized that the range of validity of the Néel ground state depends not only on the relative strengths of the sublattice exchange interactions, as measured by α and β , but also on the specific crystal structure. The results derived here are for a NaCl structure but may easily be extended to others. For example, the inequality which replaces (12) when consideration is given to a CsCl sublattice arrangement is

$$(1 + 2\alpha)(1 + 2\beta) > 0 \quad (17)$$

and is even more restrictive than (12).

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⁴The method to be described may be applied to assumed ground states of arbitrary form provided that the magnon spectra appropriate to them can be formulated.

⁵F. R. Morgenthaler, Phys. Rev. Letters **11**, 69 (1963).

⁶Strictly speaking, the dipolar energy should also be corrected by using the appropriate dipole wave sums of Cohen and Keffer [Phys. Rev. **99**, 1128 (1955)]. We do not introduce these complications here because, although the effect we are describing requires the existence of the dipolar or similar energy, its exact form is not crucial.

⁷F. R. Morgenthaler, J. Appl. Phys. **31**, 95S (1960); T. Schaug-Petersen, J. Appl. Phys. **31**, 382S (1960).

⁸(To be published.)

⁹Certain regions of the $\alpha - \beta$ plane which are unstable when $H_z = 0$ will become stabilized as H_z is first increased and then become unstable as H_z exceeds the critical value.

INTERACTION OF A DRIFT CURRENT WITH TRANSVERSE WAVES IN A SOLID STATE PLASMA

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We report here the observation of the Doppler-like interaction between the transverse plasma waves supported by a magnetized bismuth plasma (at 4.2°K) and a drift current sent through it. In the experiments, which were performed at low radio frequencies, the rotation of the plane of polarization of a damped Alfvén wave¹ was observed as a function of the applied current. The plane of polarization is rotated because the velocities of the two components of opposite circular polarization, which make up a wave launched from a plane-polarized source, are different in the presence of drift, as predicted theoretically by Bok and Nozières.² We shall calculate first the expected amount of rotation in the damped Alfvén waves and the properties of these waves themselves and then describe the experimental geometry and the results.

Bok and Nozières showed that for a plasma whose carriers drift, the contribution to the total dielectric constant from each carrier species is multiplied by the factor $[1 - (\beta + i\alpha)V_d/\omega]$, where the drift velocity V_d and complex propagation constants β and α can be positive or nega-

tive, and where ω is the experimental frequency.

We first calculate those features of the drift interaction which are independent of the anisotropy and mass differences of the carriers. We therefore assume a plasma with isotropic equal-mass carriers in which the density difference $\Delta N = (N_h - N_e)$ between the holes and electrons is small compared with the average density $N = (N_h + N_e)/2$. Choosing all symbols as positive real quantities when the wave propagation and magnetic field B_0 are both parallel to the hole current, we write the dielectric constant for the two directions of circular polarizations γ ($\gamma = \pm 1$):

$$\begin{aligned} \epsilon_\gamma &= \frac{c^2(\beta + i\alpha)^2}{\omega^2} \\ &= \epsilon_{h\gamma} \left[1 + \frac{(\beta + i\alpha)V_d}{\omega} \right] + \epsilon_{e\gamma} \left[1 - \frac{(\beta + i\alpha)V_d}{\omega} \right]. \quad (1) \end{aligned}$$

The zero-drift hole and electron contributions, $\epsilon_{h\gamma}$ and $\epsilon_{e\gamma}$, for a cyclotron frequency ω_c and relaxation time τ such that $\omega_c \gg \omega$, $\omega_c \tau \gg 1$, but

$\omega\tau \ll 1$, are

$$\begin{aligned}\epsilon_{h\gamma} &\cong (c^2/V_H^2)(\gamma - i/\omega_c\tau); \\ \epsilon_{e\gamma} &\cong (c^2/V_H^2)(-\gamma - i/\omega_c\tau)\end{aligned}\quad (2)$$

where

$$V_H^2 = \omega B_0 / k_0 e N. \quad (3)$$

The solutions to the quadratic equation in $\beta + i\alpha$ of Eq. (1) are

$$\begin{aligned}(\beta + i\alpha)/\omega &= \gamma V_d / V_H^2 \\ &\pm (1/V_H) [(V_d/V_H)^2 - 2i/\omega_c\tau]^{1/2}.\end{aligned}\quad (4)$$

Fig. 1 is essentially a plot of β and α versus drift current for $\omega_c\tau = 16$. We see that at $V_d = 0$, $|\alpha| = \beta$ and the damped Alfvén velocity exceeds

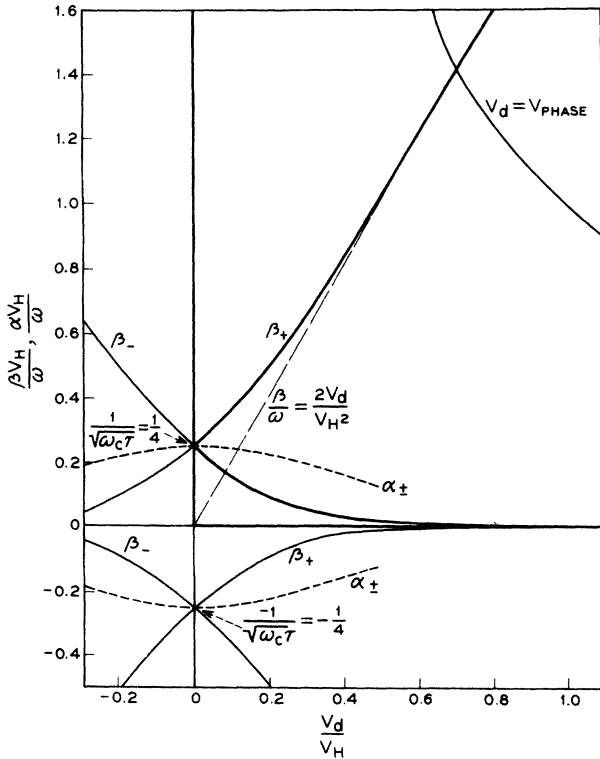


FIG. 1. Plot of the normalized propagation constant $\beta V_H/\omega$ and the corresponding normalized attenuation coefficient $\alpha V_H/\omega$ against normalized drift velocity V_d/V_H for the two senses of circular polarization β_{\pm} and two directions of propagation $-\infty < \beta < \infty$. At $V_d = 0$, $\beta_{\pm} = -\alpha_{\pm}$ since the dielectric constant is pure imaginary for transverse waves in a plasma with equal numbers of electrons and holes at a frequency such that $\omega\tau \ll 1$. The calculation is made for isotropic equal-mass carriers.

V_H by the factor $(\omega_c\tau)^{1/2}$. The half differences in β and α between the two directions of polarization are

$$\delta\beta \equiv (\beta_+ - \beta_-)/2 = \omega V_d / V_H^2 = \mu_0 j / 2B_0, \quad (5)$$

$$\delta\alpha \equiv (\alpha_+ - \alpha_-)/2 = 0, \quad (6)$$

where $j = (N_h + N_e)eV_d$. The average values $\bar{\beta}$ and $\bar{\alpha}$ do not differ appreciably from their zero-drift values until $\delta\beta$ becomes comparable with β .

The above calculation of $\delta\beta$ is also applicable to the different carrier mobilities of Bi provided we use the expression in current density j of Eq. (5). Only at large V_d ($\sim V_H$) and small $\omega_c\tau$ (~ 1) will there be any corrections to Eqs. (5) and (6).

To interpret the experiment we consider the amplitude of a transverse wave launched from a plane-polarized source and propagating in a direction z parallel to B_0 . The amplitudes in the plane parallel (A_{\parallel}) and perpendicular (A_{\perp}) to the source are

$$\begin{aligned}A_{\parallel} &= A_0 e^{-\bar{\alpha}z} (\cos^2 \delta\beta z + \sinh^2 \delta\alpha z)^{1/2} \\ &\times \cos(\bar{\beta}z - \omega t)\end{aligned}\quad (7)$$

$$\begin{aligned}A_{\perp} &= A_0 e^{-\bar{\alpha}z} (\sin^2 \delta\beta z + \sinh^2 \delta\alpha z)^{1/2} \\ &\times \cos(\bar{\beta}z - \omega t + \varphi)\end{aligned}\quad (8)$$

where $\varphi = 0$ when $\delta\alpha z = 0$.

In the low drift current limit, where $\bar{\alpha}$ and $\bar{\beta}$ are not changed appreciably from their zero-current value, we can write Eq. (8) (since $\delta\alpha = 0$) as

$$A_{\perp} \cong \sin(\delta\beta z) A_{\parallel} \Big|_{V_d=0}. \quad (9)$$

The zero-current behavior of damped Alfvén waves has been studied¹ by making measurements on $\bar{\alpha}z = \ln |A_{\parallel}|$ and $\bar{\beta}z = \omega(t - t_0)$, the wave amplitude and phase, as a function of ω , B_0 , and z . The expected dependence of $\bar{\beta}$ and $\bar{\alpha}$ on $\omega^{1/2}/B_0$ was observed. There was, however, some rotation of the plane of polarization at low magnetic fields (< 2 kG) which is not fully understood, but is probably due to mobility anisotropy. The outstanding feature of these waves is their ability to pass through a sample with a cross section several times smaller than a wavelength. With $\alpha = -\beta$ the amplitude drops to $\exp(-2\pi) \cong 2 \times 10^{-3}$ in one wavelength. Only for very large B_0 or small ω can any interference effects associated with the boundaries be observed. This feature

is very convenient in attaining large drift velocities using small cross-section samples.

In the present experiment damped Alfvén waves are passed through a large ($\sim 2 \times 3 \times 4$ cm) single-crystal Bi sample, the central area (about 2 mm^2) of which carries a large current. The transverse magnetoresistance due to B_0 keeps the current confined to an area determined by a pair of crossed "slits." The long phonon mean free path provides an intimate thermal connection between this central current filament and the extra-large sample surface.

The "slits" consist of two parallel holes (cut in the Bi using a spark cutter) which also house the source and detector rf coils [see Fig. 2(a)].

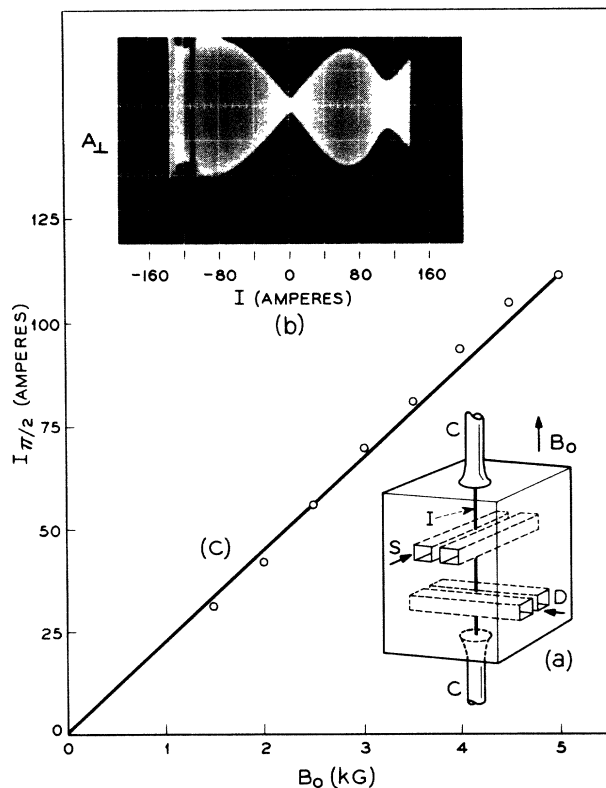


FIG. 2. (a) The large ($\sim 2 \times 3 \times 4$ cm) Bi sample contains two pairs of holes between which "slits" are formed to confine a current filament (I) which flows between current contacts (C) parallel to the magnetic field (B_0), utilizing the large magnetoresistance of Bi. The signal source (S) and detector (D), which are at right angles, are pairs of rf coils contained in the holes. (b) The amplitude A_{\perp} in the detector coils varies sinusoidally with current as the plane of polarization of the signal arriving from the source rotates in and out of parallelism. (c) The position of the maximum in current ($I_{\pi/2}$) varies linearly with B_0 .

External leakage from source to detector was minimized by closing the ends of the holes with copper sheet, so that the surface currents generated by the coils close on themselves. Large current contacts, which overlap the "slits" but not both holes, make the current path self-aligning in the field. The crystal orientation was arbitrary since the sample was cut for maximum power dissipation.

The contribution to A_{\perp} from the area surrounding the current path is zero in principle, since there the current is zero. At some values of ω and B_0 slight "tuning" of the null is possible due to the rotation in the undrifted plasma which was discussed earlier. A detuning of this null has the effect of adding a small term independent of current to $\sin(\delta\beta z)$ in Eq. (9). With this "leakage" signal present it has been easy to verify that $\delta\beta$ reverses sign with current or field direction. This test rules out any possibility that sample heating was responsible for the observed effects.

A 60-cycle transformer was used as a current source which allows a continuous display of A_{\perp} as a function of current on an oscilloscope. A typical response of A_{\perp} vs current is shown in Fig. 2(b) with $B_0 = 3$ kG and $f = 110$ kc/sec. The value of current $I_{\pi/2}$, at which $\delta\beta z = \pi/2$, is calculated from Eq. (5) using $z = 2$ cm. It agrees within the knowledge of the exact current-carrying area with the observed value. As predicted by Eq. (5) the observed value of $I_{\pi/2}$ varies linearly with B_0 as is shown in Fig. 2(c). The experimental frequency was varied from 50 kc/sec to 1 Mc/sec and, while the quality of the measurement changed slightly, the value of $I_{\pi/2}$ was unaffected over the range. This behavior is taken as a strong indication that the sample surfaces or interfaces have no effect on the damped Alfvén wave propagation.

The asymmetry in the amplitude of A_{\perp} for opposite directions of current appears to result from a change in $\bar{\alpha}$ due to the difference in the ratio of drift and Hall mobilities of the electrons and holes. The burst of "noise" starting at 100 amperes on the left in Fig. 2(b) is an oscillation which remains when the rf source is removed. With a critical current that depends on B_0 and with many harmonics in the waveform it resembles the "oscillistor effect."³

We conclude that the observed response is due to the Doppler-like interaction and that the simple infinite-medium theory of damped Alfvén waves is adequate for calculating the amount of rotation

of the plane of polarization. The interaction, which has been observed from currents of a few amperes up to large currents, is also being studied for the onset of the conversion from damped Alfvén waves to propagating helicons.

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PROPERTIES OF Ξ HYPERONS*

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In a study of interactions initiated by 1.80- and 1.95-GeV/c K^- mesons in the Lawrence Radiation Laboratory 72-in. hydrogen bubble chamber, a total of 356 Ξ^- hyperons was observed. Table I shows the pertinent reactions, the number of events observed at each momentum, and the total cross sections. The observation of the Λ from Ξ^- decay was required for the identification of reactions (a) and (b); the decay Λ was also observed in all but 24 examples of reaction (c). The cross sections are based on a τ count and were corrected for neutral Λ decays.

The Ξ^- mean life was computed by means of a maximum likelihood procedure in which each event was assigned an a priori probability

$$P(x) = N \exp(-x/L_{\Xi^-}) \times \{ \exp(-y_0/L_{\Lambda}) - \exp[-y_1(x)/L_{\Lambda}] \}, \quad (1)$$

consistent with the requirement that the Λ be seen. Here $L_i = m_i/(p_i \tau_i c)$, m_i , p_i , and τ_i being respectively the mass, momentum and mean life of particle i ; x is the decay distance of a

given Ξ^- , y_0 the minimum accepted Λ decay distance and $y_1(x)$ the maximum or the chamber wall. Because of the curvature of the Ξ^- track, the normalization N had to be calculated numerically by the requirement

$$\int_{x_0}^{x_1} P(x) dx = 1;$$

x_0 and x_1 are, respectively, the smallest and largest accepted Ξ^- flight path. The evaluation of the maximum likelihood yielded

$$\tau_{\Xi^-} = (1.77 \pm 0.12) \times 10^{-10} \text{ sec},$$

essentially independent of cut-offs, provided $x_0 > 0.5$ cm. The mean life determination does not depend sensitively on τ_{Λ} ; $d\tau_{\Xi^-}/d\tau_{\Lambda} \approx 0.04$. The result is in good agreement with other determinations.^{1,2}

Regardless of the Ξ^- spin J , the helicity of the Λ from Ξ^- decay³ in the Ξ^- rest frame is $\alpha = 2 \operatorname{Re}(A_{J-1/2} A_{J+1/2}^*)$ where A_L is the normalized amplitude for Ξ decay into a $\Lambda\pi$ system of orbital angular momentum L ; $|A_{J-1/2}|^2$

Table I. The number of Ξ^- production events at each momentum.

Final state	1.80 GeV/c		1.95 GeV/c	
	Number	Total cross section (μb)	Number	Total cross section (μb)
(a) $\Xi^- K^+$	99	113 ± 12	88	99 ± 11
(b) $\Xi^- K^+ \pi^0$	21	24 ± 5	20	23 ± 5
(c) $\Xi^- K^0 \pi^+$	71	59 ± 7	57	52 ± 7

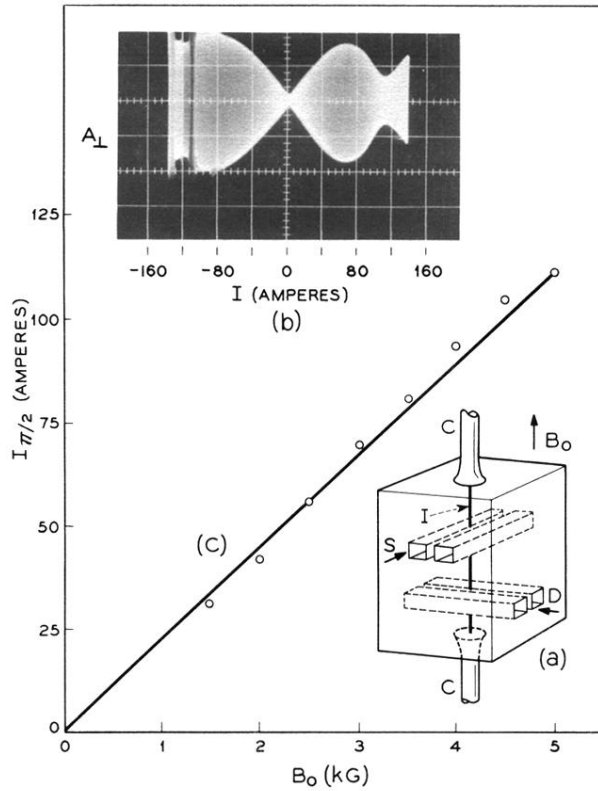


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