tive trilinear M interactions" could yield an acceptable theory.

[12]M. Gell-Mann, Cal. Tech. Report CTSL-20, 1961 (unpublished); Y. Ne'eman, Nucl. Phys. 26, 222 (1961); M. Gell-Mann, Phys. Rev. 125, 1067 (1962).

[13]Y. Fujii, Progr. Theoret. Phys. (Kyoto) 21, 232 (1959); J. Sakurai, Ann. Phys. (N.Y.) 11, 1 (1960).

[14]M. Gell-Mann, Phys. Letters 8, 214 (1964); F. Gürsey, T. D. Lee, and M. Nauenberg (to be published). For example, minimal interactions $\{3^*\} \times \{3\} \times (\{8\}$ or $\{1\}) + CP \rightarrow C/P$.

[15]M. Gell-Mann, reference 12; S. Okubo, Progr. Theoret. Phys. (Kyoto) 27, 949 (1962); 28, 24, (1962).

[16]The present work leaves open the question at what stage R invariance is broken.

[17]S. Okubo, Phys. Letters 5, 165 (1963); J. Sakurai, Phys. Rev. 132, 434 (1963).

[18]This has recently been emphasized by G. Feinberg (to be published).

[19]N. Cabibbo, Phys. Rev. Letters 12, 62 (1964).

[20]I am indebted to Dr. M. A. B. Bég for pointing out to me the interesting article by R. Gatto, Nuovo Cimento 28, 1504 (1963), which also deals with the connection between P and intrinsic quantum numbers. The present approach is different. In particular Gatto considers current interactions only.

EVOLUTION OF THE SOLAR SYSTEM AND THE EXPANSION OF THE UNIVERSE*

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Assuming Mach's principle,[1] one might expect that the general expansion of the universe had some effect on the evolution of the solar system and of the galaxy. There have appeared in recent papers[2,3] arguments that general relativity does imply an important connection between local systems and the rest of the universe. It is our contention that this is incorrect. The apparent connection is only formal, and the "effect" is unobservable. The fact that general relativity predicts no appreciable effect on the solar system, or galaxy, due to the expansion of the universe has been previously expressed by several authors.[4]

Consider a bound system such as the solar system, galaxy, or a cluster of galaxies. The problem is to find the effect, if any, of the time dependence of the cosmological background metric on the equations of motion of this dynamical system.

For a model universe containing matter in the form of a uniform continuous distribution of radiation and/or dust, the metric is given by the Robertson-Walker line element

$$d\tau^2 = dt^2 - a^2(t)[dr^2/(1-\lambda r^2) + r^2 d\Omega], \tag{1}$$

where $\lambda = +1$, 0, or -1 for a closed, flat, or hyperbolic universe respectively. This model may be a useful approximation to our universe for distances large compared with the typical separations of galaxies, and if it be supposed that the major part of the mass of the universe consists of uniformly distributed radiation, such as neutrinos, this line element should be valid to quite small distances (in the absence of nearby massive bodies). The largest localized dynamical system to be considered is the cluster of galaxies, for which the characteristic size is presently very small compared with the radius of the universe. With the origin of coordinates at the center of the localized system the term $|\lambda r^2|$ is small within the system and is negligible compared with unity. Thus to a good approximation (and without approximation for a flat cosmology) the background metric for the dynamical system can be written as

$$d\tau^2 = dt^2 - a^2(dr^2 + r^2 d\Omega). \tag{2}$$

The conformal flatness of this geometry can be exhibited by introducing a new time coordinate such that

$$d\tau^2 = a^2(\bar{t})[d\bar{t}^2 - dr^2 - r^2 d\Omega], \tag{3}$$

or making use of the Minkowski form η_{ij},

$$d\tau^2 = a^2(\bar{t})\eta_{ij}dx^i dx^j. \tag{4}$$

Suppose that in this space two charged particles are electrically bound in circular orbits, with the particle masses very small. To compute the particle orbits it is convenient to introduce a conformal transformation, interpreted not as a mapping of the space upon itself, but as a redefinition of the metric tensor at the same coordinate point in the same coordinate system, to remove the conformal factor $a^2(\bar{t})$ from Eq. (4).[5] This transformation is to be interpreted as a "units transformation,"[6] and is conveniently supplemented by the additional assumption that the unit of reciprocal mass is transformed like the units of length and time.[5,6] This units transformation leaves invariant the measures of e, h, and c. Gürsey[3] has previously noted the simplification to be obtained by introducing a conformal transformation into the cosmological problem. This conformal redefinition of the metric tensor does not affect the equations of motion of particles. Also, the redefined metric tensor (η_{ij}) is Minkowskian, and the dynamical problem can be discussed within the framework of standard special relativity, but with a time-dependent particle mass. The equations of motion for the two particles interacting electromagnetically can be derived from the variational equation

$$0=\delta\int\{\sum\int[ma(\eta_{ij}u^iu^j)^{1/2}+eA_iu^i]\,\delta^4(x-\bar{x}(\tau))d\tau$$

$$+(1/16\pi)F_{ij}F^{ij}\}d^4x, \qquad (5)$$

where $F_{ij}=A_{j,i}-A_{i,j}$, and $u^i=dx^i/d\tau$. In these equations the particle mass is $ma(\bar{t})$ where m is the (constant) particle mass expressed in the original units. The angular momentum is a constant of the motion, and

$$\mu a(\bar{t})rv=\text{const}, \qquad (6)$$

where r is the relative coordinate radius of the orbit, v is the relative coordinate velocity, assumed to be small compared with c, and μ is the reduced mass of the two particles. Because a is very nearly constant during one orbit period,

$$\mu a(\bar{t})v^2/r=e^2/r^2, \qquad (7)$$

so that

$$r\propto a(\bar{t})^{-1} \qquad (8)$$

In an expanding universe a is increasing with time, and the radius of the orbit is decreasing (compare Pachner[2]). However, in the original units the spatial metric tensor components are $g_{\alpha\alpha}=-a^2$ $(\alpha=1,2,3)$ [Eq. (4)] and the proper orbit radius is a constant. In these original units the length of a real measuring rod would be time independent.

Next suppose that the particles are uncharged and gravitationally bound in orbit. Again assuming that the masses of the particles are small, it is known[6] that in the units of Eq. (5) the gravitational constant is

$$G_0=G/a^2(\bar{t}), \qquad (9)$$

with G a constant, and Eq. (7) is replaced by

$$\mu a(\bar{t})v^2/r=[ma(\bar{t})]^2[G/a(\bar{t})^2]/r^2. \qquad (10)$$

In the new units the radius of the orbit is inversely proportional to $a(\bar{t})$, and in the old units the radius is constant. In the units provided by real measuring rods the radius of the gravitationally bound orbit is a constant, even though the orbit is in curved, expanding space.

The above calculation is valid only if the energy content of the universe is sufficiently smooth and uniformly distributed, e.g., in neutrinos. For a universe with its matter localized in galaxies, the above analysis is inappropriate. However, we also conclude for this case that, within the framework of general relativity, the expansion of the universe is without an appreciable effect on a localized system. To see this, assume that matter is concentrated in galaxies, and the galaxies in the mean uniformly distributed in the universe, with no appreciable radiation or neutrino energy density. Now suppose that the solar system, our galaxy, and the galaxies closer than perhaps seven megaparsecs from our galaxy are removed from the universe. This would remove the local group of galaxies, and several nearby groups including the M81 and Leo groups of galaxies, although the boundary of the sphere would still be well away from the Virgo cluster, at 11 megaparsecs. Within the resulting spherical, matter-free volume, the universe would appear effectively spherically symmetric to very good accuracy. Because of this symmetry, space inside the sphere (and not too near the boundary) must be flat even though the boundary is expanding with time.[7]

Having noted that this interior region is flat we may imagine that we return our galaxy (and solar system) to its rightful place at the center of the spherical cavity. The internal dynamics of this system can now be treated in the weak-field approximation, the exterior universe being without an appreciable effect. Furthermore, for a spherical cavity the other galaxies can also be returned, their effects being treated as a first-order perturbation on the rest of the universe, including our own galaxy. On this scale the first-order perturbation calculation is adequate because the perturbation to the background metric due to these nearby galaxies is very small. Assuming that the galaxies and possible intergalactic gas represent a mean density as large as 2×10^{-29} g/cc, the perturbation of the metric tensor components at the center of the sphere would amount to only $3GM/Rc^2\sim6\times10^{-6}$.

Because the unperturbed space inside the sphere was flat, the motion of the perturbing system of matter in this space is well described by Newtonian gravity theory. The gravitational force in this system is directed roughly along the radius of the sphere. It is this force which leads to the correct acceleration in the rate of separation of these galaxies. However, we emphasize that the nearby galaxies can produce only extremely small tidal stresses within our galaxy, and these stresses are further reduced because the nearby galaxies are distributed in an approximately isotropic way. That is, contrary to the claims of Pachner,[2] general relativity implies no appreciable connection between the evolution of our solar system, and our galaxy, and the rest of the matter in the universe.

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[1]See, for example, C. Brans and R. H. Dicke, Phys. Rev. 124, 925 (1961).
[2]J. Pachner, Phys. Rev. 132, 1837 (1963); Phys. Rev. Letters 12, 117 (1964).
[3]F. Gürsey, Ann. Phys. (N.Y.) 24, 211 (1963).
[4]A. Einstein, Rev. Mod. Phys. 17, 120 (1945); 18, 148 (1946); earlier papers are listed in the second of these articles.
[5]T. Fulton, F. Rohrlich, and L. Witten, Rev. Mod. Phys. 34, 442 (1962).
[6]R. H. Dicke, Phys. Rev. 125, 2163 (1962).
[7]L. Landau and E. Lifshitz, The Classical Theory of Fields (Addison-Wesley Publishing Company, Inc., Reading, Massachusetts, 1951), pp. 304-312.

ERRATA

STATISTICAL BEHAVIOR OF FINITE ISOLATED SUPERFLUIDS. M. Rich and J. J. Griffin [Phys. Rev. Letters 11, 19 (1963)].

There is a typographical error in paragraph four, page 19: $g\hbar\omega=15$ should read $g\hbar\omega=35$.

DYNAMICAL INSTABILITY OF GASEOUS MASSES APPROACHING THE SCHWARZSCHILD LIMIT IN GENERAL RELATIVITY. S. Chandrasekhar [Phys. Rev. Letters 12, 114 (1964)].

Equation (13), expressing the Eulerian change in the pressure accompanying adiabatic radial pulsation, is incorrect. The correct expression is

$$\delta p=-\xi p_0'-\gamma p_0 e^{\nu_0/2}(r^2\xi e^{-\nu_0/2})'/r^2. \qquad (13')$$

While this expression for δp can be justified by a detailed consideration of the second law of thermodynamics in the framework of general relativity, its validity is apparent when it is noted that Eq. (10) for the corresponding change in ϵ has the alternative form

$$\delta\epsilon=-\xi\epsilon_0'-(p_0+\epsilon_0)e^{\nu_0/2}(r^2\xi e^{-\nu_0/2})'/r^2; \qquad (10')$$

and the arguments in reference 4 of the Letter strictly apply to this equation.

Equation (12) together with Eq. (13′) constitute a characteristic value problem for σ^2. This problem is self-adjoint and the variational base is now provided by the equation

$$\sigma^2\int_0^R e^{(3\lambda_0-\nu_0)/2}(p_0+\epsilon_0)r^2\xi^2dr=4\int_0^R e^{(\lambda_0+\nu_0)/2}rp_0'\xi^2dr+\int_0^R e^{(\lambda_0+3\nu_0)/2}\gamma p_0[(r^2\xi e^{-\nu_0/2})']^2r^{-2}dr$$

$$+\kappa\int_0^R e^{(3\lambda_0+\nu_0)/2}p_0(p_0+\epsilon_0)r^2\xi^2dr-\int_0^R e^{(\lambda_0+\nu_0)/2}\frac{(p_0')^2r^2\xi^2}{(p_0+\epsilon_0)}dr. \qquad (15')$$

With the trial function $\xi=re^{\nu_0/2}$, Eq. (15′) gives

$$\sigma^2\int_0^R e^{(3\lambda_0+\nu_0)/2}(p_0+\epsilon_0)r^4dr=\int_0^R e^{(\lambda_0+3\nu_0)/2}(4r^3p_0'+9\gamma p_0r^2)dr$$

$$+\kappa\int_0^R e^{3(\lambda_0+\nu_0)/2}p_0(p_0+\epsilon_0)\,r^4dr-\int_0^R dr e^{(\lambda_0+3\nu_0)/2}r^4(p_0')^2/(p_0+\epsilon_0); \qquad (17')$$