It is easy to see that one such function is

$$\tilde{\Psi} = -(1 - e^{-d/r}) \exp(-x/\lambda_d), \tag{8}$$

which possesses the added convenience that (6) may be evaluated in closed form. The result is[2]

$$U(\epsilon) = -\theta\{1 - 2\epsilon K_2(2\epsilon^{1/2})\}, \tag{9}$$

from which we obtain the equation of state

$$p/n = \theta\{1 - \tfrac{1}{6}\epsilon - \tfrac{1}{12}\epsilon^2 \ln\epsilon + O(\epsilon^2)\}. \tag{10}$$

We have shown that the equation of state for an electron gas in a positive background follows straightforwardly from the truncated Liouville hierarchy. The results obtained agree (to the extent to which three-particle correlations may be neglected) with the results of Abe,[3] Friedman,[4] and Guernsey[5] obtained by other methods. We are presently considering the effects of higher order correlations.

The author is indebted to M. Kruskal and I. Bernstein for helpful discussions, and to E. Frieman for suggesting the problem.

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*This work was accomplished under the auspices of the U. S. Atomic Energy Commission.

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HEAT CAPACITY OF THE EXCHANGE BATH IN SOLID ^{3}He

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(Received 25 February 1964)

In solid ^{3}He the atomic wave functions overlap because of the finite mass of the ^{3}He atoms ("zero-point motion"). The requirement for antisymmetry of the total wave function under exchange of any two atoms then leads to a nuclear spin-dependent energy usually discussed in terms of the two-body exchange interaction J defined by the phenomenological exchange Hamiltonian

$$\mathcal{H}_{ex} \equiv \tfrac{1}{2}\hbar J \sum_{i,j}' \vec{I}^i \cdot \vec{I}^j \tag{1}$$

(where $\sum'$ runs only over the z nearest neighbors of each atom). Values of J in both bcc and hcp (3.5 Å$<a<$3.65 Å), (3.38 Å$<a<$3.63 Å) ^{3}He have recently been published as a result of T_1 and T_2 measurements and of a consistent theoretical interpretation.[1,2] In this density range, $J/2\pi \leq 2.7$ Mc/sec. Several workers have attempted to measure the heat capacity associated with this measured exchange interaction in solid ^{3}He, but have thus far been able only to set upper limits on the heat capacity measured calorimetrically[3,4] and a lower limit in a spin-echo experiment.[1] This Letter presents the first actual measurements of the exchange heat capacity C_{ex}. In terms of the Zeeman heat capacity C_Z, we find the exchange heat capacity

$$C_{ex}^{obs} = 350 C_Z,$$

for a sample and magnetic field for which the value calculated from the measured T_1 and T_2, by means of the relation

$$C_{ex} = \tfrac{3}{32} NZ(\hbar J)^2/kT^2, \tag{2}$$

gives

$$C_{ex} = 0.084 C_Z,$$

a discrepancy of a factor 4000. [We also confirm in detail the exchange-lattice relaxation mechanism (Raman effect) which was reported previously,[1] but which has remained a subject of controversy.]

In Fig. 1, the Zeeman, exchange, and lattice reservoirs are coupled as shown. The present experiment was done at a field such that the Zeeman-exchange cross-relaxation time $T_1 = T_{ZE} = 0.15$ sec, while the bath temperature T_L was low enough that the diffusion-induced Zeeman-to-lattice relaxation time $T_{ZL} \to \infty$. For the measurement of the exchange-bath heat capacity, we worked at $T_L \approx 0.45$°K, so that the exchange

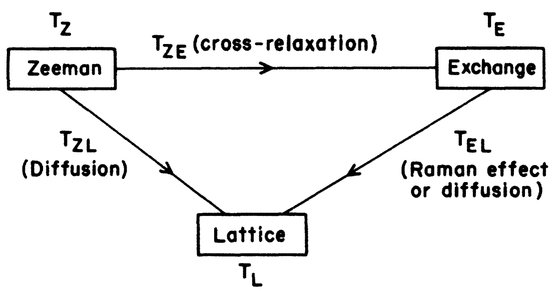

FIG. 1. The Zeeman, exchange, and lattice thermal reservoirs are shown each with its temperature T_Z, T_E, or T_L. The heat-transfer time constants T_{ZE}, T_{ZL}, and T_{EL} are defined in terms of the energy transfer rate and the heat capacity of that bath, the initial letter of which comes first in the subscript.

bath was almost decoupled from the lattice ($T_{EL} \geq 100$ minutes). The heat capacity was measured by feeding known amounts of energy ΔQ into the exchange bath and by measuring the accompanying temperature rise ΔT (using the Zeeman system as a thermometer). Then

$$C_{ex}^{obs} = \Delta Q/\Delta T.$$

The energy ΔQ is delivered to the exchange bath by applying one pulse (or a sequence of pulses) of resonant radio-frequency field of amplitude $2H_1$ and duration t_p, producing a nutation of the spins of angle

$$\varphi = \gamma H_1 t_p. \tag{3}$$

The energy of a system of N spins ($I=\frac{1}{2}$) in a magnetic field H_0 is

$$E_Z = -\tfrac{1}{4}N(\hbar\gamma H_0)^2/kT, \tag{4}$$

and the Zeeman heat capacity is thus

$$C_Z = dE_Z/dT = \tfrac{1}{4}N(\hbar\gamma H_0)^2/kT^2. \tag{5}$$

The application of a "90° pulse" to the spin system will destroy the net magnetization and will therefore transfer to the combined Zeeman and exchange systems a total energy

$$\Delta Q = +\tfrac{1}{4}N(\hbar\gamma H_0)^2/kT. \tag{6}$$

The exchange-bath heat capacity C_E^{obs} can then be calculated in terms of C_Z [known from Eq. (5)] as

$$C_Z/C_E^{obs} = T_f/T_i - 1 = h_i/h_f - 1. \tag{7}$$

T_f and T_i are the (common) Zeeman and exchange temperatures following and preceding the 90° pulse. h_i and h_f are the echo amplitudes (proportional to the net magnetization and thus $h_f \propto 1/T_f$) before and after the 90° pulse. Should it require n_e 90° pulses ($n_e \gg 1$) to reduce h_f to h_i/e, we find

$$C_Z/C_E \approx 1/n_e. \tag{8}$$

In the (attainable) approximation that $T_{EL} \rightarrow \infty$, each successive 90° pulse applied to the Zeeman system reduces the pulse amplitude h_i to

$$h_f/h_i = e^{-\mu} \text{ with } e^{-\mu} = C_E/(C_E + C_Z). \tag{9}$$

Thus n pulses will give

$$h_n = h_i e^{-\mu n} \tag{10}$$

or

$$1/T_n = (1/T_i)e^{-\mu n}, \tag{11}$$

an exponential decay of inverse spin temperature with number of 90° pulses applied.

With the aid of Eq. (10), we have determined μ from the set of runs plotted in Fig. 2 and made on hcp ^{3}He solid of $V = 19.32$ cm^3/mole. For each run the sample was first allowed to come to thermal equilibrium at 1.25°K and then cooled rapidly to $T_L = 0.45$°K in order to have T_{EL} long, so that the energy transferred from the Zeeman system to the exchange bath should not leak rapidly to the lattice. In Fig. 2 the echo amplitude is plotted vs the number of applied pulses at a pulse rate of 0.97 sec^{-1}. [This rate was chosen to have the interpulse spacing $\gg T_1$, in order to present a relaxed spin system for each new 90° pulse. The pulse rate is significant because, in truth, $T_{EL}^{-1} \neq 0$. Equations (10) and (11) are readily extended to include the effects of T_{EL}, for the case of $\mu \ll 1$, and the curves in Fig. 2 are all calculated with $T_{EL} \approx 100$ min and $n/t = 0.98$ pulses per sec.] We note that the smallest amplitude plotted in Fig. 2 corresponds to a spin temperature $T_Z = T_E \approx 6$°K.

In the sample cell used in this experiment, the ^{3}He lies outside a cylindrical rf coil and inside a copper sleeve; both surfaces are suddenly heated by the rf pulse. To be convinced that we really heat the spin system and that we do not just heat the lattice by thermal conduction from the rf coil, allowing rapid relaxation of the spins to the lattice at high temperature due to the short T_{EL} at high temperatures, note that the 180° pulses heat the exchange bath roughly twice as

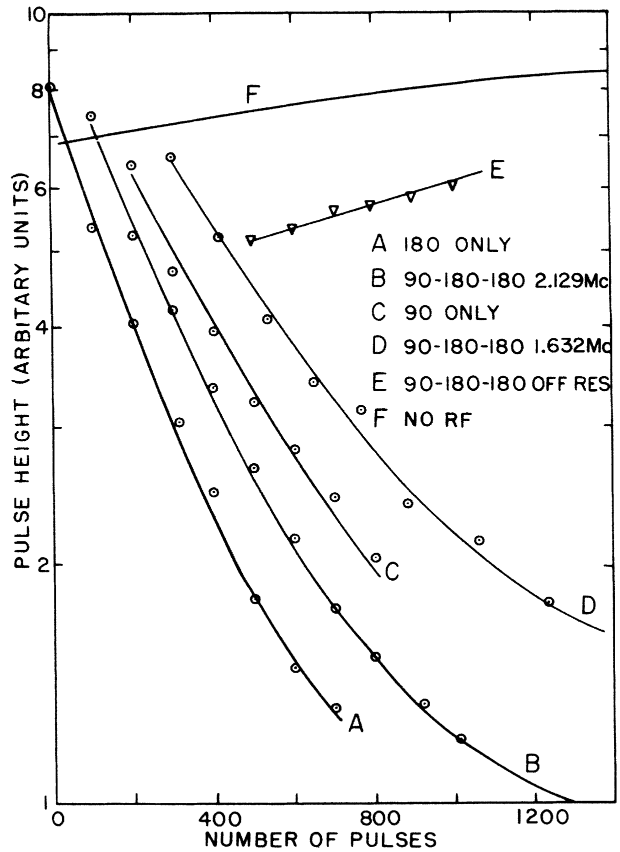

FIG. 2. The sample magnetic moment (echo amplitude) is plotted as a function of n, the number of "90° pulses" which the sample has sustained. The decrease in amplitude with n is thus a measure of the heating of the exchange-bath-plus-Zeeman system by known amounts of rf energy. Each set of data points is accompanied by a curve calculated from Eqs. (6) and (9), with $C_E^{obs}/C_Z \cong 350$ (for $f = 2.129$ Mc/sec), and $T_{EL} = 100$ minutes, and successive curves are shifted by 100. As explained in the text, the curves for magnet detuned show no heating of the exchange bath, proving that heat enters the exchange bath in this experiment through the Zeeman system, and not from the lattice. "NO pulses" refers to one sampling sequence approximately every two minutes.

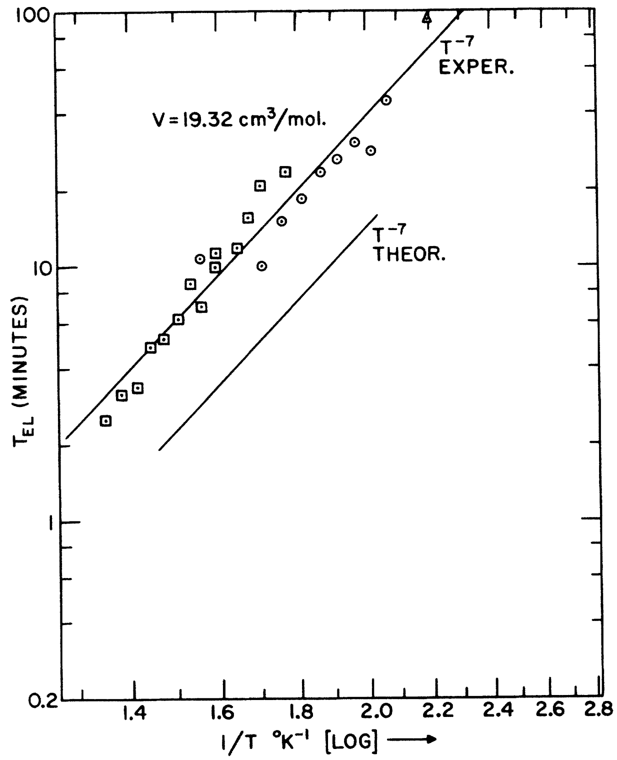

FIG. 3. The exchange-lattice relaxation time T_{EL} for the present sample of hcp ^{3}He with $V = 19.32$ cm^3 mole^{-1} is plotted for comparison with the theoretical result of reference 1 for two-phonon relaxation [as summarized in our Eq. (12)]. The agreement is remarkably good.

fast as the 90° pulses. Note also that detuning the magnet (making $\varphi = 0$ which will not change the pulsed heating of the lattice) does eliminate the warming of the exchange bath, which is thus shown to be receiving energy only in known amounts from the Zeeman system according to Eq. (6). Shown also in Fig. 2 is the warming of the exchange bath by 90° pulses at a dc field $\sim 0.77\,H_0$. The decrease of echo amplitude per 90° pulse is correspondingly smaller, as expected from Eq. (6). The reproducibility of these measurements is good, as evidenced by runs on successive days. The curves show also that n pulse triplets of the form 90°-180°-180° heat the exchange bath exactly as do n 90° pulses. This is exactly what is to be expected from rf heating of the exchange bath, and provides conclusive evidence that thermal heating of lattice is negligible, since the sensible heat due to dissipation in the triplet is five times that of a single 90° pulse.

This experiment may be summarized as having measured a "different exchange interaction" $J^{h.c.}$ ["J as measured by heat capacity" and defined by Eq. (2)] much larger than that measured in the same system and at the same time from T_1 and T_2 measurements. At 19.32 cm^3/mole, $J/2\pi = 0.29$ Mc/sec, while $J^{h.c.}/2\pi = 16$ Mc/sec. The exchange-lattice time constant T_{EL} for the sample (100 minutes at $T_L = 0.45$°K) is only a factor 3 larger than that predicted by Eq. (21)

of reference 1, which can be written for hcp ^{3}He

$$T_{EL}^{-1} = 1.55\times10^{-35}(d^2J/da^2)^2T^7/\theta_D^{10}\ \mathrm{sec}^{-1}, \quad (12)$$

(in which a is the nearest-neighbor distance and θ_D the Debye temperature of the solid), and our measurements accurately confirm the T^7 temperature dependence,[5] as is indicated in Fig. 3.

We are grateful to A. G. Redfield for his critical reading of the manuscript and to J. Krueger for measuring photographs.

Note added in proof. —After submission of this note we received a preprint by Beal, Gifford, Hatton, Richards, and Richards describing similar work on several ^{3}He-^{4}He mixtures of low ^{4}He concentration. Our results were obtained on ^{3}He containing 0.95% ^{4}He.

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[5]One might question the consistency between our results for $T_{EL} \propto T^{-7}$ as published here in Fig. 3 for hcp ^{3}He and the essentially Curie-law susceptibility for bcc ^{3}He published in Figs. 2 and 3 of A. L. Thomson, H. Meyer, and P. N. Dheer, Phys. Rev. 132, 1455 (1963). These latter indicate "prompt" cooling of the spin system to ~0.06°K in ^{3}He of molar volume 22.40 and 24.3 cm^3/mole. There is no discrepancy—first, the T_{EL} of Eq. (12) varies $\propto V^{-60}$ (locally); and, second, at low temperatures the Raman relaxation is slow compared with the single-phonon relaxation process calculated in R. B. Griffiths, Phys. Rev. 124, 1023 (1961). To illustrate these points we have calculated T_{EL} for bcc ^{3}He of $V = 24.3$ cm^3/mole. We find from the theory, $T_{EL}^{-1} = (2.0\times10^4T^7 + 0.5T)$ sec^{-1}. At 0.06°K, this material thus has $T_{EL} \sim 30$ sec, and about 5 msec at 0.5°K.

INTERACTION BETWEEN HELICON WAVES AND SOUND WAVES IN POTASSIUM

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(Received 24 February 1964)

The possibility of observing an interaction between helicon waves[1] and transverse sound waves in a metal has been recently considered by several authors.[2-4] The coupling is expected to occur when the helicon-wave phase velocity is nearly equal to the sound-wave phase velocity. We report here the results of experiments designed to measure such coupling in pure, single-crystal potassium. As shown in Figs. 1 and 2, we obtained clear evidence of the existence of the interaction and found good agreement between theory and experiment.

The dispersion relation for coupled helicon waves and sound waves in an elastically isotropic medium has been derived in references 2 through 4. The analysis used there is similar to that previously employed for study of acoustic attenuation in metals.[5] In order to interpret the results of our experiment, it was necessary to derive a more general dispersion relation applicable to the elastically anisotropic case of wave propagation along a twofold axis. We take as our model an infinite medium having n free electrons per cm^3 and a positive (ion) background of charge density ne and mass density nM. Take the magnetic field $\vec{H}_0$ to be in the z direction and assume that all field quantities vary as $e^{i(kz-\omega t)}$. Denote right- and left-circularly polarized field quantities by $E^\pm = E_x \pm iE_y$, $J^\pm = J_x \pm iJ_y$, etc. The equation of motion of the positive background becomes

$$\omega^2\vec{S}^\pm = v_1^2k^2\tfrac{1}{2}(\vec{S}^+ + \vec{S}^-) \pm v_2^2k^2(\vec{S}^+ - \vec{S}^-) - (e/M)\vec{E}^\pm - (e/Mc)\dot{\vec{S}}^\pm \times \vec{H}_0 + (e/M\sigma_0)(\vec{J}^\pm + \vec{j}^\pm), \quad (1)$$

where $\vec{S}^\pm$ is the displacement of an ion from its equilibrium position, τ is the electron relaxation time, $\vec{J}^\pm = ne\dot{\vec{S}}^\pm$ is the current density due to motion of the ions, $\vec{j}^\pm$ is the electron current density, $\vec{E}^\pm$ is the internal electric field, and $\sigma_0 = ne^2\tau/m$; v_1 and v_2 are the phase velocities of the two independent shear waves when they propagate along the twofold direction in the absence of a magnetic field. Equation (1) exhibits