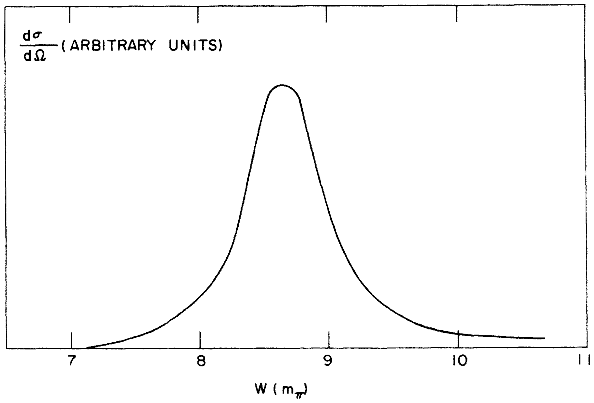

FIG. 3. Total cross section for $\pi\omega$ scattering from a 2^- resonance.

This formula is useful when one wishes to take into account only the elastic unitarity and the Born term. The subtraction is made at the middle of the short cut. A cutoff has to be introduced since the integral diverges. In the spirit of an effective-range calculation, we shall replace the integral by a constant adjusted to give a resonance at $W = 8.70$. Thus essentially we are doing a one-parameter fit to the resonance with width proportional to $(f^2/4\pi)$. The result for the total cross section is shown in Fig. 3 ($f^2/4\pi = 0.45$). It is noteworthy that we end up with a resonance whose width is roughly equal to 100 MeV. This is in good agreement with experiments.[9] Our improvement over the static calculation of Peierls comes about essentially from the steeper fall-off of $\bar{B}^J$ shown in Fig. 2.

Turning now to the 1^+ state, since the ρ exchange contributions are mainly from "far-away" singularities, it does not stand out among all the other exchanges. Thus it is not really consistent to perform a calculation with ρ exchange alone. We believe, then, that although B^{1+} is attractive, more has to be learned before one can draw conclusions on the 1^+ state of $\pi\omega$ scattering.

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[9]The uncertainty of the width should be largely due to the uncertainty in $(f^2/4\pi)$, both from the theoretical model and the experimental ω width. A change in $f^2/4\pi$ by a factor of two or three would destroy the agreement. However, the calculated B-particle width is insensitive ($< \pm 5\,\%$) to the choice of the subtraction point on the "short cut."

PROBLEM OF COMBINING INTERACTION SYMMETRIES AND RELATIVISTIC INVARIANCE

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Recently there has been discussion[1,2] concerning the possibility of combining interaction symmetries (for instance SU(3) for strong interactions) and relativistic invariance in a nontrivial way. One of the motivations is the possibility of obtaining exact mass formulas[3] for particles belonging to the same representations of the interaction group. In this paper the impossibility of such combinations, under a certain restrictive condition, is pointed out.

Consider an interaction symmetry defined by a semisimple Lie group I. When combined with Lorentz invariance, the usual assumption is that the group T that describes the full symmetry is $T = I \times L$, where L is the inhomogeneous Lorentz group. This, of course, leads to the conclusion that particles belonging to the same irreducible representation of I have the same mass since the irreducible representations of T are products of those of I and L.

It is clear that if particles belonging to the same representation of I are to have different masses, then the generators of I (denoted by I_i) do not in general commute with the translation operators, that is, in general

$$[I_i, p_\mu] \neq 0. \tag{1}$$

The problem then is to find a Lie group T that has as generators those of L and I and for which inequality (1), in general, holds. A further restriction to be imposed on T is that the generators of I commute with the generators of the homogeneous Lorentz group. This of course implies that if one applies both a homogeneous Lorentz transformation and an interaction-symmetry transformation on a state, the transformed state is independent of the order in which the two transformations are applied. It further implies that the quantum numbers associated with the interaction symmetry do not change when one performs a homogeneous Lorentz transformation. With this restriction it is shown that indeed $[I_i, p_\mu] = 0$ and thus $T = I \times L$.

The generators of the Lorentz group satisfy the familiar commutation relations

$$[M_{\mu\nu}, M_{\lambda\sigma}] = i[g_{\mu\sigma}M_{\lambda\nu} + g_{\mu\lambda}M_{\nu\sigma} + g_{\nu\sigma}M_{\mu\lambda} + g_{\lambda\nu}M_{\sigma\mu}], \tag{2}$$

$$[M_{\mu\nu}, p_\lambda] = i[p_\nu g_{\mu\lambda} - p_\mu g_{\nu\lambda}], \tag{3}$$

$$[p_\mu, p_\nu] = 0. \tag{4}$$

We will denote the generators of the group I by J_i, $i = 1, \cdots, n$, where n is the dimension of the group, and the generators of the Lorentz group by J_i, $n+1 \leq i \leq n+10$. In particular,

$$p_i = J_{n+i} \text{ for } 1 \leq i \leq 4,$$

$$M_{12} = J_{n+5},\; M_{13} = J_{n+6},\; M_{14} = J_{n+7},$$

$$M_{23} = J_{n+8},\; M_{24} = J_{n+9},\; M_{34} = J_{n+10}.$$

The full symmetry group T is determined by the commutation relations

$$[J_j, J_k] = C_{jk}{}^i J_i, \tag{5}$$

where the structure constants must satisfy

$$C_{jk}{}^i = -C_{kj}{}^i, \tag{6}$$

$$C_{is}{}^p C_{jk}{}^s + C_{ks}{}^p C_{ij}{}^s + C_{js}{}^p C_{ki}{}^s = 0. \tag{7}$$

The restriction imposed immediately after Eq. (1) implies

$$C_{jk}{}^i = 0 \text{ for } j \leq n \text{ and } k > n+4. \tag{8}$$

Consider Eq. (7) for $i \leq n$, $j > n+4$, $n+1 \leq k \leq n+4$, $p \leq n$. The restriction on T implies $C_{js}{}^p = C_{ij}{}^s = 0$ and thus Eq. (7) reduces to

$$C_{is}{}^p C_{jk}{}^s = 0.$$

From Eq. (3) there is at most one value of s in this sum for a given j and k for which $C_{jk}{}^s \neq 0$ and for a given value of s, $n+1 \leq s \leq n+4$, a pair of values j and k exist for which $C_{jk}{}^s \neq 0$. Thus

$$C_{is}{}^p = 0 \text{ for } i \leq n,\; n+1 \leq s \leq n+4,\; p \leq n. \tag{10}$$

Now consider Eq. (7) for $n+1 \leq p \leq n+4$, $i \leq n$, $j > n+4$, and $n+1 \leq k \leq n+4$. For this case $C_{ij}{}^s = 0$ and Eq. (7) reduces to

$$C_{is}{}^p C_{jk}{}^s + C_{js}{}^p C_{ki}{}^s = 0. \tag{11}$$

In Eq. (11) both s indexes need only range between $n+1$ and $n+4$. Consider the particular case $j = n+5$ and $k = n+1$. With the aid of Eq. (3), Eq. (1) can be reduced to

$$C_{i,n+2}{}^p C_{jk}{}^{n+2} + C_{j,n+1}{}^p C_{k,i}{}^{n+1} + C_{j,n+2}{}^p C_{ki}{}^{n+2} = 0,$$

where

$$C_{j,n+1}{}^p = 0 \text{ unless } p = n+2,$$

$$C_{j,n+2}{}^p = 0 \text{ unless } p = n+1.$$

This implies $C_{i,n+2}{}^p = 0$ for all i unless $p = n+1$ or $p = n+2$. Letting $p = n+2$, one obtains

$$C_{ik}{}^k = C_{i,n+2}{}^{n+2}.$$

If one considers Eq. (7) for the case $j = n+8$ and $k = n+3$ one concludes $C_{i,n+2}{}^{n+1} = 0$. Continuing in a similar way we find

$$C_{ik}^{\ \ j} = C_i \delta_{jk}, \quad i \leq n, \ n+1 \leq j, \ k \leq n+4. \qquad (12)$$

Now consider Eq. (7) with $n+1 \leq p$, $k \leq n+4$, and i, $j \leq n$. From the previous discussion for this case we can write

$$C_{jk}^{\ \ s} = \delta_{sk} C_j, \quad C_{js}^{\ \ p} = \delta_{sp} C_j,$$

$$C_{ks}^{\ \ p} = -\delta_{pk} C_s, \quad C_{is}^{\ \ p} = \delta_{ps} C_i,$$

$$C_{ki}^{\ \ s} = -\delta_{sk} C_i.$$

Inserting these relations in Eq. (7) leads to the conclusion that

$$C_s C_{ij}^{\ \ s} = 0 \text{ for } i, \ j \leq n. \qquad (13)$$

If one introduces a standard coordinate system[4] of the group I, it is easy to see that Eq. (13) implies $C_S \equiv 0$. Thus

$$C_{ij}^{\ \ k} = 0 \text{ for } i \leq n, \ n+1 \leq j, \ k \leq n+4. \qquad (14)$$

Now consider Eq. (7) with i, $j < n$, $n+1 \leq k \leq n+4$, $p > n+4$. From the original assumption and the previous discussion, it follows that $C_{js}^{\ \ p} = C_{is}^{\ \ p} = 0$ for this case. Thus Eq. (7) reduces to

$$C_{ks}^{\ \ p} C_{ij}^{\ \ s} = 0 \qquad (15)$$

Since this must be true for all i, $j \leq n$, using again the standard coordinate system of the group I, one sees that

$$C_{ks}^{\ \ p} \equiv 0 \text{ for } p > n+4, \ n+1 \leq k \leq n+4, \ s \leq n. \qquad (16)$$

Combining Eqs. (8), (10), (13), and (14) leads to

$$C_{kj}^{\ \ i} = 0 \text{ for } k \leq n, \ j \geq n+1, \text{ all } i: \qquad (17)$$

that is, $T = I \times L$. Thus if one demands that the interaction symmetry commute with the homogeneous Lorentz transformation and requires the existence of a Lie group T whose generators are those of the interaction-symmetry group and the Lorentz group, then it follows that $T = I \times L$. This applies in particluar to the group SU(3).

In conclusion, if one wishes to combine such an interaction symmetry with Lorentz invariance to form a larger group that will give mass splitting, one must accept not only lack of commutation of the symmetry-interaction generators with the Lorentz translation generators but also their lack of commutation with the homogeneous Lorentz generators. It is felt that this will, in general, lead to interpretation difficulties.

It should be noted that Eqs. (13) and (15) are true even if the interaction-symmetry group is not semisimple; for some such groups one can still deduce Eqs. (14), (16), and (18).

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