event space. This connection is generally nonintegrable and the internal curvature need not be covariant constant.

de Broglie et al.[5] and Allcock[6] have considered models where internal states of elementary particles are represented by points in the group space of the rotation and Lorentz groups. There are two important differences between their approach and the one presented here. They formulated their theory in a flat Minkowski space; their internal space has nothing to do with the Riemann curvature tensor. For our hypnothesis the nonintegrability of the Riemann connection is essential; for if the connection is integrable, the event space is a Minkowski space, the holonomy group is the identity and the internal state space disappears. The second difference is that they have used the full group space rather than the tangent space as we have done. The possibility has not been discussed here, but we have the freedom within the mathematical framework to consider the entire holonomy group space as an internal state space.

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UNITARY SYMMETRY AND WEAK INTERACTIONS*

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A necessary condition for the success of any theory of weak interactions is that it be consistent with the observed relations[1-3]

$$\alpha_- \approx 0, \tag{1}$$

$$\alpha_+ \approx 0, \tag{2}$$

$$\alpha_0 \approx -\alpha_\Lambda \approx \alpha_\Xi \tag{3}$$

between the asymmetry parameters for the nonleptonic modes of Σ, Λ, and Ξ decay. Here we wish to show that, within the framework of unitary symmetry,[4,5] it is possible to derive (2) and (3) from the $\Delta T = \frac{1}{2}$ rule, time-reversal invariance, and two symmetry principles. Equation (1) cannot be derived directly from the theory, but is consistent with it.

At least one of the symmetries used for nonleptonic decays can be applied to leptonic decays, and, with the aid of certain dynamical assumptions, it predicts a ratio of the rates for $K^+ \to \mu + \nu$ and $\pi^+ \to \mu + \nu$ which agrees with experimental data. In addition, electromagnetic interactions obey both symmetry principles and a prediction can be made about the weak electromagnetic decays $\Sigma^+ \to p + \gamma$ and $\Xi^- \to \Sigma^- + \gamma$.

We begin by noting that the members of any unitary multiplet can be classified in three different ways.[6,7] One classification is based on isotopic spin T and hypercharge

$$Y_T = B + S,$$

where B denotes baryon number and S strangeness; the second is based on K spin and electric charge Q, and the third on L spin and a corresponding hypercharge

$$Y_L = Q - Y_T. \tag{4}$$

The third components of all three spins can be expressed in terms of Q and Y_T[8]:

$$T_3 = Q - \tfrac{1}{2}Y_T, \quad K_3 = Y_T - \tfrac{1}{2}Q, \quad L_3 = -\tfrac{1}{2}(Q + Y_T); \tag{5}$$

but the relation between the total spins, T, K, and L, is more complicated and varies from one multiplet to another.[9]

The interaction Hamiltonian for nonleptonic decays can be written as a sum of two parts,

$$H_{NL} = H_{(+1)} + H_{(-1)}, \tag{6}$$

where $H_{(-1)}$ is the Hermitian conjugate of $H_{(+1)}$. Their quantum numbers are

$$T_3 = -\tfrac{1}{2}, \quad Y_T = 1; \quad K_3 = 1, \quad Q = 0;$$

$$L_3 = -\tfrac{1}{2}, \quad Y_L = -1; \tag{7}$$

for $H_{(+1)}$, and

$$T_3=\tfrac{1}{2},\quad Y_T=-1;\quad K_3=-1,\quad Q=0;$$

$$L_3=+\tfrac{1}{2},\quad Y_L=1; \tag{8}$$

for $H_{(-1)}$. Since the transformation

$$(T,T_3,Y_T)\to(L,-L_3,Y_L),$$

$$(K,K_3,Q)\to(K,-K_3,Q) \tag{9}$$

interchanges the quantum numbers in (7) and (8), it follows that H_{NL} may, in certain circumstances, be invariant under (9); here we shall assume that this is indeed the case. The transformation (9) is referred to as the T-L transformation, and H_{NL} is said to be T-L invariant.

Under (9), baryons transform according to the rule[10]

$$T\text{-}L:\ (B)\to(-p,-\tfrac{1}{2}(\Sigma^0-\sqrt{3}\Lambda^0),\Xi^-,\tfrac{1}{2}(\sqrt{3}\Sigma^0+\Lambda^0);$$

$$-\Sigma^+,\Xi^0,n,\Sigma^-);$$

$$(B)\equiv(\Sigma^+,\Sigma^0,\Sigma^-,\Lambda;p,n,\Xi^0,\Xi^-). \tag{10}$$

The corresponding rule for pseudoscalar mesons is obtained from the substitution

$$(B)\to(\pi^+,\pi^0,\pi^-,\eta;K^+,K^0,-\bar{K}^0,K^-). \tag{11}$$

Following Lee,[11] we shall also assume that H_{NL} is invariant under the product $R\times P$ of R-conjugation[4] and the parity transformation. The effects of R conjugation upon baryons are[12]

$$R:\ (B)\to\lambda(-\Sigma^-,\Sigma^0,-\Sigma^+,\Lambda;\Xi^-,-\Xi^0,-n,p), \tag{12}$$

where the phase factor λ is indeterminate; for the moment we choose

$$\lambda=+1. \tag{13}$$

The corresponding effects for pseudoscalar mesons follow from (11).

Now consider the process[13] $\Xi^0\to n+\bar{K}^0$. It follows from invariance under the T-L and $R\times P$ transformations [see Eqs. (10)-(13)] that

$$\langle\Xi^0|n\bar{K}^0\rangle_A=0, \tag{14}$$

where the subscript A denotes the axial-vector interaction.[14] When combined with a prediction of the $\Delta T=\frac{1}{2}$ rule, namely,

$$-\langle\Xi^0|n\bar{K}^0\rangle+\langle\Xi^0|pK^-\rangle=\langle\Xi^-|nK^-\rangle,$$

Eq. (14) implies

$$\langle\Xi^0|pK^-\rangle_A=\langle\Xi^-|nK^-\rangle_A. \tag{15}$$

The T-L transform of (15) is

$$\langle n|\Sigma^+\pi^-\rangle_A=-\langle\Sigma^-|\Xi^0\pi^-\rangle_A. \tag{16}$$

Now from $R\times P$ invariance,

$$\langle\Sigma^-|\Xi^0\pi^-\rangle_A=-\langle\Sigma^+|n\pi^+\rangle_A, \tag{17}$$

and from time-reversal invariance together with the neglect of final-state interactions,[15]

$$\langle n|\Sigma^+\pi^-\rangle=-\langle\Sigma^+|n\pi^+\rangle. \tag{18}$$

Therefore, by substituting (17) and (18) into (16), we obtain

$$\langle\Sigma^+|n\pi^+\rangle_A=0 \tag{19}$$

and hence Eq. (2).

When applied to Σ decay, the $\Delta T=\frac{1}{2}$ rule yields

$$\sqrt{2}\langle\Sigma^+|p\pi^0\rangle=\langle\Sigma^-|n\pi^-\rangle-\langle\Sigma^+|n\pi^+\rangle, \tag{20}$$

$$2\langle\Sigma^0|n\pi^0\rangle=\langle\Sigma^-|n\pi^-\rangle+\langle\Sigma^+|n\pi^+\rangle, \tag{21}$$

for the pionic decay modes, and

$$\langle\Sigma^+|p\eta\rangle=\sqrt{2}\langle\Sigma^0|n\eta\rangle, \tag{22}$$

for the η-decay modes.[13] By taking the T-L transforms of (22) and of the left-hand sides of (20) and (21), we obtain three new equations which can be combined to give

$$-\sqrt{2}\langle p|\Sigma^+\pi^0\rangle+\langle\Sigma^+|n\pi^+\rangle=\langle\Sigma^0-\sqrt{3}\Lambda^0|\Xi^0\pi^0\rangle. \tag{23}$$

Time-reversal invariance and the neglect of final-state interactions[15] enable us to replace the first term in (23) by $\langle\Sigma^+|p\pi^0\rangle$. Applying $R\times P$ invariance to the right-hand side of (23) and using (21) and (22), we obtain a relation

$$\sqrt{3}\langle\Sigma^+|p\pi^0\rangle_V=-\langle\Lambda|p\pi^-\rangle_V,$$

$$\langle\Sigma^+|p\pi^0\rangle_A=\sqrt{3}\langle\Lambda|p\pi^-\rangle_A, \tag{24}$$

which holds for all values of $\langle\Sigma^-|n\pi^-\rangle_V$, including zero. This theory is therefore consistent with Eq. (1), but does not necessarily predict it.

It also follows from time-reversal and $R\times P$ invariance that

$$\langle\Lambda|p\pi^-\rangle_V=-\langle\Xi^-|\Lambda\pi^-\rangle_V,$$

$$\langle\Lambda|p\pi^-\rangle_A=+\langle\Xi^-|\Lambda\pi^-\rangle_A. \tag{25}$$

Taken together, Eqs. (24) and (25) lead to Lee's relation,[11]

$$\sqrt{3}\langle\Sigma^+|p\pi^0\rangle=\langle\Lambda|p\pi^-\rangle+2\langle\Xi^-|\Lambda\pi^-\rangle. \tag{26}$$

The roles of the vector and axial-vector interactions in the above discussion can easily be interchanged by taking the phase factor of (12) to be $\lambda = -1$ instead of $+1$ [see (13)]. This interchange of V and A has no effect upon the predictions of (25), namely,[16,17]

$$\alpha_{\Xi} \approx -1.1\alpha_{\Lambda}; \tag{27}$$

but, because of the Σ - Λ mass difference and the different kinematical factors associated with the vector and axial-vector interactions,[17] it does affect the predictions of (24). As it stands, Eq. (24) predicts

$$\alpha_{\Lambda} \approx -0.7\alpha_{0}, \tag{28}$$

but with the opposite assignment of V,A, it gives

$$\alpha_{\Lambda} \approx -2\alpha_{0}. \tag{29}$$

Since (28) is in better agreement with experiment[1-3] than (29), we tentatively conclude that the V,A assignments of (19), (24), and (25) are correct, and in particular that $\Sigma^+ \to n + \pi^+$ is a pure vector interaction (i.e., s-wave).

In deriving the numerical estimates in (27)-(29), we have interpreted Eqs. (24) and (25) not as precise equalities between matrix elements, but rather as equalities between coupling constants. This interpretation provides one method of allowing for mass differences among members of the baryon octet; it can, however, be generalized in a manner more suitable for the discussion of leptonic decays. We shall treat (24) and (25) as "functional equalities": Each equation

$$\langle A|BC\rangle = \mu\langle A'|B'C'\rangle$$

is assumed to imply that the matrix element $\langle A|BC\rangle$ is the same function, up to a multiplicative constant μ, of the masses, momenta, and spins of A, B, C, as is $\langle A'|B'C'\rangle$ of the masses, momenta, and spins of A', B', C'. It is easy to see that our initial interpretation of (24) and (25) is equivalent to "functional equality" as long as the coupling constants in nonleptonic decay are independent of baryon masses.

Consider now the decays $K \to \mu + \nu$ and $\pi \to \mu + \nu$. If the leptons are unitary singlets and if the leptonic interaction Hamiltonian H_L is T-L invariant, then from (10) and (11)

$$\langle K^+|\mu^+\nu\rangle = \langle \pi^+|\mu^+\nu\rangle. \tag{30}$$

These matrix elements are written in the usual V-A form

$$\langle K^+|\mu^+\nu\rangle = (2K_0)^{-1/2} f_K(-m_K^2) K_\lambda [\bar{u}_\nu \gamma_\lambda (1+\gamma_5) v_\mu],$$
$$\langle \pi^+|\mu^+\nu\rangle = (2\pi_0)^{-1/2} f_\pi(-m_\pi^2) \pi_\lambda [\bar{u}_\nu \gamma_\lambda (1+\gamma_5) v_\mu], \tag{31}$$

where K_λ, π_λ ($\lambda = 0, 1, 2, 3$) are the four-momenta of the K and π mesons, respectively, and m_K, m_π are their masses. If (30) is a "functional equality," then $f_K(-m_K^2)$ is the same function of $-m_K^2$ as is $f_\pi(-m_\pi^2)$ of $-m_\pi^2$:

$$f_K(-m_K^2) = F(-m_K^2),$$
$$f_\pi(-m_\pi^2) = F(-m_\pi^2). \tag{32}$$

The function $F(-m^2)$ must have the dimensions of inverse mass, and so the simplest form it can take is

$$F(-m^2) = g/m, \tag{33}$$

where g is a dimensionless constant. Assuming that (33) represents a good approximation to the true behavior of $F(-m^2)$,[18] we predict the ratio of the rates for $K^+ \to \mu + \nu$ and $\pi^+ \to \mu + \nu$ to be $R \approx 1.4$; this compares favorably with the observed ratio of 1.35.[19] Notice that had (30) been interpreted as an equality of coupling constants, i.e., $f_K(-m_K^2) = f_\pi(-m_\pi^2)$, our predicted ratio would have been wrong by a factor $(m_K/m_\pi)^2 \approx 10$.

It still remains to be seen whether T-L invariance, together with "functional equality," can account for other properties of leptonic decays: for example, the suppression of Λ beta decay compared with neutron beta decay.

We shall show elsewhere[9] that the combination of T-L invariance and $\Delta T = \frac{1}{2}$ requires H_{NL} to be a member of an octet, and H_L to belong to either an octet or a decuplet. Since Lee's theory starts by assuming H_{NL} to belong to an octet, it may well be asked why this theory contains more predictions, namely (19) and (24), than his. The answer can best be seen by constructing the phenomenological form of H_{NL}: In Lee's theory, observable processes will depend on two independent axial-vector coupling constants and three vector ones, whereas in this theory, they depend on one axial-vector and two vector coupling constants. In other words, the assumption that H_{NL} belongs to an octet does not necessarily imply that it is T-L invariant.

It is easy to see that electromagnetic interactions are T-L invariant, and that they will be $R \times P$ invariant if the condition $A_\mu \to -A_\mu$ is imposed on the electromagnetic field. Therefore, if weak interactions are also T-L and $R\times P$ invariant, these symmetries will show up in weak electromagnetic decays $A\to B+\gamma$. One simple consequence is

$$\langle \Sigma^+ | p\gamma\rangle_\pm = \pm\langle \Xi^- | \Sigma^-\gamma\rangle_\pm, \tag{35}$$

where the suffices + and - denote the parity-conserving and parity-nonconserving parts of $\langle A|B\gamma\rangle$, respectively. Equation (35) implies that the asymmetry parameters for $\Sigma^+\to p\gamma$ and $\Xi^-\to\Sigma^-\gamma$ are roughly equal in magnitude but opposite in sign.[20] Notice that the same prediction can be made in the global symmetry scheme.[21]

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[8]The definition of L spin used here differs slightly from that in reference 7, and leads to a change in the sign of L_3.

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[10]The T-L transformation is a special case of a more general transformation discussed by B. d'Espagnat and J. Prentki, Nuovo Cimento 24, 497 (1962). It is also related to one of the Weyl reflections; see A. J. Macfarlane, E. C. G. Sudarshan, and C. Dullemond, Nuovo Cimento 30, 845 (1964).

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[12]Because R conjugation changes the signs of T_3 and Y_T, and because these quantum numbers are associated with two of the infinitesimal generators of SU(3), we require $R^{-1}A_\nu{}^\mu R = -A_\mu{}^\nu$. Notice that the T-L transformation commutes with R.

[13]Although this process is forbidden by energy conservation, terms corresponding to it will generally appear in any Hamiltonian constructed from the product of antibaryon, baryon, and pseudoscalar meson octets.

[14]The space-time structure of the interaction Hamiltonian is taken to be $[\bar\psi_1 i\gamma_\mu(g_V+g_A\gamma_5)\psi_2]\partial_\mu\varphi$. The coupling constants g_V and g_A are real if time reversal invariance holds.

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[16]Note that if we had taken a scalar-pseudoscalar type interaction instead of vector-axial vector, $R\times P$ invariance would imply $\alpha_\Lambda \approx +\alpha_\Xi$.

[17]The numerical estimates of Eqs. (27), (28), and (29) are based on the formulas of S. A. Bludman, Phys. Rev. 115, 468 (1959), and on the value of γ_Λ given in reference 3.

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