In this plot a pure helicon wave would be represented by the straight dashed line $u/v_S = v_H/v_S$. We see that the lower branches of the dispersion relation represent excitations which are helicon waves for $v_H/v_S \ll 1$ and which resemble sound waves for $v_H/v_S \gg 1$. The upper branches represent excitations which change from sound waves for $v_H/v_S \ll 1$ to helicon waves for $v_H/v_S \gg 1$. The experimental data points contain an adjustable parameter in the following sense: The magnitude of the wave number, k, corresponding to a given peak is not accurately known. However, in a series of peaks in the beat pattern two successive peaks differ in wave number by $2\pi/d$, where d is the thickness of the specimen. Since k decreases with increasing H_0, we arbitrarily assign a k value to one, and only one, peak in each series of peaks. In Fig. 2 we have arbitrarily assigned a k value to that peak in each series of peaks which appears at the highest magnetic field. In each case the k value was chosen to make the data point lie near the corresponding theoretical curve. The adjustable parameter enters essentially as a translation of each series of data points along the ordinate in Fig. 2. Consequently, the data points are to be compared only with the slope and curvature of the theoretical curves.

From a study of the 20- and 30-Mc/sec data discussed here and additional data obtained at frequencies ranging from 5 Mc/sec to 50 Mc/sec, we conclude that the coupling of helicon waves and transverse sound waves exists in potassium, and that there is good agreement of the theory with experiment.

It is a pleasure to acknowledge the technical assistance of G. Adams. Miss B. B. Cetlin computed the theoretical curves.

[1]P. Aigrain, Proceedings of the International Conference on Semiconductor Physics, Prague, 1960 (Czechoslovakian Academy of Sciences, Prague, 1961), p. 224; R. Bowers, C. Legendy, and F. Rose, Phys. Rev. Letters 7, 339 (1961).

[2]G. Akramov, Fiz. Tverd. Tela 5, 1310 (1963) [translation: Soviet Phys.—Solid State 5, 955 (1963)].

[3]T. Kjeldaas, Jr., Bull. Am. Phys. Soc. 8, 428, 446 (1963); and private communication (to be published).

[4]D. N. Langenberg and J. Bok, Phys. Rev. Letters 11, 549 (1963); J. J. Quinn and S. Rodriguez, Phys. Rev. Letters 11, 552 (1963); Phys. Rev. 133, A1589 (1964).

[5]T. Kjeldaas, Jr., Phys. Rev. 113, 1473 (1959); M. H. Cohen, M. J. Harrison, and W. A. Harrison, Phys. Rev. 117, 937 (1960).

[6]At 4.2°K the approximate value of $\omega_c\tau$ where ω_c is the electron cyclotron frequency ranged from 70 to 220 at the magnetic fields employed in the experiments.

[7]Previous experimental arrangements used on metals have allowed only observation of standing-wave resonances corresponding to an odd number of half-wavelengths contained in the specimen [F. E. Rose, M. T. Taylor, and R. Bowers, Phys. Rev. 127, 1122 (1962); R. G. Chambers and B. K. Jones, Proc. Roy. Soc. (London) A270, 417 (1962)]. Our traveling-wave experimental technique is readily extended to much higher frequencies than can be employed using the standing-wave technique. The cyclotron damping effects first observed by Taylor, Merrill, and Bowers [M. T. Taylor, J. R. Merrill, and R. Bowers, Phys. Letters 6, 159 (1963)] are more clearly resolved using the traveling-wave technique at high frequencies and will be discussed in a future publication.

[8]The calculated phase velocities of the two shear waves which propagate in the [110] direction in potassium are 0.67×10^5 cm/sec and 1.70×10^5 cm/sec. Here we have used the elastic constants listed by Mason [W. P. Mason, Physical Acoustics and the Properties of Solids (D. Van Nostrand Company, Inc., Princeton, New Jersey, 1948)] and the lattice constant measured by Barrett [C. S. Barrett, Acta Cryst. 9, 671 (1956)] at 5°K.

NONEXTREMAL ORBITS IN MAGNETOACOUSTIC GEOMETRIC RESONANCES*

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In a recent Letter, Langenberg, Quinn, and Rodriguez[1] discussed the possibility of measurement of nonextremal Fermi-surface orbits by observing giant quantum oscillations in the attenuation of sound, for sound waves parallel to the direction of the magnetic field. In this Letter we wish to point out that these same nonextremal orbits (as well as other nonextremal orbits) may also be observed in geometric resonance experiments, and furthermore, the conditions for observation of these orbits are much easier to attain in the case of geometric resonance than in the quantum oscillation case. Since geometric resonances measure linear dimensions of the electron orbit in real space, and giant quantum oscillations measure cross-sectional areas of the Fermi surface, it will be possible to make both these complementary measurements upon electrons of nonextremal orbits. We shall calculate the period of oscillation for geometric resonances for a general ellipsoidal energy surface for arbitrary angle between magnetic field and direction of propagation of sound. As a special case, the geometric resonance effect for fields parallel to the sound waves will be found. This effect was discussed by Quinn[2] in a recent Letter, using a quantum mechanical formalism, which is actually unnecessary for this semiclassical effect; for the special case considered by Quinn, our results agree with his.

For spherical energy surfaces, there are no parallel-field geometric resonances. Therefore it is necessary to consider ellipsoidal energy surfaces. This has the added advantage of clearly showing the linear dimensions which are experimentally measured in geometric resonances, which have never been previously demonstrated theoretically; and, of course, ellipsoidal energy surfaces are quite realistic for a number of materials, e.g., semimetals.

Suppose that a portion of the energy surface is given by $2mE_F = \vec{p}\cdot\alpha\cdot\vec{p}$. Then it is easy to show that the Hamiltonian, the Boltzmann equation, and the equations of motion of the electrons have the same form as for spherical energy surfaces, if the following transformation is made:

$$\begin{aligned}&\vec{p} = \alpha^{-1/2}\cdot\vec{p}', \quad \vec{v} = \alpha^{1/2}\cdot\vec{v}', \quad \vec{r} = \alpha^{1/2}\cdot\vec{r}',\\ &\vec{E} = \alpha^{1/2}\cdot E', \quad \vec{H} = |\alpha|^{-1/2}\alpha^{-1/2}\cdot\vec{H}',\\ &\vec{q} = \alpha^{-1/2}\cdot\vec{q}', \quad \vec{u} = \alpha^{-1/2}\cdot\vec{u}',\end{aligned} \tag{1}$$

where $\vec{p}$, $\vec{v}$, and $\vec{r}$ are the momentum, velocity, and coordinate of the electron; $\vec{E}$ and $\vec{H}$ the electric and magnetic fields; $\vec{q}$ the sound-wave vector, u the velocity of ions in the lattice; and $|\alpha|$ is the determinant of α.[3] The solution for the primed quantities is the same as for electrons with spherical energy surfaces; the inverse transformation gives the solution for electrons with ellipsoidal energy surfaces.

Thus, if the inverse transformation is applied to the solution of the equation of motion for an electron with spherical energy surfaces in a constant magnetic field, one finds the following solution for ellipsoidal energy surfaces:

$$\vec{r}(t) = r(0) + m(\hat{H}\cdot\alpha^{-1}\cdot\hat{H})^{-1}\Big\{\hat{H}(\hat{H}\cdot\vec{p})t + \left[\frac{\vec{p}}{\omega_c}(\cos\omega_c{}'t - 1) + \frac{H\times m\vec{v}}{\omega_c{}'}\sin\omega_c{}'t\right]\times\alpha^{-1}\cdot\hat{H}\Big\}, \tag{2}$$

where $\vec{p} = \vec{p}(0)$, $\vec{v} = \vec{v}(0)$, $\omega_c = eH/mc$, $\omega_c{}' = \omega_c[|\alpha|\hat{H}\cdot\alpha^{-1}\cdot\hat{H}]^{1/2}$, and $\hat{H}$ is a unit vector in the direction of the magnetic field. Thus, the motion of the electron is an ellipse in the plane perpendicular to $\alpha^{-1}\hat{H}$; and at the same time the electron moves with uniform motion in the direction of the magnetic field. The physical explanation[4] of geometric resonances for the field perpendicular to the direction of the sound-wave vector shows that there is a maximum in the attenuation whenever the extremal dimension of the orbit of the electron in real space in the direction of the sound-wave vector is equal to an integral multiple of the wave length. This is also the case for arbitrary angle between sound wave and magnetic field, provided that the sound-wave vector has a nonzero projection upon the plane of the orbit, i.e., $\vec{q}$ is not parallel to $\alpha^{-1}\hat{H}$. Thus, only for spherical energy surfaces will no oscillations be observed for $\vec{q}\parallel\hat{H}$; in fact, geometric resonances have been observed for this geometry.[5,6]

The exact analysis of the oscillation of the attenuation may be made by transforming the spherical case to the ellipsoidal case. This analysis will also enable us to identify the orbits which contribute to the attentuation, that is, the $(\vec{p}\cdot\hat{H})$ values of the orbits observed.

According to Cohen, Harrison, and Harrison,[4] the conductivity tensor for spherical energy surfaces has the following form:

$$\sigma = \sum_{\nu=-\infty}^{\infty}\int\frac{[\vec{O}J_\nu(\xi)][\vec{O}^\dagger J_\nu(\xi)]}{1 + i[\nu\omega_c + (\vec{v}_F\cdot\hat{H})(\vec{q}\cdot\hat{H}) - \omega]\tau}d\Omega, \tag{3}$$

where $\vec{O}$ is a differential vector operator, $J_\nu(\xi)$ the Bessel function of νth order, $d\Omega$ an element of solid angle of $\vec{v}_F$, the Fermi velocity, and

$$\begin{aligned}\xi &= |\vec{q}\times\hat{H}||\vec{v}_F\times\hat{H}|/\omega_c\\ &= [\{\vec{q}^2\vec{H}^2 - (\vec{q}\cdot\vec{H})^2\}\{\vec{v}_F{}^2\vec{H}^2 - (\vec{v}_F\cdot\vec{H})^2\}]^{1/2}/(\omega_cH^2).\end{aligned} \tag{4}$$

For $(\vec{q}\cdot\hat{H})v_F\tau \equiv (\vec{q}\cdot\hat{H})l \gg 1$, the function $\{1 + i[\nu\omega_c + (\vec{v}_F\cdot\hat{H})(\vec{q}\cdot\hat{H}) - \omega]\tau\}^{-1}$ which appears in (3) effectively differs from zero only when $\cos\theta \equiv (\hat{v}_F\cdot\hat{H})$ (where $\hat{v}_F$ is a unit vector) is in a small neighborhood of

$$\cos\theta^* = (\omega - \nu\omega_c)/v_F(\vec{q}\cdot\hat{H}) = (v_s/v_F)[1 - (\nu\omega_c/\omega)](\hat{q}\cdot\hat{H})^{-1}, \qquad (5)$$

where v_S is the velocity of sound and $\hat{q}$ is a unit vector. The half-width of this neighborhood is $[(\vec{q}\cdot\hat{H})l]^{-1}$. Values of ν for which $|(\omega - \nu\omega_c)/v_F(\vec{q}\cdot\hat{H})| > 1$ give negligible contributions to the summation. In the limit $(\vec{q}\cdot\hat{H})l \gg 1$, the integration in (3) may be performed, to a good approximation, by evaluating the Bessel functions for $\cos\theta = \cos\theta^*$. Thus,

$$\sigma \approx \sum_\nu [\vec{O}J_\nu(\xi_\nu^*)][\vec{O}^\dagger J_\nu(\xi_\nu^*)] \times \int \{1 + i[\nu\omega_c + (\vec{v}_F\cdot H)(\vec{q}\cdot\hat{H}) - \omega]\tau\}^{-1} d\Omega, \qquad (6)$$

where

$$\xi_\nu^* = \omega_c H^{-2}\left\{[\vec{q}^2\vec{H}^2 - (\vec{q}\cdot\vec{H})^2] \times \left[\vec{v}_F^{\,2}\,\vec{H}^2 - \frac{(\omega - \nu\omega_c)^2\vec{H}^4}{(\vec{q}\cdot\vec{H})^2}\right]\right\}^{1/2} \qquad (7)$$

and the summation is over all ν such that ξ_ν^* is real.

We now transform to the ellipsoidal case by using the transformation inverse to (1). Then ξ_ν^* becomes

$$\xi_\nu^* = \frac{q}{\omega_c}\left\{\left(\frac{2E_F}{m}\right)\frac{(\hat{q}\cdot\alpha\cdot\hat{q})(\hat{H}\cdot\alpha^{-1}\cdot\hat{H}) - (\hat{q}\cdot\hat{H})^2}{|\alpha|\,(\hat{H}\cdot\alpha^{-1}\cdot\hat{H})^2} \times \left[1 - \frac{m}{2E_F}\left(\frac{\omega - \nu\omega_c{}'}{\vec{q}\cdot\hat{H}}\right)^2(\hat{H}\cdot\alpha^{-1}\cdot\hat{H})\right]\right\}^{1/2}. \qquad (8)$$

This shows that the conductivity, and hence, the attenuation, is a sum of oscillatory functions of the arguments ξ_ν^*. Each period is due to a different orbit, i.e., a different value of $(\vec{p}_F\cdot\hat{H})$. These $(\vec{p}_F\cdot\hat{H})$ values are found by transforming (5)

$$(\vec{p}_F\cdot\hat{H}) = [m/(\vec{q}\cdot\hat{H})](\hat{H}\cdot\alpha^{-1}\cdot\hat{H})(\omega - \nu\omega_c{}'). \qquad (9)$$

For $\nu = 0$, $(\hat{p}_F\cdot\hat{H}) = (v_S/v_F)(\hat{q}\cdot\hat{H})^{-1}(\hat{H}\cdot\alpha^{-1}\cdot\hat{H})$. Therefore, except for $\vec{q}$ almost perpendicular to $\vec{H}$, $(\hat{p}_F\cdot\hat{H}) \approx 0$, and the extremal orbit will be observed. For $\nu \neq 0$, nonextremal orbits will be observed. These orbits are identical to those discussed in reference 1 for $\vec{q} \parallel \vec{H}$; but in addition to these orbits others will be observed for other values of angle. The reason that the identical orbits contribute in the quantum oscillation case is that these effects are all due to the same energy denominators, which correspond to Doppler-shifted cyclotron resonance conditions.[7]

It may be shown that $\xi_\nu^* = \frac{1}{2}q(R_{\max} - R_{\min})$, where $R_{\max}$ is equal to the maximum value of the oscillatory part of $\vec{r}(t)$ in the direction of the wave vector, and $R_{\min}$ the corresponding minimum value, for that orbit whose $(\vec{p}\cdot\hat{H})$ value is given by (9). The condition for a maximum in attenuation is $\xi_\nu^* = n\pi$, where n is an integer. This means that the condition for a maximum in geometric resonances is the well-known "matching" condition:

$$D = R_{\max} - R_{\min} = n\lambda. \qquad (10)$$

The conditions for observation of these oscillations are $\omega_c\tau > 1$; and also ξ_ν^* must be of order 1 (but greater than 1). This latter condition requires $\omega > (v_S/v_F)\omega_c$. This condition is also required in the quantum oscillation case, but in order for quantum oscillations to be observed, ω_c must be much greater than for geometric resonances. Hence, significantly lower frequencies are satisfactory for the observation of nonextremal orbits for the case considered here than for the quantum oscillation case.

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[1]D. N. Langenberg, J. J. Quinn, and S. Rodriguez, Phys. Rev. Letters 12, 104 (1964).

[2]J. J. Quinn, Phys. Rev. Letters 11, 316 (1963).

[3]Although there is freedom of choice in determining the square root of a matrix, this freedom is of no consequence here, because $\alpha^{1/2}$ never appears in a final result, but only in intermediate calculations. The analysis given above is correct only for symmetric $\alpha^{1/2}$; but this condition is guaranteed by symmetric α.

[4]For a discussion of the physical explanation, see M. H. Cohen, M. J. Harrison, and W. A. Harrison, Phys. Rev. 117, 937 (1960).

[5]L. McKinnon, M. T. Taylor, and M. R. Daniel, Phil. Mag. 7, 523 (1962); M. R. Daniel and L. McKinnon, Phil. Mag. 8, 537 (1963).

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[7]T. Kjeldaas, Phys. Rev. 113, 1473 (1959).