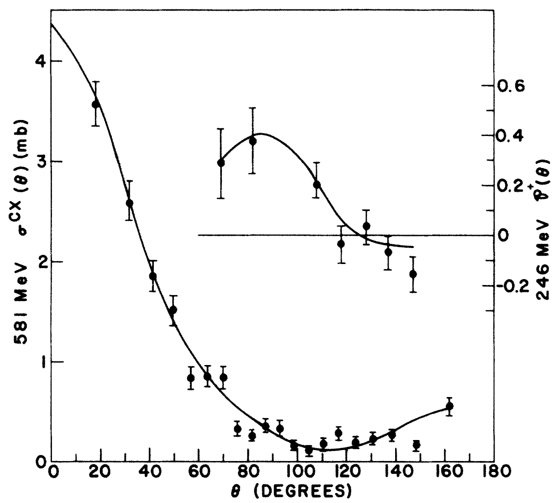

FIG. 2. Prediction of the original energy-dependent phase-shift analysis versus some new data that were not used in the original analysis. The charge-exchange differential cross section data, $\sigma^{cx}(\theta)$, are preliminary results from a recent Berkeley Bevatron experiment [C. Chiu (private communication)], and the π^+-p polarization data, $P^+(\theta)$, are the values obtained by means of the Berkeley polarized target [C. Schultz (private communication)].

Recently much new data have become available through private communication. The solution obtained without the new data is in good agreement with the new data. Examples of such agreement are shown in Fig. 2.

This work was begun while the author was with the Laboratory for Nuclear Science, Massachusetts Institute of Technology, Cambridge, Massachusetts. The computations were done at the Lawrence Radiation Laboratory, University of California, Livermore, California. The details of the analysis will be published in a report presently in preparation.

The author wishes to thank Professor B. T. Feld of the Massachusetts Institute of Technology for initially suggesting this problem and for much advice during the course of the analysis. Dr. Michael Moravcsik of Lawrence Radiation Laboratory has lent much advice and encouragement to the completion of this work. Mr. Robert M. Wright, Mr. Richard A. Arndt, and Mr. Eldon J. Halda were the principals among many who were responsible for the actual computer computations.

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DETERMINATION OF THE SPIN OF THE f^0 RESONANCE*

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In studying π^--p interactions above 3 GeV/c, several groups[1-6] have reported the existence of the f^0 resonance in the $\pi^+\pi^-$ system from the reaction

$$\pi^- + p \to n + \pi^+ + \pi^- . \quad (1)$$

The fact that the resonance does not seem to show up in the $\pi^-\pi^0$ spectrum from the reaction

$$\pi^- + p \to p + \pi^- + \pi^0 \quad (2)$$

nor in the $\pi^+\pi^0$ spectrum from the reaction[7,8] $\pi^+ + p \to p + \pi^+ + \pi^0$ has led to the conclusion[9] that the f is an isotopic singlet ($T = 0$) and, therefore, that its spin (L) is even. This latter conclusion arises from the fact that the f^0 width is large and therefore it conserves isotopic spin in its decay.

The $L = 0$ assignment has definitely been ruled out by the published data[2,4-6] on the decay angular distribution of the f^0, thus indicating an $L = 2$ assignment, which seems to agree with the data to within the error allowed by the somewhat limited statistics available. Hagopian and Selove[4] also mentioned the fact that, disregarding the isotopic spin restriction, their data were incompatible with $L=1$.

However, the possibility of an $L=1$, $T=1$ assignment has been revived by a recent suggestion of Frazer, Patil, and Xuong[10] that the newly discovered ω-π resonance[11] could be a decay mode of the $T_Z=+1$ member of a $T=1$, $L=1$ f meson. In fact, Frazer, Patil, and Xuong conclude that the f^0 decay angular distributions of references 5 and 6 give a better fit to $L=1$ than to $L=2$.

The data presented in this Letter definitely favor $L=2$ as opposed to $L=1$ for the f^0.

We consider a total of 1398 events of type (1) taken from a 3.7-GeV/c π^- exposure in the BNL 20-inch hydrogen chamber. In order to investigate the purity of the f^0 sample, we examine the Dalitz plots for Reactions (1) and (2) which are shown in Fig. 1. The $p\pi^-\pi^0$ plot shows a definite vertical band at the value $T_{\pi^-}=1.0$ GeV indicating production of $N^{*+}(1238)$ in its $p\pi^0$ decay mode. From a projection of the events in this band onto the T_{π^0} axis, we find a constant density of points outside the ρ^- overlap region. By subtracting a small over-all constant background and neglecting possible interference effects from the ρ^- overlap region, we estimate 80 events are due to $\pi^- + N^{*+}$ production. The N^* branching ratio into $n+\pi^+$ then predicts that there will be 40 N^{*+} events in the $n\pi^-\pi^+$ plot, which is consistent with the appearance of Fig. 1(a). 120 N^{*+} correspond to a cross section of 0.35 mb for the reaction $\pi^- + p \rightarrow N^{*+} + \pi^-$. The lack of evidence for $N^{*-}(1238) \rightarrow n + \pi^-$ can possibly be explained by the necessity of exchange of a doubly charged particle for the reaction $\pi^- + p \rightarrow N^{*-} + \pi^+$ if it is to go via one-meson exchange. This cannot be the explanation for the lack of $N^{*0} \rightarrow \pi^- + p$ in Fig. 1(b). However, if the N^* production is due to ρ exchange, one expects only $\frac{1}{4}$ as much N^{*0} as N^{*+} in Fig. 1(b).

In Fig. 2 are shown the distributions in $\cos\theta$ for the two reactions. The angle θ is taken between the outgoing and incoming π^- momentum vectors as seen in the di-pion rest system. In order to increase the purity of the f^0 sample we have only used the events in the di-pion mass interval 1250 ± 80 MeV and with $\Delta^2 < 20\mu^2$ (μ = pion mass). This mass interval corresponds to the f^0 peak region seen in Fig. 3(a), and is chosen to be just slightly wider than 1250 ± 70 MeV, which is the position and half-width we find from a Breit-Wigner fit to the data.

Since N^{*+} events tend to have large positive values of $\cos\theta$, we have tried to make a correction for this distortion of the $\cos\theta$ plot by making

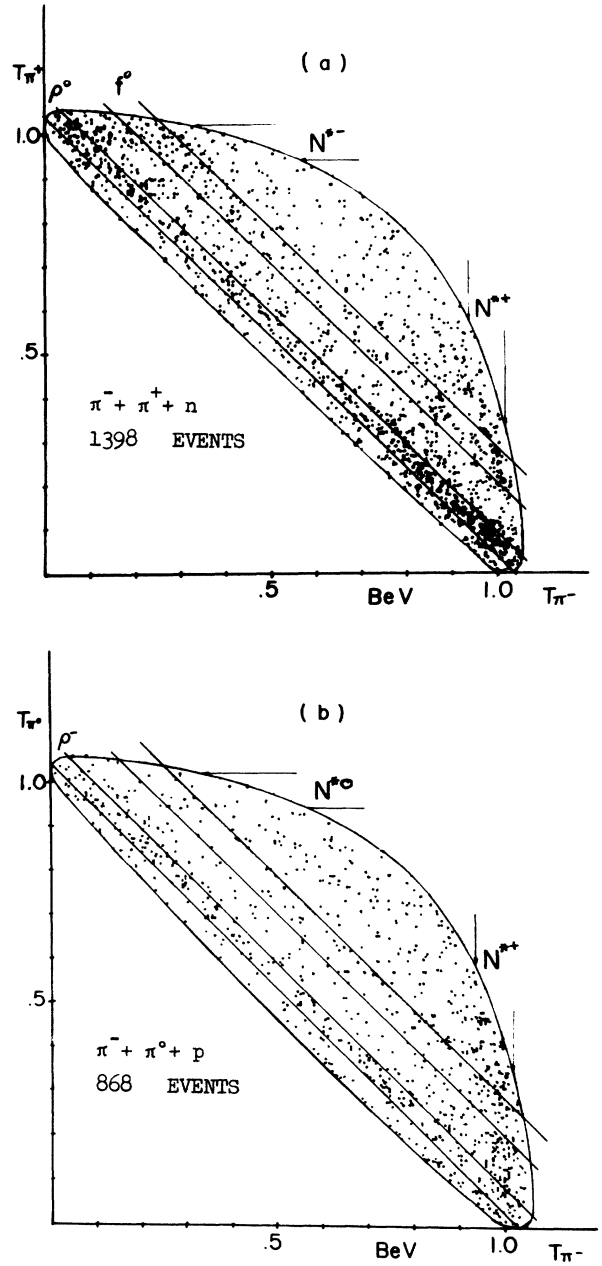

FIG. 1. Dalitz plots for Reactions (1) and (2). The vertical and horizontal arrows indicate N^* mass regions of 1238 ± 90 MeV. The slanted bands indicate di-pion mass regions at the center of the ρ (765 ± 75 MeV) and f (1250 ± 80 MeV) peaks.

a random subtraction of events due to N^{*+} production, taking constant density along the vertical N^* bands. The effects of this subtraction are shown in Fig. 2. The subtraction of the N^* events seems reasonable on the basis of the Dalitz plots, although the conclusions we draw below are not

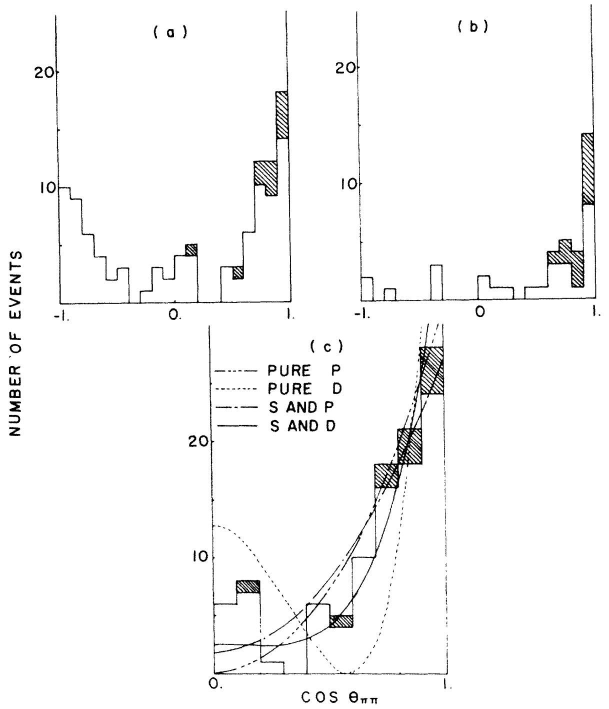

FIG. 2. Distributions of $\cos\theta$ (see text). The effect of a random subtraction of N^* events is shown by the shaded areas. (c) is (a) folded about $\cos\theta = 0$. The curves for pure s wave, pure d wave, along with the least-squares best fits to $(s+p)$ wave and $(s+d)$ wave are shown in (c).

weakened appreciably if we do not make this subtraction. It appears that the forward peak in Fig. 2(b) could be due entirely to N^* production.

The almost perfect symmetry of Fig. 2(a) seems in itself to be evidence against the f^0 being a $T=1$, $L=1$ resonance since the $T=1$, $L=1$ ρ^0 from the same reaction shows a very large asymmetry throughout the ρ^0 resonance region.[12]

Because of the symmetry of Fig. 2(a), we have folded it about $\cos\theta = 0$ to produce Fig. 2(c). The normalized least-squares best fits to $A + B\cos^2\theta$ and to $A + B\cos^2\theta + C\cos^4\theta$ are shown. For comparison, $\cos^2\theta$ (pure p wave) and $1 - 6\cos^2\theta + 9\cos^4\theta$ (pure d wave) are also shown. The $A + B\cos^2\theta + C\cos^4\theta$ curve gives a χ^2 probability of 9%, whereas $A + B\cos^2\theta$ gives less than 0.1%. The hump around $\cos\theta = 0$ is the most striking evidence for the necessity of a $\cos^4\theta$ term, although such a term is also indicated by the steepness of the data near large values of $\cos\theta$. The fact that the central hump is not nearly as high as would be required by pure d-wave π-π scattering is not surprising in view of the large s-wave background that can be present to interfere. Such a decrease is quite feasible even for no s-wave background if one considers that the f^0 production may not go entirely through one-pion exchange. This could introduce Y_2^m terms (with $m \neq 0$) into the distribution so that practically any values for

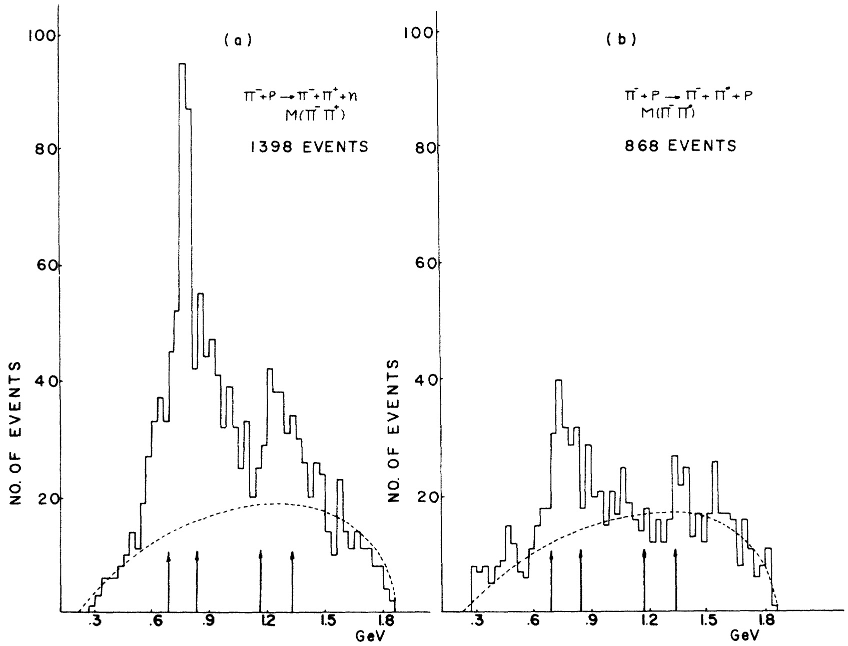

FIG. 3. Distributions of invariant mass of the di-pion systems from Reactions (1) and (2). The arrows indicate the regions 765 ± 75 MeV and 1250 ± 80 MeV.

A, B, C would be allowed. An examination of the data for azimuthal dependence (Treiman-Yang test) that such terms might induce gives only a fair fit to isotropy, so that no conclusions for or against one-pion exchange can be drawn.

We conclude that the spin of the f^0 resonance is greater than one. Spin values of three or greater cannot be ruled out on the basis of our angular distributions.

An independent argument against $T=1$ for the f^0 (and therefore in favor of even spin) can be made on the basis of the di-pion mass distributions (Fig. 3). Assuming that f production occurs predominantly through one-pion exchange with the pion-pion vertex in the $T=1$ channel, one would expect $\frac{1}{2}$ as many events in an f^- "peak" as there are in the f^0 peak. (The factor of $\frac{1}{2}$ comes from the nucleon vertex.) If one subtracts half the number of events in our f^0 peak above phase space from the f^- region, a definite dip occurs which falls far below the general appearance of the background. Thus our data are inconsistent with the combined assumptions of $T=1$ and predominance of one-pion exchange. If the one-pion-exchange restriction is relaxed, then the $T=1$ cross sections must only obey the familiar triangle relations.[9,10] In such a case, no conclusions can be drawn from our data alone except that a fairly large f^+ peak should show up in $\pi^+ + p \to p + \pi^+ + \pi^0$ at the same energy.

Taking the f^0 spin to be even, one still might ask whether $T=2$ is a possibility. The lack of any evidence for a resonance in the total cross section or angular distribution in the $\pi^-\pi^0$ data seems to rule against this even more strongly since pure $T=2$ π-π scattering would require a 4.5 times bigger bump in $\pi^-\pi^0$ than in $\pi^+\pi^-$.

We are greatly indebted to Dr. Ralph Shutt and the members of the bubble chamber group at Brookhaven National Laboratory for their generous help in taking the pictures and providing us with computer programs.

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APPROXIMATE SOLUTION TO THE N/D EQUATIONS*

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(Received 13 January 1964)

In the study of partial-wave dispersion relations, it has become increasingly clear from the search for self-consistent or "bootstrap" solutions to the N/D equations[1] that even for the ρ resonance one must go beyond the one-channel approximation.[2] On the other hand, numerically solving these equations for several channels may require a prohibitive amount of computer time. Hence, recently published calculations[2,3] for the coupled-channel problem have used an approximate solution to the N/D equations developed by Baker,[4] the determinantal method, which is an enormous simplification computationally.

Whereas the determinantal method is easy to use, it has several serious drawbacks: (i) It contains an arbitrary subtraction point to which the solutions are sensitive (see Fig. 1).[5] (ii) In the multichannel problem, the solutions violate time-reversal invariance, i.e., they are not symmetric.[6] The purpose of this note is to present an approximate solution to the N/D equations which has the exact same degree of simplicity as the determinantal method but (a) does not have drawbacks (i) and (ii), and (b) for a number of interesting cases is a closer approximation to the actual solution. After developing the approximate solu-