[2]F. Zachariasen and C. Zemach, Phys. Rev. 128, 849 (1962).

[3]A. W. Martin and K. C. Wali, Phys. Rev. 130, 2455 (1962).

[4]M. Baker, Ann. Phys. (N.Y.) 4, 271 (1958). We refer to the first-order approximation of Baker's method.

[5]Physical arguments lead Zachariasen and Zemach (reference 2) to suggest that the subtraction point S_0 should be chosen to correspond roughly to the start of the left-hand cut. In the example shown in Fig. 1, this could correspond to an $S_0 = -25$.

[6]As discussed by Zachariasen and Zemach (reference 2) there are a number of ways in which one can arbitrarily force the solution to be symmetric. Also see reference 3.

[7]G. F. Chew and S. C. Frautschi, Phys. Rev. 124, 264 (1961).

[8]J. L. Uretsky, Phys. Rev. 123, 1459 (1961); D. Y. Wong, Phys. Rev. 126, 1220 (1960).

[9]J. D. Bjorken and M. Nauenberg, Phys. Rev. 121, 1250 (1961).

OCTET-OCTET MIXING*

Peter G. O. Freund
The Enrico Fermi Institute for Nuclear Studies, The University of Chicago, Chicago, Illinois
(Received 3 February 1964)

In a recent paper[1] in collaboration with Balachandran and Schumacher, we have shown that the resonance theory of form factors and the unitary symmetry scheme of strong interactions require the existence of two octets of vector mesons which we identify as $V=[\varphi(1020), \rho(750), K^*(880)]$ and $V'=[\varphi'(780), \rho'(1220), K^{*\prime}(990)]$. Very suggestive theoretical and experimental arguments in favor of our ρ' assignment have recently been advanced by the La Jolla group,[2] who advocate the view that the earlier observed f meson[3] is, in fact, the 2π mode of ρ'. With these encouraging results in mind and in reference to the good fit to the form-factor data obtained in reference 1, it seems to be warranted to submit the two-octet scheme to a more detailed investigation. In particular, we shall consider here the appreciable V-V' octet-octet mixing[1] suggested by the failure of the Gell-Mann-Okubo[4] mass formula for each octet individually.[5] We shall show that the inclusion of V-V' mixing leads to good values for the coupling strengths of vector mesons to pairs of pseudoscalar mesons as well as to an explanation of the absence of the $K\pi$ mode for our proposed $K^{*\prime}(990)$.

We can regard the physical masses squared φ, φ', ρ, ρ', K^*, and $K^{*\prime}$ as the eigenvalues of the nondiagonal mass-squared matrix

$$M_0^2 = \begin{pmatrix} \varphi_0 & \gamma_\varphi & & & & \\ \gamma_\varphi & \varphi_0' & & & & \\ & & \rho_0 & \gamma_\rho & & \\ & & \gamma_\rho & \rho_0' & & \\ & & & & K_0^* & \gamma_{K^*} \\ & & & & \gamma_{K^*} & K_0^{*\prime} \end{pmatrix}. \qquad (1)$$

If SU(3)-symmetry breaking is due, as commonly assumed,[4] to an interaction that transforms like the eighth component of an SU(3) vector, then we can write the mass formulas

$$3(\varphi_0+\varphi_0')+\rho_0+\rho_0' = 4(K_0^*+K_0^{*\prime}) \qquad (2)$$

and

$$3(\varphi_0'-\varphi_0)+(\rho_0'-\rho_0) = 4(K_0^{*\prime}-K_0^*) \qquad (3)$$

for the six diagonal elements of (1). A similar sum rule is then expected to hold for the three off-diagonal elements of (1) which therefore must have the form[6]

$$\gamma_\varphi = a+\gamma,$$
$$\gamma_\rho = a-\gamma,$$
$$\gamma_{K^*} = a+\tfrac{1}{2}\gamma. \qquad (4)$$

Since in the limit of exact SU(3) symmetry the off-diagonal elements should vanish, we have the additional condition

$$a=0 \qquad (5)$$

and hence

$$\gamma_\varphi = -\gamma_\rho = 2\gamma_{K^*} = \gamma. \qquad (4')$$

The number of independent parameters in (1) thus reduces to seven. Equation (3) in conjunction with the six eigenvalue conditions for (1) determine these seven parameters[7] up to a sign ambiguity for γ. We obtain

$$\gamma = \pm 0.2\ (\text{BeV})^2, \qquad (6)$$

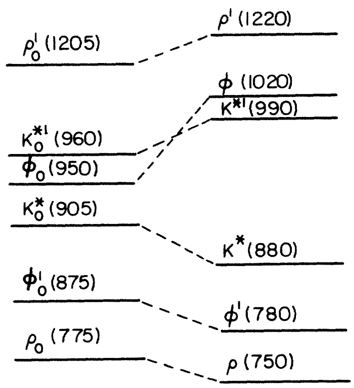

FIG. 1. Level displacements in octet-octet mixing.

and the corresponding solution for the unmixed levels is presented in Fig. 1. The mixing angles λ can then also be calculated up to an over-all sign ambiguity, and we find

$$\begin{pmatrix} \lambda_\varphi \\ \lambda_0 \\ \lambda_{K*} \end{pmatrix} = \pm \begin{pmatrix} 36^\circ \\ 13^\circ \\ -32^\circ \end{pmatrix}. \tag{7}$$

Each of the unmixed octets V_0 and V_0' couples to pairs of pseudoscalar mesons by pure F coupling. Therefore, the decays of V and V' mesons into two pseudoscalars can be described in terms of only two coupling constants f and f' and of the above calculated mixing angles. In terms of

$$G_x = \cos\lambda_x f + \sin\lambda_x f',$$
$$G_{x'} = \sin\lambda_x f + \cos\lambda_x f',$$
$$x = \varphi, \rho, K*, \tag{8}$$

the partial widths are given by

$$\Gamma(\rho \to 2\pi) = \tfrac{4}{3}(p^3/\rho) G_\rho^{\,2},$$
$$\Gamma(\rho' \to 2\pi) = \tfrac{4}{3}(p^3/\rho') G_{\rho'}^{\,2},$$
$$\Gamma(\rho' \to K\bar{K}) = \tfrac{2}{3}(p^3/\rho') G_{\rho'}^{\,2},$$
$$\Gamma(K* \to K\pi) = (p^3/K*) G_{K*}^{\,2},$$
$$\Gamma(K*' \to K\pi) = (p^3/K*') G_{K*'}^{\,2},$$
$$\Gamma(\varphi \to K_1^{\,0}K_2^{\,0}) = 0.65\,\Gamma(\varphi \to K^+K^-)$$
$$= (p^3/\varphi) G_\varphi^{\,2}; \tag{9}$$

where p is the three-momentum of one of the pseudoscalar mesons in the rest frame of the vector meson and the symbol of a vector meson stands as above for its mass squared. The factor 0.65 in the last relation (9) stands for the difference in phase space due to the K^+-K^0 mass difference. We determine f and f' from the known values of the first two widths in (9).[8] The last three widths can then be predicted. Our results are listed in Table I. The improved agreement one finds for the $K*$ and φ decay rates, as compared to the predictions of unitary symmetry without octet-octet mixing [$\Gamma(K*) = 31$ MeV], leads one to believe that V-V' mixing might be a major symmetry-breaking agent.[9] The most important result that one reads off Table I is, however, that even though $K*'(990) \to K + \pi$ has a favorable phase space when compared with $K*(880) \to K + \pi$, it has a width smaller by a factor 50 or so, i.e., $K*'$ couples very weakly to pseudoscalar mesons. This might well be the reason why this particle has not been observed experimentally so far. In view of the extremely small partial width for the $K\pi$ mode of $K*'$, we suggest that the search for this particle be directed towards three- and more-particle decay modes (such as $K\pi\pi$).

The author has been stimulated to this work largely by an enlightening conversation with Professor S. L. Glashow and Professor W. R. Frazer. He wishes also to acknowledge discussions with C. R. Schumacher and F. von Hippel.

Table I. Partial widths for vector-meson decays into two pseudoscalar mesons.[a]

$\Gamma(\rho \to 2\pi)$	110	110	110
$\Gamma(\rho' \to 2\pi)$	70	100	150
Sign in (7)	±	±	±
f	1.16	1.18	1.22
f' [b]	±0.36	±0.48	±0.62
$\Gamma(\rho' \to K\bar{K})$	7.5	10.7	16.1
$\Gamma(K* \to K\pi)$ [c]	38	43	52
$\Gamma(K*' \to K\pi)$	4	1.76	0.44
$\Gamma(\varphi \to K_1^{\,0}K_2^{\,0})$ [c]			
$=0.65\,\Gamma(\varphi \to K^+K^-)$	2.4	2.68	3.2

[a]All widths are listed in MeV, whereas f and f' are dimensionless. The three columns allow for three possible input values of $\Gamma(\rho' \to 2\pi)$.

[b]The ± signs in this and the third line of the table are to be correlated (see reference 8).

[c]Experimental values: $\Gamma(K* \to K\pi) = 50$ MeV, $\Gamma_{tot}(\varphi \to K\bar{K}) = 3.1 \pm 1.0$ MeV; see, e.g., R. H. Dalitz, Ann. Rev. Nucl. Sci. 13, 339 (1963).

*Work supported by the U. S. Atomic Energy Commission.

[1]A. P. Balachandran, P. G. O. Freund, and C. R.

Schumacher, The Enrico Fermi Institute for Nuclear Studies Report No. EFINS 64-2 (to be published). Note our reversed assignments of the $T=0$, $Y=0$ mesons to the two octets as compared with this reference. Our convention avoids crossings in Fig. 1.

[2]W. R. Frazer, S. H. Patil, and N. Xuong (to be published); F. R. Halpern (to be published); D. D. Carmony et al. (to be published).

[3]See, e.g., V. Hagopian and W. Selove, Phys. Rev. Letters 10, 533 (1963), and the literature quoted there.

[4]M. Gell-Mann, California Institute of Technology Report No. CTSL-20, 1961 (unpublished); S. Okubo, Progr. Theoret. Phys. (Kyoto) 27, 949 (1962).

[5]In the "old" scheme in which the φ is considered to be a unitary singlet, ω-φ mixing has been studied by many authors, especially by J. J. Sakurai, Phys. Rev. Letters 9, 472 (1962); Phys. Rev. 132, 434 (1963); S. L. Glashow, Phys. Rev. Letters 11, 48 (1963).

[6]J. J. de Swart, Rev. Mod. Phys. 35, 916 (1963).

[7]The equation (2) becomes an identity once the mass formula (14) of reference 1 is satisfied by the physical masses.

[8]We inserted 70, 100, and 150 MeV for $\Gamma(\rho' \rightarrow 2\pi)$ as possible values suggested by reference 2. It is easy to see that there are four solutions for f and f' (two by two indistinguishable) due to the above-mentioned sign ambiguities and to the second power to which the parentheses in (8) appear. We list in Table I, for each value of $\Gamma(\rho' \rightarrow 2\pi)$, only the two reasonable solutions [the other two solutions lead to very bad values for $\Gamma(K^* \rightarrow K\pi)$].

[9]An alternative proposal that uses the pseudoscalar mass differences as the major symmetry-breaking agency has been discussed by J. J. Sakurai, Phys. Rev. Letters 12, 79 (1964).