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CROSS-FIELD MAGNETOABSORPTION IN SEMICONDUCTORS

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The cross-field interband magnetoabsorption of the interband transition in germanium has been observed at room temperature and at 77°K in magnetic fields as low as 15 kilo-oersteds. The most significant result of these experiments is that we can now also study the normally forbidden transitions $\Delta n = \pm 1$ which permit the measurement of the effective masses of holes and electrons separately rather than the reduced or combined mass.

It was originally shown theoretically by Aronov[1] that the properties of interband transitions between Landau states are altered by the presence of a dc electric field transverse to the magnetic field. He predicted an oscillatory behavior of the absorption as a function of the electric field. For the usual semiconductor, in order to observe this effect it would require electric fields $\sim 10^4$ V/cm for a field of 10^5 oersteds. Such values of a steady electric field are too large, particularly at low temperatures, to permit reliable observation of the effect because of heating and possibly impurity or interband ionization at 4°K. Therefore, in order to observe some of the predicted properties of the cross-field magnetoabsorption we have used a modulation technique in which the signals are detected at twice the fundamental frequency of the modulation.

The experiments were carried out on germanium samples 7 microns thick with H perpendicular to (110) plane and the infrared radiation propagating parallel to H. Periodic electric fields of 200 V/cm to 500 V/cm at 700 cps were applied perpendicular to the magnetic field. Variations in the intensity of the transmitted radiation were detected with a phase-sensitive detector at 1400 cps. The contribution to the second harmonic signal has several physical origins, i.e., periodic variations in the band gap due to heating, variations in the band gap due to the electric field alone,[2] and the cross-field effect considered here. The latter consists of two parts, namely a cross-field shift of the Landau levels and a variation of the matrix elements with electric field. The temperature fluctuations due to heating gave large contributions at room temperature. Since these variations were out of phase with the electric field, the temperature effect could be made to disappear by proper setting of the phase-sensitive detector.

In the cross fields a number of lines were found that were absent in the normal magnetoabsorption observations.[3] Most of the lines were "positive" (increased absorption); some lines were "negative" (decreased absorption). The "negative" lines coincided with normally allowed transitions.

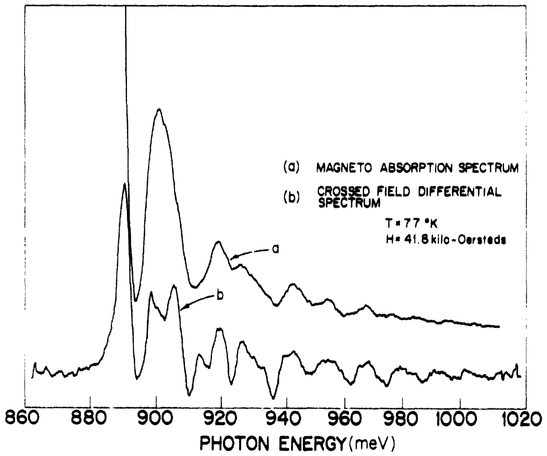

FIG. 1. (a) Magnetoabsorption in germanium at $H = 41.8$ kilo-oersteds and zero electric field. (b) Differential cross-field magnetoabsorption at second harmonic (i.e., 1400 cps) at same value of H and $E = 500$ V/cm. Note the existence of "positive" and "negative" lines; $T = 77°K$.

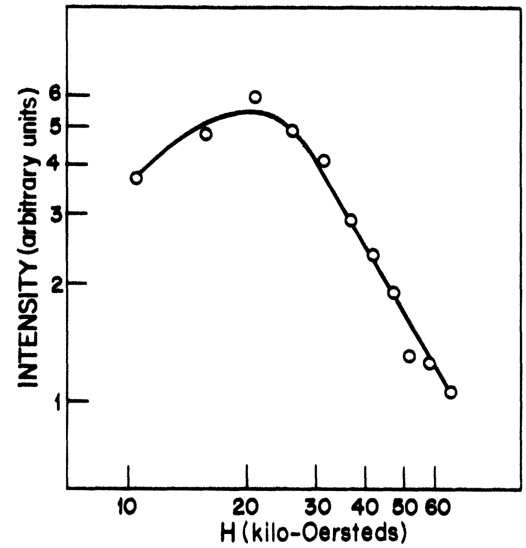

FIG. 2. The variation of the amplitude of the cross-field magnetoabsorption for the forbidden transitions $\Delta n = \pm 1$ as a function of magnetic field at room temperature. $E \approx 200$ V/cm.

Curve (a) of Fig. 1 shows the normal magnetoabsorption at zero electric field. Curve (b) of Fig. 1 shows the cross-field differential spectrum at the second harmonic. New additional lines appear in the latter spectrum. The amplitude variation of the cross-field lines as a function of magnetic field showed the behavior indicated in Fig. 2; at the higher fields the intensity decreased with H^2 in agreement with theory. Finally, Fig. 3 shows the plot of the transitions with and without electric field. The unequal spacings are consistent with the quantum effects predicted by Luttinger and Kohn[4] for the valence band.

The existence of variations in the absorption coefficient at even harmonics can be derived by extending the theory of Aronov. He has shown that the absorption coefficient for cross fields takes the form

$$\alpha = A \exp(-a^2/2) \sum_{n,n'} \left| \sum_{m=0}^{n'+1} b_m(n',n) a^{n+n'-2m} \right|^2 \times (\hbar\omega - \mathcal{E}_{nn'})^{-1/2} \tag{1}$$

where $A = 4\pi^2 e^3 H (2/\mu)^{1/2} P^2 / \hbar^2 n_0 c^2 m_0^2 \omega$, P is the momentum matrix element, μ the reduced effective mass of holes and electrons $\mu^{-1} = m_1^{*-1} + m_2^{*-1}$; $a = eEL_M/\hbar\omega_c$; $\omega_c = eH/c(m_1^* + m_2^*)$; $L_M = (\hbar c/eH)^{1/2}$; $\mathcal{E}_{nn'} = \mathcal{E}_g + \hbar\omega_{c1}(n+\frac{1}{2}) + \hbar\omega_{c2}(n'+\frac{1}{2}) - (m_1^* + m_2^*)c^2E^2/2H^2$, $\mathcal{E}_g$ is the energy gap, n_0 the index of refraction, and $b_m(n',n) = (-1)^{n-m} \times n!\,n'!\,2^m/m!\,(n'-m)!\,(n-m)!$. The significance of Eq. (1) is that the magnetoabsorption for interband transitions is modified in that in addition to the allowed transitions $\Delta n = 0$, the forbidden transitions $\Delta n = \pm 1, \pm 2, \cdots$ can now also occur. For an electric field $E = E_0 \cos\omega t$ the absorption coefficient becomes $\alpha = \alpha_0 + \Delta\alpha \times (1 + \cos 2\omega t) + \cdots$. Expanding Eq. (1) in powers of a^2 and retaining only terms in a^2 (the condition $a < 1$ is well satisfied in our experiments) we find that for the allowed transitions $\Delta\alpha$ takes the form

$$\Delta\alpha = -A \sum_n \frac{(m_1^* + m_2^*)c^2E_0^2}{8} [H^{-2}(\hbar\omega - \mathcal{E}_n)^{-3/2} + (4n+2)(m_1^* + m_2^*)ce^{-1}\hbar^{-1}H^{-3} \times (\hbar\omega - \mathcal{E}_n)^{-1/2}] 2^n n!\,n!. \tag{2}$$

Similarly we can show that for the transitions $\Delta n = \pm 1$ the differential absorption at the second harmonic of the modulation frequency becomes

$$\Delta\alpha = A \sum_n 2^{2m-1} (m+1)!\,(m+1)! \times (\hbar\omega - \mathcal{E}_{nn'})^{-1/2} \frac{E_0^2 c^3 (m_1^* + m_2^*)^2}{\hbar e H^3} \tag{3}$$

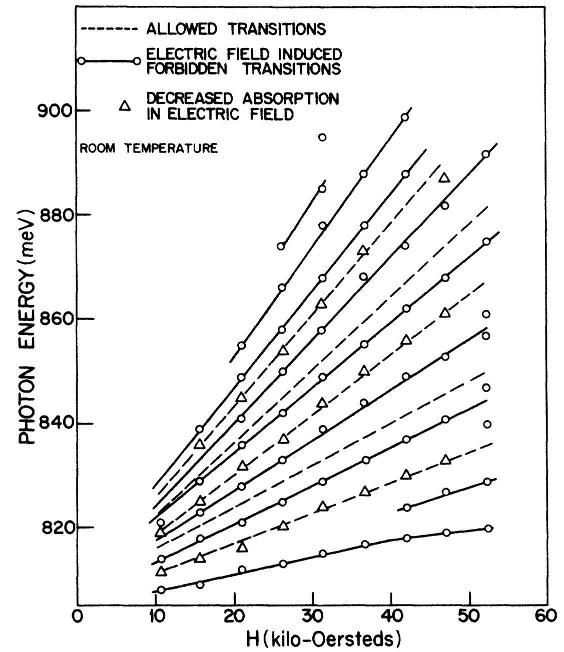

FIG. 3. The energy of transitions for cross-field magnetoabsorption vs magnetic field. The dotted lines are the normally allowed transitions $\Delta n = 0$, the solid lines are the forbidden transitions $\Delta n = \pm 1$.

where $m = n$ for $\Delta n = +1$ and $m = n - 1$ for $\Delta n = -1$. Higher order transitions can also be detected but they will be weaker when $a < 1$ since they will appear as second and higher even harmonics for $\Delta n = \pm 2, \pm 3, \cdots$ with a higher order power in a. From Kramers-Kronig relations it is also possible to obtain expressions for the differential dispersion for modulation harmonics associated with the forbidden transitions. These can be detected by studying the Faraday and Voigt effects.[5] The results of Eq. (2) and Eq. (3) show that the differential cross-field magnetoabsorption is proportional to the square of the amplitude of the electric field and inversely to the square of the magnetic field for terms containing $(\hbar\omega - \mathcal{E}_{nn'})^{-1/2}$ and inversely to H for the term in $(\hbar\omega - \mathcal{E}_n)^{-3/2}$. The latter singularity predicts an increase of the amplitude of the oscillatory effect of the allowed transitions with a ratio of $\omega_c\tau$, where τ is the scattering time.

The significance of the above experiment is that we can observe the interband transitions even at room temperature quite easily with the cross-field modulation technique. The modulation method which permits the detection of the "forbidden" transitions represents an increased resolving capability over the previous method. Furthermore, these new transitions now permit the determination of the individual spacings of the Landau levels in the conduction or valence band or the measurement of the effective masses of holes and electrons separately rather than the reduced or combined mass. Another interesting result of these experiments is that we have found a new method of modulating light even in modest fields, or the order of 10^4 oersteds. It should be relatively easy to extend the frequency to the microwave region by placing the sample in a resonant cavity in the region of maximum electric field.

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