three figures, possibly more.

The ground-state ^3P wave function has been reported previously[5]; wave functions for all excited states of the configurations $1s^2 2s^2 2p^n (3s, 3d)$ ($n = 0, 1, 2, 3$) of O and N have been computed and are being prepared for publication.

In view of Garstang's careful and extensive discussion of this OI transition, we would like to note some additional calculations we have performed. We computed the σ^2 value using our ^3P ground-state function and our ^5S wave function for the $3s$ electron. The value 0.1052 obtained confirms the value 0.11 obtained by Garstang using the Hartree, Hartree, and Swirles function and Nicklas and Treanor's function, respectively, for these two states.[2] The results show that, as one might expect, the value of σ^2 is relatively insensitive to the difference between the ^5S and ^3S wave functions. In addition, for the $2p - 3s$ transition (neglecting coupling) we have obtained the value $\sigma^2 = 0.15$ from the Hartree-Fock-Slater (HFS) wave functions computed by one of us.[6] Our experience in comparing values of σ^2 obtained from HFS wave functions with more accurate values (theoretical and experimental) indicates that for transitions involving excitation of a $2p$ electron they are somewhat too large. The ratio of positive to negative contributions to the integral for the analytic SCF functions is 10.1, indicating that while some cancellation does occur in the integration, it is not severe.

Finally, we would like to point out that the value of σ^2 for the NI $2p^2 3s(^4P) - 2p^3(^4S)$, reported in an earlier paper,[7] was listed incorrectly as 0.12. The correct value is $\sigma^2 = 0.131$ and this value leads to the f number of 0.10 as quoted in that paper.

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ENERGY GAP IN FINITE NUCLEI CALCULATED IN THE SLAB MODEL*

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Impressive success has been achieved in understanding the nuclear energy gap and associated phenomena in terms of the Belyaev[1] adaptation of the superconducting theory to finite nuclei. The calculations, however, generally employ simplified internucleonic potentials, with a few adjustable parameters empirically fitted to a large quantity of nuclear data. On the other hand, calculations for infinite nuclear matter, utilizing "realistic" internucleonic potentials, have shown that the energy gap is very small or nonexistent.[2-4] In order to bridge the difference between these two approaches, we have performed calculations for the energy gap of a finite system with internucleonic potentials which reproduce singlet, s-wave scattering data.

To simulate a finite nucleus, we have used a slab model in which the system is of infinite extent in the x and y directions, but is confined to a region of thickness L in the z direction. The basis functions for the system are taken to be the product of plane waves in the x and y directions, and standing waves (vanishing outside the slab) in the z direction. The energy gap is found from the BCS[5] integral equation

$$\Delta_{\vec{k}} = -\tfrac{1}{2}\sum_{\vec{k}'} \frac{\{\vec{k}-\vec{k}\,|V|\vec{k}'-\vec{k}'\}\,\Delta_{\vec{k}'}}{(\hat{\epsilon}_{\vec{k}'}^{\,2} + \Delta_{\vec{k}'}^{\,2})^{1/2}}, \qquad (1)$$

where $|\vec{k}\}$ denotes a state described above and $|-\vec{k}\}$ is the time-reversed state of $|\vec{k}\}$. In this work, $\hat{\epsilon}_{\vec{k}}$ is represented by the effective mass approximation

$$\hat{\epsilon}_{\vec{k}} = (\hbar^2/2m^*)(k^2 - k_F^{\,2}). \qquad (2)$$

The $\sum_{\vec{k}'}$ in Eq. (1) is properly replaced by in-

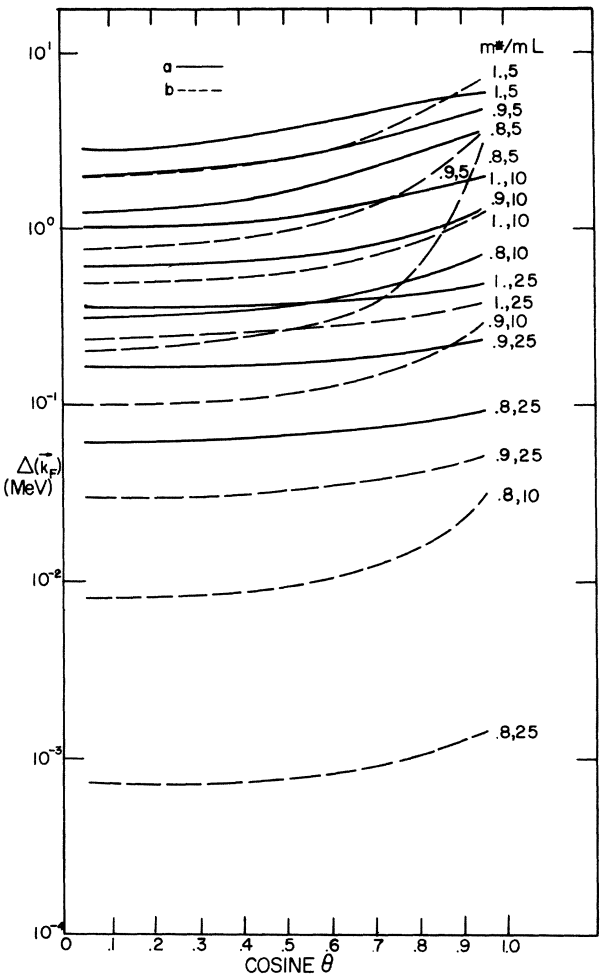

FIG. 1. Anisotropy of the gap parameter $\Delta(\vec{k}_F)$ as function of $\cos\theta = (\vec{k}_F)_z/k_F$ for potentials (a) and (b) and various combinations of $(m^*/m, L)$.

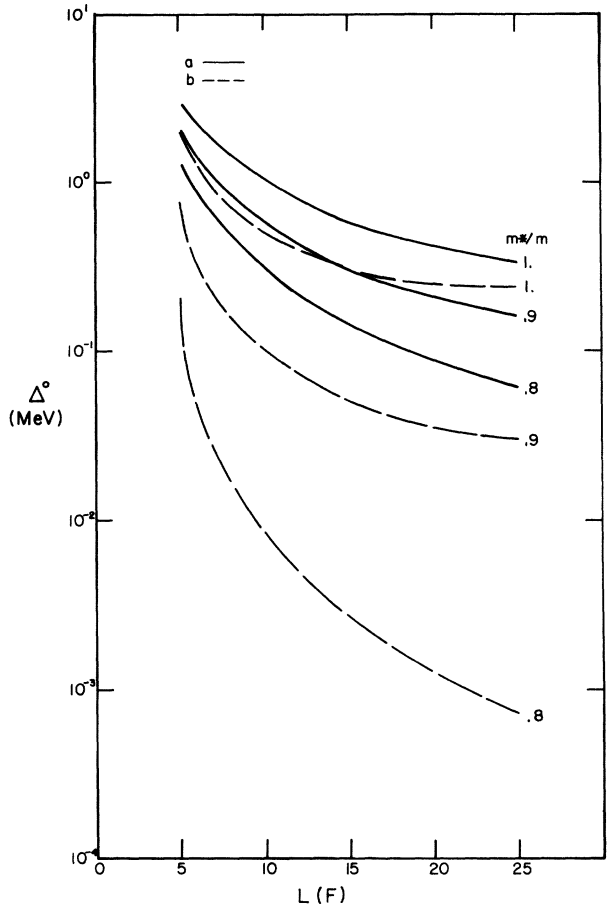

FIG. 2. Minimum value of $\Delta(\vec{k}_F)$ as a function of L for various values of m^*/m. The solid curves are for potential (a), the dashed curves for potential (b). The minimum is achieved at $\cos\theta = 0$, and the value at minimum is denoted by Δ^0.

tegrals over k_x and k_y, and a discrete sum over k_z. We have replaced the sum over k_z by an integral as well, and in doing so have explicitly eliminated the possibility of oscillations in the solutions to Eq. (1). Such oscillations have been found by Blatt and Thompson,[6] who studied a superconducting slab with a BCS-type ("shell") interaction. However, our results correspond roughly with the mean of the oscillatory solution. Although the nuclear pairing energy data (see Fig. 3) exhibit oscillations suggestive of those found by Blatt and Thompson, there is no hope of reproducing them in detail in any slab model since they arise from details of the spherical or spheriodal shell structure.

The two-body interactions employed reproduce the singlet s-wave scattering data (scattering length, effective range, and 310-MeV phase shift) as described in Kennedy, Wilets, and Henley.[3] Briefly, the interactions are represented by back-to-back Yamaguchi potentials,

$$\langle \tfrac{1}{2}\vec{K}+\vec{k}, \tfrac{1}{2}\vec{K}-\vec{k} \mid V \mid \tfrac{1}{2}\vec{K}+\vec{k}', \tfrac{1}{2}\vec{K}-\vec{k}' \rangle = \sum_{i=1}^{2} \lambda_i w_i(k) w_i(k'), \qquad (3)$$

where $w_i(k) = (k^2+\beta_i^2)^{-1}$ and $|\vec{k}\rangle$ is a plane-wave state. The parameters for the two distinct potentials used here, denoted by (a) and (b), are[3]

	β_1 (F^{-1})	λ_1 (F^3)	β_2 (F^{-1})	λ_2 (F^3)
(a)	3	7.655	1.766	1.731
(b)	6	123.1	1.620	0.9530.

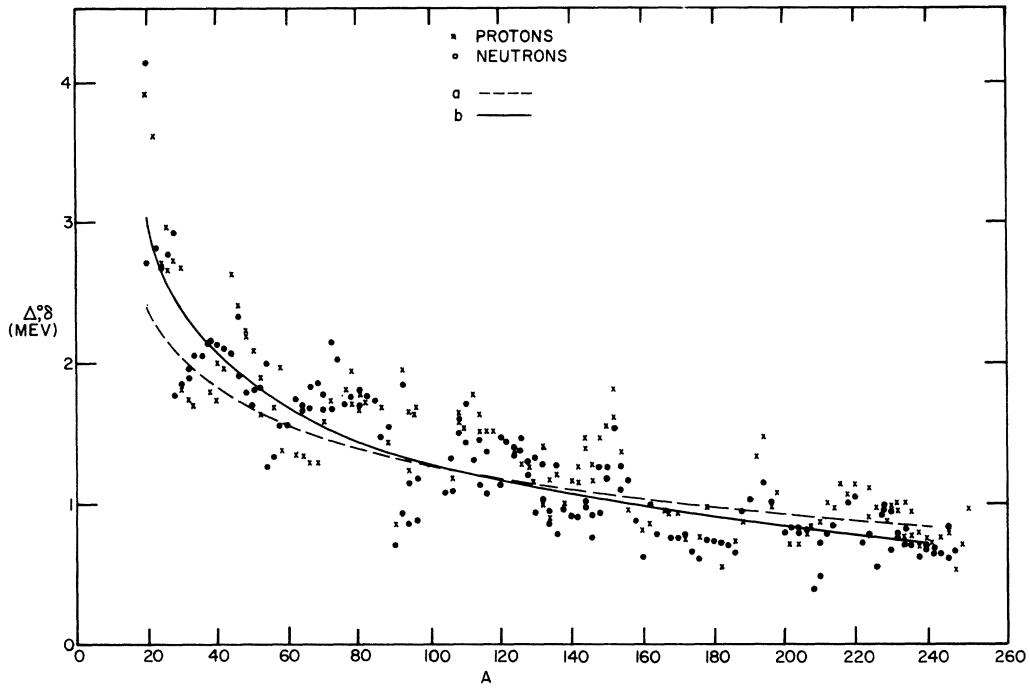

FIG. 3. A comparison of theoretical and semiempirical values of the energy gap as a function of A. The crosses and circles are proton and neutron pairing energies, δ, deduced by Nemirovsky and Adamchuk from even-odd mass differences. The solid and dashed lines are calculated values of Δ^0 (see Fig. 2) fitted to the data by adjusting the single parameter m^*/m for potentials (a) and (b); the infinite-matter values of Δ^0 for these parameters are 0.016 and 0.10 MeV, respectively. The parameters Δ^0 and δ are not identical, but are related by $\delta \approx (\hat{\epsilon}^2+\Delta^{02})^{1/2}$, where $\hat{\epsilon}$ is the normal single-particle energy of the odd nucleon measured relative to the Fermi energy. Since $\hat{\epsilon}$ is roughly half the mean level spacing, the distinction between δ and Δ^0 is usually small.

Between the two, (b) corresponds to the harder repulsive core. The numerical method of solution of Eq. (1) will be reported in a later publication together with a generalization of the method to a finite square well in the z direction.

The results displayed here are for $k_F = 1.4$ F^{-1}, which characterizes the central density of the nucleus. Because of the slab geometry, the gap [which is interpreted as $2\Delta(\vec{k}_F)$] is anisotropic. Figure 1 shows the gap as a function of $(\vec{k}_F)_z/k_F$ $(=\cos\theta)$ for various combinations of the effective mass and slab thickness for both potentials. For $m^*/m = 1$ and $L = 25$ F, the value of the gap at $\cos\theta = 0$ is roughly twice that for infinite nuclear matter. It is obvious that the gap is a very sensitive function of the effective mass and slab thickness. Figure 2 is simply the data of Fig. 1 replotted for $\cos\theta = 0$, the angle at which the gap is smallest.

In order to obtain a correspondence between the slab and a finite nucleus, we fix the slab thickness such that the same effective surface-to-volume ratio obtains. Thus the slab has a surface $2S$ $(S \to \infty)$ and a volume SL^*. Here L^* $(<L)$ is the effective thickness of the slab and is defined so that $SL^* \times$(central density) gives the number of particles. For $k_F L \gg 1$, we have $L^* = L - 3\pi/4k_F$; this is the relationship used below. Then, representing the nucleus by a uniformly dense sphere, we set

$$(3/2)L^* = R = r_0 A^{1/3}, \tag{4}$$

with $r_0 = 1.09$ F.

In Fig. 3 are shown the gap parameters for neutrons and protons deduced from even-odd mass differences by Nemirovsky and Adamchuk.[7] The two curves superimposed on these data represent calculations using potentials (a) and (b) and a single value of m^* fitted to the data for each of the two potentials. [Curve (b) is almost identical with the least-squares empirical fit of Nemirovsky and Adamchuk: $\Delta = 16.2A^{-0.561}$ MeV.] The values of m^*/m required to fit the data of Fig. 3 are 0.806 for potential (a) and 0.950 for potential (b). These values of m^*/m are somewhat larger than the usually accepted value for

the effective mass,[8] which is about 0.7. Since the density surface thickness in the model is sharper than that of an actual nucleus, we cannot expect better agreement. Calculations based upon a finite square-well potential (now in progress) will yield a more gradual falloff in nuclear density, and are expected to reduce the effective mass required to fit the data.

The identification of the energy gap as a function of surface-to-volume ratio indicates that the gap should also be an increasing function of nuclear deformation as Griffin[9] deduced from the anisotropies of fission fragments. If we assume that the gap in spherical nuclei is given by

$$\Delta_{\text{sph}} = cA^{-1/2}, \tag{5}$$

then for a spheroidal nucleus characterized by a deformation parameter β, we have

$$\Delta \approx cA^{-1/2}[1 + (3/4\pi)\beta^2], \tag{6}$$

with $c \approx 12.8$ MeV. This estimate, of course, ignores specific shell structure effects.

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$K_1 - K_2$ MASS DIFFERENCE AND A POSSIBLE DI-PION RESONANCE AT ABOUT 400 MeV†

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Recently Brown and Singer[1] found a good agreement with all the available data concerning energy spectra and branching ratios in the three-pion decays of the η and K mesons by assuming the existence of an $I = J = 0$ di-pion resonance of mass about 400 MeV and of full width 75 to 100 MeV reported earlier by Samios et al.[2] The purpose of the present paper is to point out that the presence of such a resonance would make the K_1 meson heavier than the K_2 meson by the right order of magnitude.[3]

The dispersion-theoretical formulation of this problem was first presented by Barger and Kazes,[4] and we shall briefly recapitulate their approach. Within the framework of the I_3-conserving strong interactions the rest masses of the K_1 and K_2 mesons are degenerate and their mass difference is generated by weak interactions. In order to study this problem let us introduce the proper self-energy operator or the polarization operator $\Pi(s)$ of the K^0 meson due to weak interactions, where s is the square of the virtual K-meson mass w. Then the self-energy of the K^0 meson due to weak interactions is given by[5]

$$\delta\mathfrak{M}^2 = \Pi(M^2), \tag{1}$$

where M is the degenerate K-meson mass and $\delta\mathfrak{M}^2$ is the complex self-energy given more explicitly by

$$\delta\mathfrak{M}^2 = 2M\,\delta\mathfrak{M}, \tag{2}$$

and

$$\delta\mathfrak{M} = \delta M - \tfrac{1}{2}i\Gamma. \tag{3}$$

δM and Γ are the mass shift and full width of the K meson. These formulas give a physical interpretation of the quantity $\Pi(s)$.

Next we shall introduce an unsubtracted dispersion relation for $\Pi(s)$, i.e.,

$$\mathrm{Re}\Pi(s) = \frac{\mathrm{P}}{\pi}\int_0^\infty ds' \frac{\mathrm{Im}\Pi(s')}{s' - s}. \tag{4}$$