and by Lee[10] on the basis of R invariance. It is interesting that the d/f ratio for $\bar{B}_8 B_8 PS_8$ coupling (d_0/f_0) has been found[11] to be nearly equal to the d/f ratio for $\bar{B}_8 B_8{}^* PS_8$ coupling (d_1/f_1).

The parity-violating parts of the amplitudes of the γ decays of the strange baryons can be written as

$$\{a(\bar{B}_2{}^3 O_\alpha B_1{}^1 - \bar{B}_1{}^1 O_\alpha B_3{}^2) + b(\bar{B}_1{}^3 O_\alpha B_2{}^1 - \bar{B}_1{}^2 O_\alpha B_3{}^1) + c(\bar{B}_2{}^1 O_\alpha B_1{}^3 - \bar{B}_3{}^1 O_\alpha B_1{}^2) + d(\bar{B}_1{}^1 O_\alpha B_2{}^3 - \bar{B}_3{}^2 O_\alpha B_1{}^1)\} A_\alpha, \qquad (2)$$

making use of the tensor notation,[12] where[13] $\bar{B}O_\alpha B = \bar{B}\gamma_5 \sigma_{\alpha\beta} B \partial_\beta$ and $A_1{}^1$ should be read as $A_1{}^1 - \frac{1}{2}(A_2{}^2 + A_3{}^3)$. From the invariance of the interaction[5] under the exchange of the indices 2 and 3, we find $a = -d$ and $b = c = 0$. Since the configurations $\bar{\Xi}^- \Sigma^-$ $(= \bar{B}_1{}^3 B_2{}^1)$ and $\bar{\Sigma}^+ p$ $(= \bar{B}_2{}^1 B_1{}^3)$ are not found in the first and fourth terms in the bracket of (2), the parity-violating amplitudes of the γ decays, $\Sigma^+ \to p + \gamma$ and $\Xi^- \to \Sigma^- + \gamma$, are zero in the limit of unitary symmetry and their asymmetry parameters are zero. This will be checked by experiments in the future.

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[1]N. Cabibbo, Phys. Rev. Letters 12, 62 (1964).

[2]M. Gell-Mann, Phys. Rev. Letters 12, 155 (1964).

[3]M. Gell-Mann, California Institute of Technology Synchrotron Laboratory Report No. CTSL-20, 1961 (unpublished); Phys. Rev. 125, 1067 (1962).

[4]Y. Ne'eman, Nucl. Phys. 26, 222 (1961).

[5]Assumption (ii) can be replaced by the following: (ii') The interaction is invariant under the exchange of the indices 2 and 3, when matrices are written in terms of tensor notations. [See S. Okubo, Progr. Theoret. Phys. (Kyoto) 27, 947 (1962).] This corresponds to the assumption (a) of Cabibbo (reference 1).

[6]G. Feldman, P. T. Mathews, and A. Salam, Phys. Rev. 121, 302 (1961).

[7]For example, the amplitude for "$\bar{B} + B \to$ vector meson $\to$ p-v-spurion and pion" process is proportional to $\bar{\psi}(p')\gamma_\mu \psi(p)(p' - p)_\mu$ and vanishes in the limit of the unitary symmetry.

[8]For example, see S. Glashow and A. H. Rosenfeld, Phys. Rev. Letters 10, 184 (1963).

[9]H. Sugawara (to be published).

[10]B. Lee (to be published).

[11]For example, see reference 8 and Y. Hara, Phys. Rev. (to be published).

[12]S. Okubo, Progr. Theoret. Phys. (Kyoto) 27, 947 (1962).

[13]G. Calcuci and G. Furlan, Nuovo Cimento 21, 679 (1961).

UNITARY SYMMETRY MODEL OF WEAK INTERACTIONS*

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In this note we present a model of weak interactions based on unitary symmetry of strong interactions[1] and on the V-A theory of weak interactions. The basic assumption made here is that the weak interaction currents of strongly interacting particles, J_μ, have positive RC conjugation parity,

$$RCJ_\mu(RC)^{-1} = J_\mu, \qquad (1)$$

where R and C are the R-conjugation and charge-conjugation operator, respectively.[2]

We first consider the representation of J_μ as a sum of baryon-antibaryon products in the octet model of unitary symmetry in which baryons (mesons) are members of the "8" representation of the SU(3) group. Therefore, J_μ is expressed by a direct product of two octets which has the well-known decomposition into the direct sum "1" + "8_S" + "27" + "8_a" + "10" + "10*". Under the interchange of two octets in the product, the first three representations ("1", "8_S", "27") are symmetric while the last three ("8_a", "10", "10*") are antisymmetric. If we divide J_μ into a vector (V) and an axial-vector part (A), we expect from the assumption (1) that the V current and the A current have different SU(3) coupling schemes because of their different properties under charge conjugation. RC conjugation gives the interchange of two octets in the product with a minus sign for the V and with a plus sign for the A. The permissible candidates for representation of J_μ are therefore "8_a", "10", and "10*" for the V part and "8_S" and "27"[3] for the A part.

The Goldberger-Treiman (G-T) relation[4] for the π meson is well satisfied by experiments. If we apply the same derivation of G-T relation as that of Gell-Mann,[5] we arrive at a conclusion that the axial-vector current should be octet only and the type of SU3 coupling should be the same as the strong $\pi\bar{N}N$ coupling, which is almost pure "8_S" due to the 33 resonance analysis.[6] Since the "8" part of the A current should be "8_S" in our model, the G-T relation is consistent with it and suggests that the coupling of "27" current is very weak. For this reason we do not include the "27" current in what follows.[7] Since the "8" current contains only the $\Delta S = \Delta Q$ strangeness changing current, the A part should satisfy the $\Delta S = \Delta Q$ rule.[8]

There are some experimental indications[9] that there exists a $\Delta S = -\Delta Q$ V current. If we take these experiments for granted, we must include a small amount of "10" and "10*" current into our scheme.

Our model based on assumption (1) is, therefore, to take the "8_a" together with small amounts of "10" and "10*" currents[10] as the V current, and to take the "8_S" current as the A current. If we disregard the "10" and "10*" current, we obtain the same current as that of Cabibbo,[11] which beautifully explains almost all leptonic decays of strange particles. We shall therefore not discuss these decays here.

To discuss the nonleptonic decay of strange particles, let us assume that these representations of currents appear in the weak-interaction Hamiltonian in the current × current form[12] (or equivalently in the form of an interaction with intermediate vector bosons) to produce the nonleptonic decay interaction. And if we include the "10" and "10*" current into our scheme, we must also assume at least two uncoupled currents (or equivalently two charged intermediate vector bosons) to avoid the $\Delta S = 2$ nonleptonic transition. Since the currents satisfy (1), we can derive

$$RPH_w(RP)^{-1} = H_w \qquad (2)$$

for the nonleptonic decay interaction H_w, by using CP invariance which we always assume. Therefore, the parity-conserving part and -nonconserving part have different R-parity. It is worthwhile to notice that if we assume a derivative type of effective Hamiltonian for nonleptonic decay and R invariance of strong interactions, Eq. (2) gives $\alpha_\Lambda = -\alpha_\Xi$.[13]

As has been suggested,[14] a natural extension of the $|\Delta I| = \frac{1}{2}$ rule for nonleptonic decays would be to take $H_w(\Delta S = 1)$ to be a member of an "8" representation of SU(3). The nonleptonic decay Hamiltonian is given by $H_w = H_w(\Delta S = 1) + H_w^*(\Delta S = 1)$, the star denoting the Hermitean conjugate. The Hamiltonian $H_w(\Delta S = 1)$ induced by the current × current form, however, is not always a member of "8" representation. There are two points of view on the $|\Delta I| = \frac{1}{2}$ rule. One is that this rule has its origin in the dynamics,[15] while the other is to guarantee it by introducing a sufficient number of intermediate vector bosons to enable $H_w(\Delta S = 1)$ to be an irreducible representation of SU(3).[16] Let us assume here that this rule can be explained in either way so that effectively $H_w(\Delta S = 1)$ is a member of an "8" representation. Neglecting the contribution from the "10" and "10*" current to $H_w(\Delta S = 1)$,[17] we obtain $UH_w(\Delta S = 1)U^{-1} = H_w^*(\Delta S = 1)$, so that

$$UH_w U^{-1} = H_w, \qquad (3)$$

where U is the operator associated with a unitary transformation $x_1 \to x_1$, $x_2 \to x_3$, $x_3 \to -x_2$ in SU(3) space.

Although R invariance of strong interactions is not exact, the analysis of the 33 resonance[6] suggests that it should be satisfied approximately. In what follows, we shall assume strong interactions to be R invariant and analyze the nonleptonic decay of hyperons. As mentioned previously we shall take the derivative type of effective Hamiltonian $i\bar{\psi}\gamma_\mu(a + b\gamma_5)\psi\partial_\mu\varphi$ to insure that $\alpha_\Lambda = -\alpha_\Xi$. In general, there exist 8 independent SU(3) coupling schemes. With the aid of Okubo's notation,[18] they are easily constructed as follows:

$$\bar{N}_2{}^a N_b{}^3 f_a{}^b, \;\; \bar{N}_a{}^3 N_2{}^b f_b{}^a, \;\; \bar{N}_a{}^b N_b{}^3 f_2{}^a,$$

$$\bar{N}_a{}^3 N_2{}^b f_b{}^a, \;\; \bar{N}_a{}^b N_2{}^3 f_b{}^a,$$

$$\bar{N}_2{}^3 N_a{}^b f_b{}^a, \;\; \bar{N}_2{}^a N_a{}^b f_b{}^3, \;\; \bar{N}_a{}^b N_2{}^a f_b{}^3 + \text{H.c.},$$

in which, for example, $\bar{N}_2{}^a N_a{}^b f_b{}^3$ is an abbreviation of

$$\bar{N}_2{}^a i\gamma_\mu(a_7 + b_7\gamma_5)N_a{}^b \partial_\mu f_b{}^3.$$

Each coupling scheme has a set of two real parameters[19] which we label in order: $a_i, b_i, i = 1, \cdots, 8$. The last two couplings do not contribute to the interesting decay modes. In fact, if we write the effective decay Hamiltonian in the above form for $\Lambda^0 \to p\pi^-$, $\Sigma^- \to p\pi^0$, $\Sigma^+ \to n\pi^+$, $\Sigma^- \to n\pi^-$, and $\Xi^- \to \Lambda^0\pi^-$ by specifying (a_Λ, b_Λ), (a_0, b_0), (a_+, b_+), (a_-, b_-), and (a_Ξ, b_Ξ), respectively, we obtain

$$a_\Lambda = 6^{-1/2}(-a_1 - a_3 + 2a_4),$$
$$a_0 = 2^{-1/2}(a_1 - a_3),$$
$$a_+ = a_1 + a_5,$$
$$a_- = a_3 + a_5,$$
$$a_\Xi = 6^{-1/2}(-a_2 + 2a_3 - a_4); \quad (4)$$

and the same equations for the b's. Equation (3) requires that

$$a_3 = a_7, \quad a_4 = a_8, \quad a_5 = a_6;$$
$$b_3 = b_7, \quad b_4 = b_8, \quad b_5 = b_6; \quad (5)$$

while Eq. (2) requires

$$a_1 = -a_2, \quad a_3 = -a_4, \quad a_5 = -a_6;$$
$$b_1 = b_2, \quad b_3 = b_4, \quad b_5 = b_6. \quad (6)$$

From these two restrictions we obtain

$$a_5 = a_6 = 0$$

and

$$a_\Lambda = -a_\Xi = -6^{-1/2}(a_1 + 3a_3), \quad b_\Lambda = b_\Xi = 6^{-1/2}(-b_1 + b_3),$$
$$a_0 = 2^{-1/2}(a_1 - a_3), \quad b_0 = 2^{-1/2}(b_1 - b_3),$$
$$a_+ = a_1, \quad b_+ = (b_1 + b_5),$$
$$a_- = a_3, \quad b_- = (b_3 + b_5).$$

Experimentally the Gell-Mann–Rosenfeld triangle[20] for Σ decay is along the S and P coordinate axis, which means that either a_1 or a_3 must be negligible. We therefore have two cases:

(1) $a_3 \approx 0$, $a_\Lambda = -a_\Xi$, $b_\Lambda = b_\Xi$,

$$\sqrt{3}\,(a_0, b_0) + (a_\Lambda, b_\Lambda) + 2(-a_\Xi, b_\Xi) = 0;$$

(2) $a_1 \approx 0$, $a_\Lambda = -a_\Xi$, $b_\Lambda = b_\Xi$,

$$\sqrt{3}\,(a_0, b_0) + (a_\Lambda, b_\Lambda) + 2(a_\Xi, b_\Xi) = 0.$$

The first solution gives $\alpha_\Lambda \approx \alpha_{\Sigma^0}$, which contradicts the experimental results. On the other hand, the second solution satisfies Lee's triangle relation[21] in our (a, b) plane which is essentially equivalent to his plane. But we have obtained it here from different assumptions. Furthermore, we predict that Σ^- decay is almost pure S wave (not P) decay.

We wish to thank E. McCliment for his reading of the manuscript. After this note was written, the author became aware of a revised version of a paper by B. W. Lee, in which he arrived at the same conclusion for hyperon decays.

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[1]M. Gell-Mann, Phys. Rev. 125, 1067 (1962); Y. Ne'eman, Nucl. Phys. 26, 222 (1961).

[2]RC conjugation used here is the same as A conjugation used by other authors for η-meson decay. In our model we must take RC-parity of pseudoscalar mesons to be even. J. B. Bronzan and F. Low (to be published).

[3]Of course, the inclusion of "1" is not profitable because it is neutral.

[4]M. L. Goldberger and S. B. Treiman, Phys. Rev. 110, 1178 (1958).

[5]M. Gell-Mann, Phys. Rev. 125, 1067 (1962).

[6]A. W. Martin and K. C. Wali, Phys. Rev. 130, 2455 (1963); R. Cutkosky, Ann. Phys. (N.Y.) 23, 415 (1963).

[7]See also reference 16.

[8]This result is consistent with the K_{e4} decay experiments.

[9]Robert P. Ely et al., Phys. Rev. Letters 8, 132 (1962); Gideon Alexander, Silverio P. Almeida, and Frank S. Crawford, Jr., Phys. Rev. Letters 9, 69 (1962).

[10]We must admit that the omission of "10" and "10*" currents is simpler theoretically and elegant and directly relates to the conserved vector-current theory, but we include them for insurance sake. See D. Horn and Y. Ne'eman, Nuovo Cimento 29, 760 (1963).

[11]N. Cabibbo, Phys. Rev. Letters 10, 531 (1963).

[12]R. P. Feynman and M. Gell-Mann, Phys. Rev. 109, 193 (1958).

[13]Compare with J. J. Sakurai, Phys. Rev. Letters 7, 426 (1962).

[14]For example, Benjamin W. Lee (to be published).

[15]For example, E. R. McCliment and K. Nishijima, Phys. Rev. 4, 1970 (1962).

[16]A simple model of this type is to assume that the Hamiltonian belongs to the "3" representation which is accomplished by assigning an SU(3) representation to the intermediate vector bosons. The number of complex bosons is nothing but the dimension of the representation to which these bosons belong. "8"⊗"3" contains "3" in its direct sum so at least 3 complex

bosons are necessary if we neglect the "10" and "10*" current. We need at least 6 complex bosons for the "10" and "10*" current and 15 for "27". Notice that "8"⊗"15*" does not contain "3", while "8"⊗"6*" does.

[17]This contribution is supposed to be small, and in some models there is no contribution from either "10" or "10*" currents to $H_w(\Delta S=1)$.

[18]With the aid of Okubo's notation, this transformation can be written as $UT_2{}^3U^{-1}=T_3{}^2$. This transformation also corresponds to a rotation through 180° around the Y axis in the U-spin space. S. Okubo, Progr. Theoret. Phys. (Kyoto) 27, 949 (1962).

[19]We shall neglect the phase shift of the final states of the decay. The CP invariance assures the reality of these parameters.

[20]M. L. Gell-Mann and A. H. Rosenfeld, Ann. Rev. Nucl. Sci. 7, 407 (1957).

[21]It should be noticed that the assumptions used in Lee's paper are invariance of H_w under RU as well as the "8" SU(3) tensor nature of H_w.