the effective mass,[8] which is about 0.7. Since the density surface thickness in the model is sharper than that of an actual nucleus, we cannot expect better agreement. Calculations based upon a finite square-well potential (now in progress) will yield a more gradual falloff in nuclear density, and are expected to reduce the effective mass required to fit the data.

The identification of the energy gap as a function of surface-to-volume ratio indicates that the gap should also be an increasing function of nuclear deformation as Griffin[9] deduced from the anisotropies of fission fragments. If we assume that the gap in spherical nuclei is given by

$$\Delta_{\rm sph} = cA^{-1/2}, \tag{5}$$

then for a spheroidal nucleus characterized by a deformation parameter β, we have

$$\Delta \approx cA^{-1/2}[1+(3/4\pi)\beta^2], \tag{6}$$

with $c \approx 12.8$ MeV. This estimate, of course, ignores specific shell structure effects.

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K_1-K_2 MASS DIFFERENCE AND A POSSIBLE DI-PION RESONANCE AT ABOUT 400 MeV[†]

K. Nishijima
Department of Physics, University of Illinois, Urbana, Illinois
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Recently Brown and Singer[1] found a good agreement with all the available data concerning energy spectra and branching ratios in the three-pion decays of the η and K mesons by assuming the existence of an $I=J=0$ di-pion resonance of mass about 400 MeV and of full width 75 to 100 MeV reported earlier by Samios et al.[2] The purpose of the present paper is to point out that the presence of such a resonance would make the K_1 meson heavier than the K_2 meson by the right order of magnitude.[3]

The dispersion-theoretical formulation of this problem was first presented by Barger and Kazes,[4] and we shall briefly recapitulate their approach. Within the framework of the I_3-conserving strong interactions the rest masses of the K_1 and K_2 mesons are degenerate and their mass difference is generated by weak interactions. In order to study this problem let us introduce the proper self-energy operator or the polarization operator $\Pi(s)$ of the K^0 meson due to weak interactions, where s is the square of the virtual K-meson mass w. Then the self-energy of the K^0 meson due to weak interactions is given by[5]

$$\delta\mathfrak{M}^2 = \Pi(M^2), \tag{1}$$

where M is the degenerate K-meson mass and $\delta\mathfrak{M}^2$ is the complex self-energy given more explicitly by

$$\delta\mathfrak{M}^2 = 2M\delta\mathfrak{M}, \tag{2}$$

and

$$\delta\mathfrak{M} = \delta M - \tfrac{1}{2}i\Gamma. \tag{3}$$

δM and Γ are the mass shift and full width of the K meson. These formulas give a physical interpretation of the quantity $\Pi(s)$.

Next we shall introduce an unsubtracted dispersion relation for $\Pi(s)$, i.e.,

$$\mathrm{Re}\Pi(s) = \frac{P}{\pi}\int_0^\infty ds' \frac{\mathrm{Im}\Pi(s')}{s'-s}. \tag{4}$$

For the present purpose it is more convenient to consider a function $m(w)$ defined by

$$m(w) = \Pi(w^2)/2w; \tag{5}$$

then on the mass shell $w = M$ we find

$$m(M) = \delta\mathfrak{M} = \delta M - \tfrac{1}{2}i\Gamma. \tag{6}$$

If we write

$$\mathrm{Im}\,m(w) = -\tfrac{1}{2}\Gamma(w), \tag{7}$$

$\Gamma(w)$ expresses the full width of the virtual K meson of mass w so that it is positive-definite, and the dispersion relation (4) reads

$$w\,\mathrm{Re}\,m(w) = \frac{\mathrm{P}}{\pi}\int_0^\infty \frac{w'^2\Gamma(w')dw'}{w^2 - w'^2}. \tag{8}$$

In particular, on the mass shell $w = M$, we obtain

$$\delta M = \frac{\mathrm{P}}{\pi}\int_0^\infty \frac{w'^2\Gamma(w')dw'}{M(M^2 - w'^2)}. \tag{9}$$

If we distinguish the K_1 and K_2 mesons by subscripts 1 and 2, we have the following inequality on the mass shell:

$$\Gamma_1(M) = \hbar/\tau_1 \gg \Gamma_2(M) = \hbar/\tau_2. \tag{10}$$

It would not be unreasonable to assume that the above inequality persists over a wider energy region, i.e.,

$$\Gamma_1(w) \gg \Gamma_2(w). \tag{11}$$

The dispersion relation (9), then, leads to another inequality

$$|\delta M_1| \gg |\delta M_2|, \tag{12}$$

so that the $K_1 - K_2$ mass difference is given approximately by

$$\Delta M = \delta M_1 - \delta M_2 \approx \delta M_1. \tag{13}$$

Furthermore, the K_1 meson is known to decay predominantly into two pions, so that we can approximate the full width $\Gamma_1(w)$ by the corresponding partial width $\Gamma_{2\pi}(w)$. Then the lower limit of the dispersion integral (9) is given by 2μ, since a virtual K_1 meson cannot decay into two pions unless $w > 2\mu$. Thus we arrive at an approximate formula

$$\Delta M \approx \frac{\mathrm{P}}{\pi}\int_{2\mu}^\infty \frac{w^2\Gamma_{2\pi}(w)dw}{M(M^2 - w^2)}. \tag{14}$$

Since $\Gamma_{2\pi}(w)$ is positive-definite, this formula shows that the domain $2\mu < w < M$ gives a positive contribution to ΔM, whereas the domain $w > M$ gives a negative contribution. The sign of the mass difference ΔM is determined, therefore, by the competition between the two energy regions, below and above M. It is clear, therefore, that for a stable spin-zero meson there is only a negative contribution so that the self-energy is always negative. This theorem was first given by Lehmann.[6]

To be more specific, it is necessary to know the final-state interactions in this two-pion channel. If we take the selection rule $|\Delta I| = \frac{1}{2}$ for granted, the quantum numbers $I = 0$, $J = 0$ are uniquely assigned to the final two-pion state, and the energy dependence of $\Gamma_{2\pi}(w)$ is determined by the phase shift for pion-pion scattering in the $I = J = 0$ state by solving an integral equation of the Omnês type. This can be done in the same way as done for the calculation of the electromagnetic form factors of the nucleons.

Now coming back to the original problem, it is intuitively clear that the assumed 400-MeV resonance slightly below the K-meson mass would enhance the positive contribution to ΔM. In order to evaluate its effect on ΔM, we have assumed an S-wave resonance formula[7] for $\Gamma_{2\pi}(w)$ instead of solving the Omnês equation,[4] i.e.,

$$\Gamma_{2\pi}(w) = \frac{(s - 4\mu^2)^{1/2}}{s}\,\frac{c}{(s - s_R)^2 + \gamma^2(s - 4\mu^2)/s}, \tag{15}$$

where the parameters s_R and γ are related to the di-pion mass M_R and full width Γ_R by

$$s_R = M_R^2, \quad \gamma[1 - (2\mu/M_R)^2]^{1/2} = M_R\Gamma_R. \tag{16}$$

The normalization constant c is determined subject to the condition

$$\Gamma_{2\pi}(M) = \hbar/\tau_1. \tag{17}$$

Insertion of Eq. (15) into the dispersion relation (14) gives the mass difference ΔM. Using $M = 500$ MeV, $\mu = 140$ MeV, and $M_R = 400$ MeV, we find

$$\Delta M = 1.4\hbar/\tau_1, \quad \text{for } \Gamma_R = 70 \text{ MeV};$$
$$= 0.7\hbar/\tau_1, \quad \text{for } \Gamma_R = 100 \text{ MeV}. \tag{18}$$

These values are consistent with the experimental ones.[3] The basic assumptions made in this calculation are the existence of the $I = J = 0$ di-pion resonance at about 400 MeV and the rapid decrease of the width $\Gamma_{2\pi}(w)$ as is manifestly seen in the resonance formula (15). Should this prediction fail, that would be an indication either against the assumed resonance or for a slow decrease of the width $\Gamma_{2\pi}$ at high energies, and the 400-MeV enhancement would be due to final state interactions.

The resonance formula (15) corresponds to an effective-range formula

$$[(s-4\mu^2)/s]^{1/2}\cot\delta = (M_R{}^2 - s)/M_R\Gamma_R. \quad (19)$$

This formula gives a rather small scattering length, i.e., half the pion Compton wavelength or less, and cannot account for the ABC anomaly[8] near the threshold. Inclusion of such an effect is expected to increase ΔM further, but it is less important than the resonance under consideration because of the smaller kinematical factor and of the larger dispersion demoninator. It is instructive, however, to recall the qualitative conclusion of Barger and Kazes[4] that negative and small scattering lengths will give a negative ΔM, and a positive large scattering length can yield positive mass difference if the phase shift becomes as large as 0.5-1.8 for large momenta. The existence of the ABC anomaly seems to rule out the former possibility.[9]

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We thank R. N. Hall for pointing out that Eq. (2) should read

$$v = (1 + Re^{-2\alpha l})^2/(1 - Re^{-2\alpha l})^2.$$

This does not affect the conclusions of the paper.