10 millioersteds or less. Again a susceptibility approximating complete diamagnetism was observed. The first critical field was 0.4 ± 0.1 Oe. The susceptibility of the powder specimen was very nearly reversible in fields up to several kilo-oersted.

The reduced samples were prepared from single crystals of $SrTiO_3$, and subsequently lapped and cleaned to remove surface material. Two of the samples were reduced by prolonged heating in a vacuum of 10^{-5} to 10^{-7} mm Hg. Sample TiR2a' was reduced in the presence of titanium metal, and VR8 by vacuum alone. The third sample, HR6, was reduced in a dynamic flow of hydrogen at 950°C. Samples TiR2a' and VR8 were quenched more rapidly than HR6 after reduction, and thus may be more strained which could alter the transition temperatures of these samples.[1,2] The resistivities and Hall coefficients of the specimens were measured over the temperature range 2°K-300°K. All three samples were degenerate at liquid helium temperatures.

X-ray diffractometer measurements on single-crystal specimen HR6 showed the material to be a good single crystal of the perovskite structure. The same sample when powdered was shown to be of one phase, viz. perovskite; the sensitivity for the detection of second phases was better than $\frac{1}{2}\%$. Special care was taken in looking for the phases Ti and TiO.

The superconducting transitions were observed by cooling the specimens by thermal contact through Apiezon N grease with single crystals of chromium potassium alum in an adiabatic demagnetization apparatus, and the magnetic susceptibility was observed in a similar way, except that copper wires were interposed between the specimen and the cooling salt to avoid interference with the specimen mutual inductance coils. The rate of temperature increase at 0.25°K during the resistance measurements was typically 3 millideg/min.

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STABILITY OF THE NÉEL GROUND STATE IN A TWO-SUBLATTICE FERRIMAGNET*

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Most literature dealing with ferromagnetic resonance and related topics assumes that the static magnetization vector associated with each magnetic sublattice of the crystal is spatially uniform. Since it is now known that such a configuration is not always the ground state,[1-3] it is worth while to re-examine the conditions under which the assumption is valid.

A convenient approach to this problem is to consider the spin-wave (magnon) spectra $\omega_k(\vec{k})$ associated with the assumed Néel ground state.[4] A necessary condition that this configuration be stable is that $\omega_k^2 \geq 0$ for all allowed values of $\vec{k}$. Since the spin waves form a complete set, any

deviation from the ground state may be expanded in terms of these modes; therefore, the condition is sufficient as well.

The spin-wave spectra for a cubic, two-sublattice crystal including the volume dipolar energy have recently been derived for small $\vec{k}$.[5] The dipolar energy was initially included because it provides certain interesting nonlinear couplings which were being studied. It has turned out, however, that this term, while very small compared to the exchange energy, is important in determining the range of stability of the Néel configuration.

We begin our discussion by reviewing the conditions under which the Néel theory predicts a stable configuration—one with both sublattices fully saturated in the event that $T \to 0°\mathrm{K}$. The appropriate Hamiltonian is given by

$$\mathcal{H} = -\mu_0\{\tfrac{1}{2}[w_{11}|\vec{M}_1|^2 + w_{22}|\vec{M}_2|^2 + 2w_{12}\vec{M}_1\cdot\vec{M}_2] + (\vec{M}_1 + \vec{M}_2)\cdot\vec{H}_z\} \quad (1)$$

where $\vec{M}_i$ is the magnetization of the ith sublattice, $w_{ij} = w_{ji}$ are dimensionless exchange constants, and $\vec{H}_z$ is the internal dc magnetic field oriented along the z axis. Although the treatment holds in general, we consider the case $w_{12} < 0$. Then $\vec{M}_i = M_i\vec{i}_z$, $i = 1, 2$ where, by convention, $M_1 > 0$, and $M_2 < 0$ unless H_z is very large. We also have

$$M_i = M_{is}B_i\left[\frac{\mu_0 M_{is}}{n_i KT}\left(H_z + \sum_{j=1}^{2} w_{ij}M_j\right)\right], \quad i = 1, 2, \quad (2)$$

where n_i, M_{iS}, and B_i are, respectively, the magnetic ion density, the saturation value of the magnetization, and the appropriate Brillouin or Langevin function of the ith sublattice. Both sublattices are saturated when $T \to 0°\mathrm{K}$ provided $(\alpha+1)|w_{12}|M_{2S} + H_z > 0$ and $(\beta+1)|w_{12}|M_{1S} - H_z > 0$ where $\alpha = (w_{11}M_1)/(w_{12}M_2)$ and $\beta = (w_{22}M_2)/(w_{12}M_1)$. These stability conditions are valid for fields such that $0 < H_z < |w_{12}|(M_{1S} - M_{2S})$. As the upper bound is overreached (1) is no longer minimized when $\vec{M}_1$ and $\vec{M}_2$ are antiparallel and the simple ground state breaks down. The boundaries of the region correspond to the case where, according to the Néel theory, only one or the other of M_i is saturated.

The spin-wave spectra previously given are only valid when $2\pi/|\vec{k}|$ is large compared to the nearest neighbor spacing l; we wish now to correct this deficiency. For simplicity, we assume two interpenetrating face-centered cubic magnetic sublattices arranged like a NaCl lattice. The parameter w_{12} is a measure of the strength of the exchange interaction between any magnetic moment and its six nearest neighbors (belonging to the other sublattice), whereas w_{11} and w_{22} arise from the twelve nearest neighbors of the same sublattice. If we carry the molecular field expansion to higher orders,[6] there results for the effective exchange field acting on the ith sublattice

$$\vec{H}_i = w_{ii}[\vec{M}_i + \tfrac{1}{3}(xy + xz + yz - 3)\delta\vec{M}_i\cos\vec{k}\cdot\vec{r}_i] + w_{ij}[\vec{M}_j + \tfrac{1}{3}(x + y + z - 3)\delta\vec{M}_j\cos\vec{k}\cdot\vec{r}_j], \quad (3)$$

$$i = 1, 2, \quad j = 1, 2 \ (i \neq j),$$

where

$$\vec{M}_i \simeq M_i\vec{i}_z + \delta\vec{M}_i\cos\vec{k}\cdot\vec{r}_i; \quad \delta\vec{M}_i\cdot\vec{i}_z = 0;$$

and $\vec{r}_i$ are the positions (treated continuously) of the magnetic moments giving rise to $\vec{M}_i$. The wave vector $\vec{k} = \vec{i}_x k_x + \vec{i}_y k_y + \vec{i}_z k_z$ is the same for both sublattices; $x = \cos k_x l$, $y = \cos k_y l$, and $z = \cos k_z l$.

The spin-wave equations previously derived are easily modified to take care of the complete molecular fields of (3). The spin-wave spectra associated with the Néel ground state are then given by

$$(\omega_k^2 - \omega_1^2)(\omega_k^2 - \omega_2^2) = 0 \quad (4)$$

where

$$\omega_1^2 + \omega_2^2 = (A^2 + 2BC + D^2) + \mu_0[\gamma_1 M_1(A + C) + \gamma_2 M_2(B + D)]\sin^2\Psi; \quad (5)$$

$$\omega_1^2\omega_2^2 = (AD - BC)^2 + \mu_0(AD - BC)[\gamma_2 M_2(A - B) + \gamma_1 M_1(D - C)]\sin^2\Psi;$$

$$A = \gamma_1\mu_0 w_{12}M_2\{1 + [1 - \tfrac{1}{3}(xy + xz + yz)]\alpha\} + \gamma_1\mu_0 H_{z1};$$

$$B = -\gamma_1\mu_0 w_{12}M_1\tfrac{1}{3}(x + y + z);$$

$$C = -\gamma_2\mu_0 w_{12}M_2\tfrac{1}{3}(x + y + z);$$

$$D = \gamma_2\mu_0 w_{12}M_1\{1 + [1 - \tfrac{1}{3}(xy + xz + yz)]\beta\} + \gamma_2\mu_0 H_{z2};$$

$$H_{zi} = H_0 - N_z(M_1 + M_2) + H_{ai}, \quad i = 1, 2;$$

$$\sin^2\Psi = (k_x^2 + k_y^2)/(k_x^2 + k_y^2 + k_z^2). \quad (6)$$

Here, H_0 is the applied dc field, N_z the z demag-

netizing factor, and γ_i the gyromagnetic ratio (negative) of the ith sublattice. For the sake of generality, the anisotropy fields H_{ai}, appropriate to each sublattice, are also included. Notice that if $|\vec{k}|l \ll 1$ and x, y, and z are expanded to second order, the equations reduce to those previously published.[5]

The stability criterion $\omega_i^2 \geq 0$, $i = 1, 2$, implies that the right-hand side of (5) and (6) must be ≥ 0. Although these inequalities are necessary, they are not sufficient to ensure that the ω_i^2 are real; the latter condition is guaranteed provided that in addition,

$$(\omega_1^2 - \omega_2^2)^2 = (\omega_1^2 + \omega_2^2)^2 - 4\omega_1^2\omega_2^2 \geq 0. \quad (7)$$

The inequality based on (6) is automatically satisfied provided we neglect the dipolar energy (by setting $\Psi = 0$), and the apparent stability criterion (7), which involves only the exchange energy, becomes

$$(A + D)^2 + 4(BC - AD) \geq 0. \quad (8)$$

Notice that the inequality based on (5) is satisfied wherever (8) is fulfilled. The situation is quite different if we include the dipolar terms because, independent of (8), whenever $(AD - BC)^2$ is very near zero (which is possible) and the second term in (6) is negative, stability is violated. The dipolar stability condition which is more restrictive than (8) may be shown to be

$$AD - BC \leq 0 \ (M_2 < 0) \quad (9a)$$

$$\geq 0 \ (M_2 > 0) \quad (9b)$$

for all allowed $\vec{k}$, since in the stable regions $\gamma_2 M_2(A - B) + \gamma_1 M_1(D - C) \leq 0$ (≥ 0) whenever $AD - BC \leq 0$ (≥ 0). Notice that the inequalities based on (5) and (7) are automatically satisfied whenever (9) is satisfied. [The dipolar terms in (5) and (7) are negligible except when $A + D \approx 0$; even then they are only of minor importance because $|w_{12}| \gg 1$.]

The region of dipolar instability associated with (9) for a particular value of $\vec{k}$ is bounded by the hyperbolas which are the roots of (6). These are given by

$$\left(1 + \frac{H_{z1}}{w_{12}M_2} + \eta\alpha\right)\left(1 + \frac{H_{z2}}{w_{12}M_1} + \eta\beta\right) - \sigma^2 = 0 \quad (10)$$

and

$$\left(1 + \frac{H_{z1} + M_1 \sin^2\Psi}{w_{12}M_2} + \eta\alpha\right)\left(1 + \frac{H_{z2} + M_2 \sin^2\Psi}{w_{12}M_1} + \eta\beta\right) - \left(\sigma - \frac{\sin^2\Psi}{w_{12}}\right)^2 = 0 \quad (11)$$

where $\eta = 1 - \frac{1}{3}(xy + xz + yz)$, $\sigma = \frac{1}{3}(x + y + z)$, and only the upper branches of (10) and (11) need to be considered. A sketch of this region, for the case $H_{zi} = 0$, is shown in the α-β plane of Fig. 1. The width of the region is $\sim |w_{12}|^{-1} \sin^2\Psi$ and is therefore very narrow. Although only a small area of the plane is unstable for this $\vec{k}$, the hyperbolas sweep out large areas of the plane as $\vec{k}$ is varied. Possible values of η as a function of σ are shown in Fig. 2. Curves appropriate to $\vec{k}$ directed along the principal directions of the NaCl lattice are also shown in this figure. It is seen that the region given by the upper branch of

$$(1 + \tfrac{4}{3}\alpha)(1 + \tfrac{4}{3}\beta) > \tfrac{1}{9} \quad (12)$$

is stable for all $\vec{k}$, $H_{zi} = 0$. This region is shown in Fig. 1 and should be compared to that predicted by the simple Néel theory which is also shown in the figure. We note in passing that a given region in the α-β plane may simultaneously be unstable on account of dipolar and exchange instabilities.

The role of the long-wavelength magnons is of particular interest. For simplicity we take $H_{ai} = 0$ and define $H_{zi} = H_z$; the spectrum then re-

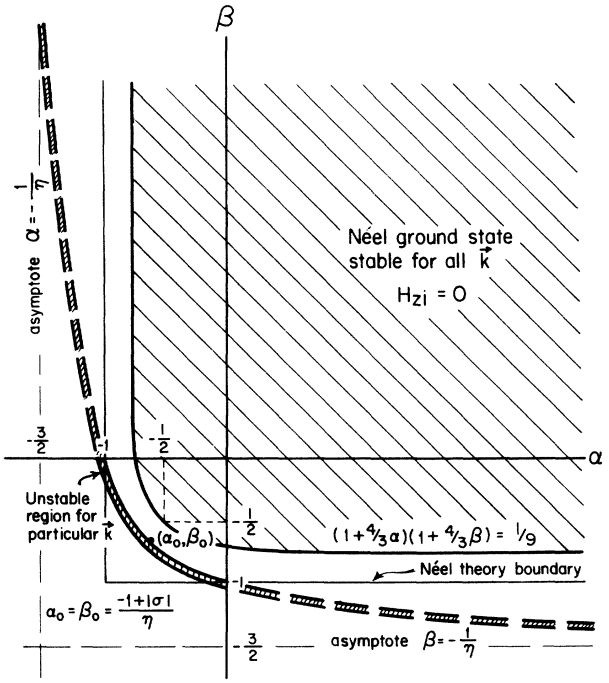

FIG. 1. The region of dipolar instability, associated with Eq. (9) of the text, for a particular value of $\vec{k}$ when $H_{zi} = 0$. The region of stability for all $\vec{k}$ when $H_{zi} = 0$ is also shown.

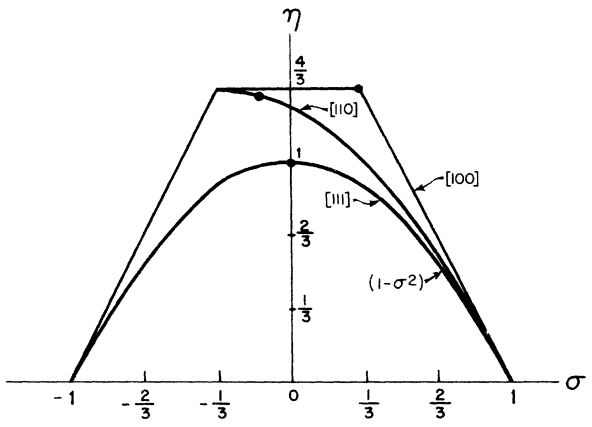

FIG. 2. Allowed values of η as a function of σ for the NaCl sublattice arrangement. The circles indicate the edges of the first Brillouin zone for the principal crystallographic directions of the fcc lattice.

duces to

$$\omega_k^2 \simeq \gamma_{\rm eff}^2 \mu_0 (H_z + \lambda_{\rm eff} k^2 M) \times (H_z + \lambda_{\rm eff} k^2 M + M \sin^2\Psi), \qquad (13)$$

where $\gamma_{\rm eff} = (M_{1S} - M_{2S})(M_{1S}/\gamma_1 - M_{2S}/\gamma_2)^{-1}$, $M = M_{1S} - M_{2S}$, and $\lambda_{\rm eff} = \frac{1}{3}|w_{12}|l^2(\alpha+\beta+1) \times M_{1S}M_{2S}/(M_{1S} - M_{2S})^2$. For negative values of $\lambda_{\rm eff}$, the modes which have propagation vectors in the range

$$\frac{H_z}{-\lambda_{\rm eff}} \leq k^2 \leq \frac{H_z + M\sin^2\Psi}{-\lambda_{\rm eff}} \qquad (14)$$

are unstable and lead to a collapse of the Néel ground state. As $H_z \to 0$, the stability condition is evidently $\lambda_{\rm eff} = 0$; this also follows from (10) in the limit as $k \to 0$.

The physics underlying the long-wavelength spin-wave instabilities is best understood by noting that, in the absence of loss, the spin-wave amplitudes are modulated by the transverse dipolar field according to

$$\frac{d(\delta M_k)}{dt} = -\gamma_{\rm eff}\mu_0(M_{1S} - M_{2S})\frac{\sin^2\Psi}{2}\sin 2\varphi_k \delta M_k \qquad (15)$$

where φ_k is the spin-wave precessional phase angle. It is precisely this modulation which gives rise to elliptically—rather than circularly—polarized normal modes. The precessional frequencies are zero for spin waves in the critical $\vec{k}$ range; this implies that $\sin 2\varphi_k$ is a constant which, if positive, leads to an instability. Similar remarks also apply to the shorter wavelength spin waves. The amplitude of the maximally unstable modes is seen to be proportional to

$$\exp[\gamma_1\gamma_2\mu_0(M_{1S} - M_{2S})^2 t/(2|\gamma_2 M_{1S} - \gamma M_{2S}|)]$$

and is related to the situation sometimes found when the magnetization undergoes transient reversal[7]; the connection will be analyzed in detail in a later paper.[8] The dipolar energy, although small, is important because the Zeeman and exchange energies are balanced against one another; in such cases, a small perturbation of the proper form can have a large effect. It should be noted, in this connection, that certain forms of anisotropy energy, those which cause magnons to be elliptically polarized, can play a role very similar to the dipolar energy. Perhaps it is not too surprising, in retrospect, that Kaplan[3] found the effects of anisotropy in his spiral ground-state calculations rather striking.

The picture is much the same for arbitrary $\vec{k}$ with the stable region shifting according to (10), where for simplicity we again take $H_{ai} = 0$. A given region in the α-β plane, stable when $H_z = 0$,[9] becomes unstable when the field reaches a critical value given by

$$H_z = \frac{|w_{12}|}{2}\{[(1+\eta\beta)M_{1S} - (1+\eta\alpha)M_{2S}] \pm [(\{1+\eta\beta\}M_{1S} + \{1+\eta\alpha\}M_{2S})^2 - 4M_{1S}M_{2S}\sigma^2]^{1/2}\}_{\rm min} \leq |w_{12}|(M_{1S} - M_{2S}), \qquad (16)$$

where H_z (>0) is evaluated for that $\vec{k}$ which gives the minimum value; this will always occur when the positive square root is taken and $\eta = 2(1 - |\sigma|)$, $\frac{1}{3} \leq |\sigma| \leq 1$. If $\vec{k} = 0$, $\sigma = 1$ and, as expected, the upper bound is obtained. This agrees with the analysis based on (1). For very large H_z, the Néel state will again be stable with $M_2 = +M_{2S}$; in this case (9b) applies.

Notice that $H_z \leq 0$ is a solution of (16) provided $\vec{k} \to 0$ and the minus sign in front of the square root is taken. It may prove that the unstable spin waves, which are generated when the applied field (H_0) is reduced below $N_z(M_{1S} - M_{2S})$, take part in domain wall nucleation. This speculation might possibly be checked, in crystals which have a spin compensation point, by studying the temperature dependence of the nucleation time. For such materials, the spin-wave growth rates, given previously, depend on temperature through the

sublattice magnetizations and vary rapidly in the vicinity of the compensation temperature.

In conclusion, it should be emphasized that the range of validity of the Néel ground state depends not only on the relative strengths of the sublattice exchange interactions, as measured by α and β, but also on the specific crystal structure. The results derived here are for a NaCl structure but may easily be extended to others. For example, the inequality which replaces (12) when consideration is given to a CsCl sublattice arrangement is

$$(1+2\alpha)(1+2\beta)>0 \qquad (17)$$

and is even more restrictive than (12).

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INTERACTION OF A DRIFT CURRENT WITH TRANSVERSE WAVES IN A SOLID STATE PLASMA

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We report here the observation of the Doppler-like interaction between the transverse plasma waves supported by a magnetized bismuth plasma (at 4.2°K) and a drift current sent through it. In the experiments, which were performed at low radio frequencies, the rotation of the plane of polarization of a damped Alfvén wave[1] was observed as a function of the applied current. The plane of polarization is rotated because the velocities of the two components of opposite circular polarization, which make up a wave launched from a plane-polarized source, are different in the presence of drift, as predicted theoretically by Bok and Nozières.[2] We shall calculate first the expected amount of rotation in the damped Alfvén waves and the properties of these waves themselves and then describe the experimental geometry and the results.

Bok and Nozières showed that for a plasma whose carriers drift, the contribution to the total dielectric constant from each carrier species is multiplied by the factor $[1-(\beta+i\alpha)V_d/\omega]$, where the drift velocity V_d and complex propagation constants β and α can be positive or negative, and where ω is the experimental frequency.

We first calculate those features of the drift interaction which are independent of the anisotropy and mass differences of the carriers. We therefore assume a plasma with isotropic equal-mass carriers in which the density difference $\Delta N=(N_h-N_e)$ between the holes and electrons is small compared with the average density $N=(N_h+N_e)/2$. Choosing all symbols as positive real quantities when the wave propagation and magnetic field B_0 are both parallel to the hole current, we write the dielectric constant for the two directions of circular polarizations γ ($\gamma=\pm1$):

$$\epsilon_\gamma=\frac{c^2(\beta+i\alpha)^2}{\omega^2}=\epsilon_{h\gamma}\left[1+\frac{(\beta+i\alpha)V_d}{\omega}\right]+\epsilon_{e\gamma}\left[1-\frac{(\beta+i\alpha)V_d}{\omega}\right]. \qquad (1)$$

The zero-drift hole and electron contributions, $\epsilon_{h\gamma}$ and $\epsilon_{e\gamma}$, for a cyclotron frequency ω_c and relaxation time τ such that $\omega_c\gg\omega$, $\omega_c\tau\gg1$, but