potential, but also by modulation of the number of each type of state. The number of 2p1h states gives the degeneracy splitting of the virtual level. The effect of the 3p2h states on the 2p1h states is to change their strength. It is therefore necessary to know where the 3p2h states occur before one can estimate the width of the 2p1h "doorway states."

We may summarize by noting that states which require excitations over only one energy gap can be plentiful. When two such excitations are necessary, the number of 3p2h states is severely curtailed. The region of immediate experimental interest for study of the neutron-proton interaction would therefore be at and near the doubly magic nuclei.

This survey gives one some feeling for the complexity and systematics of the compound states that may be built out of particle-hole interactions. It also predicts regions where the "compound nucleus" is simple enough to lend itself to computation.

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EVIDENCE FOR A π-ρ INTERACTION PRODUCED IN THE π^+p REACTION AT 3.65 BeV/c†

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In this note we wish to discuss our results on the study of the π^+p reaction at 3.65 BeV/c leading to four charged particles in the final state. In particular, we wish to point out an anomaly we have observed in the mass distribution of the $\pi\rho$ system. This work was carried out in the 20-inch BNL hydrogen bubble chamber[1] exposed in the Yale-Brookhaven beam[2] at the AGS. The analysis[3] of the four-prong events permitted us to identify the three reactions[4]

$$\pi^+p\to\pi^+\pi^-\pi^+p,\quad 1784 \text{ events},\qquad \sigma=3.85\pm0.30 \text{ mb}, \tag{1}$$

$$\pi^+p\to\pi^+\pi^-\pi^0\pi^+p,\quad 1998 \text{ events},\qquad \sigma=4.3\pm0.35 \text{ mb}, \tag{2}$$

$$\pi^+p\to\pi^+\pi^+\pi^+\pi^-n,\quad 359 \text{ events},\qquad \sigma=0.76\pm0.07 \text{ mb}. \tag{3}$$

In what follows we discuss Reaction (1) and, in particular, those events leading to ρ^0 formation. Table I gives the summary of the cross section for all the channels identified in Reaction (1). The result of our analysis of the reaction products indicates that the majority of the events involve

Table I. Partial cross sections for channels leading to ρ^0, f^0, and $N^{*++}(1238)$ formation in the reaction $\pi^+p\to\pi^+\pi^-\pi^+p$ at 3.65 BeV/c.[a]

Channel	Final state	Branching ratio (%)	Cross section (mb)
(1a)	ρ^0N^{*++}	30.5	1.17 ± 0.12
(1b)	$\rho^0\pi^+p$	23.0	0.86 ± 0.09
(1c)	$\pi^+\pi^-N^{*++}$	30.1	1.16 ± 0.12
(1d)	f^0N^{*++}	3.4	0.13 ± 0.04
(1e)	$\pi^+\pi^-\pi^+p$ ("nonresonant")	13.0	0.53 ± 0.1
	Total	100.0	3.85 ± 0.30[b]

[a]This table refers only to the most prominent features. Finer features such as the A^+ effect, $N^{*0}(1238)$ formation, etc., are not explicitly incorporated.

[b]The cross sections were calculated by calibration of the total number of π^+p interactions against the cross-section measurements by M. J. Longo and B. J. Moyer, Phys. Rev. 125, 701 (1962).

some resonance phenomenon between the outgoing particles.

One of the dominant processes in Reaction (1) is the formation of the N^{*++} resonance.[5] We found it expedient to classify our events into types according to whether or not a π^+p mass value falls inside the N^{*++} band, defined by $1.12 \leq M(p\pi^+) \leq 1.32$ BeV. We call the group of events for which one π^+p mass falls inside the N^{*++} band "type 1" (1024 events). Those events for which both π^+ mesons form a π^+p mass which lies inside the N^{*++} band we call "type 2" (118 events). Finally, "type 3" (642 events) refers to events for which both π^+p mass combinations lie outside the N^{*++} band.[6] This breakdown into types is illustrated in Fig. 1. In Fig. 1(a) we present a scatter plot of the $\pi_a{}^+p$ mass against the $\pi_b{}^+\pi^-$ mass. As we pointed out in an earlier communication[7,8] the kinematical limits for the resulting scatter plot are given by an isosceles triangle. Since each event has two π^+ mesons, it is represented by two points in this triangle plot. For clarity we have left out from Fig. 1(a) those points of type-1 events which correspond to the "reflection" of the N^{*++} band. These points are shown separately in Fig. 1(e), with their projections [Figs. 1(f) and 1(g)]. These correspond to the alternate π^+p mass combination $(\pi_2{}^+p)$ not yielding masses in the N^{*++} band. It is noteworthy that the corresponding $\pi_1{}^+\pi^-$ masses do not show any marked ρ^0 formation either [Fig. 1(g)]. The most prominent feature of the $M(\pi^+\pi^-)$ distribution is ρ^0 production,[9,10] defined here by $0.65 \leq M(\pi^+\pi^-) \leq 0.85$ BeV [Figs. 1(c) and 1(d)]. This proceeds with comparable cross sections through the channels

$$\pi^+ + p \to \rho^0 + N^{*++}, \qquad (1a)$$

$$\pi^+ + p \to \rho^0 + \pi^+ + p. \qquad (1b)$$

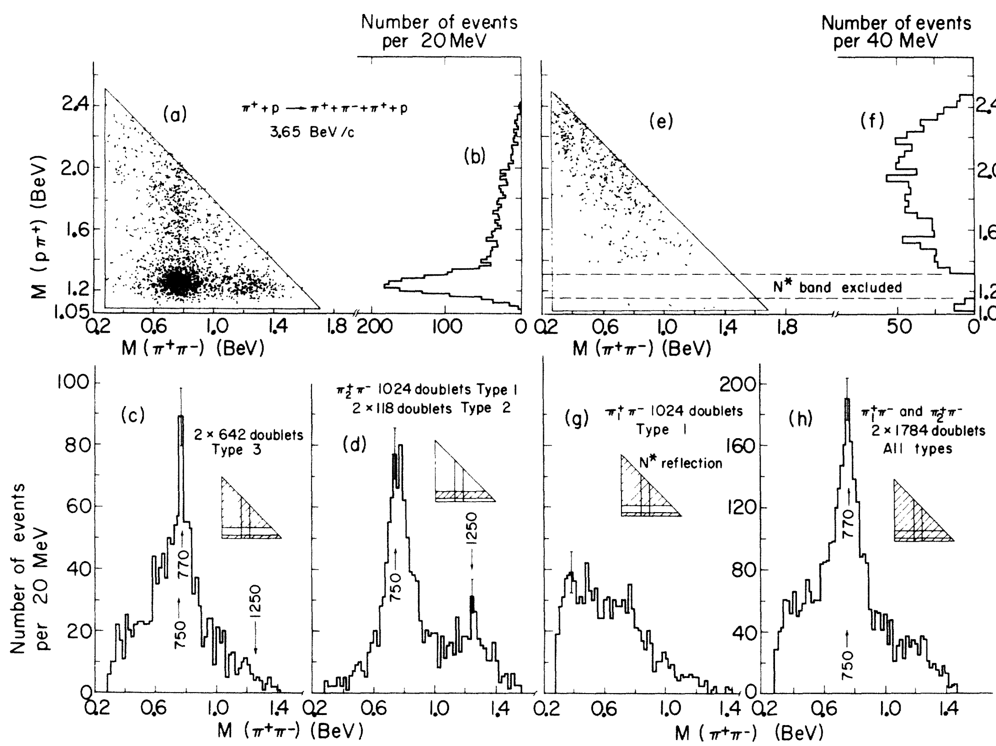

FIG. 1. Scatter plots of the effective-mass distributions for the two-particle composites in the reaction $\pi^+p \to \pi^+\pi^-p\pi^+$. The mass projections also are shown. In the present reaction each event contributes two points to the scatter plot. Those points corresponding to the reflection of the N^{*++} band (a) are shown separately in (e). The projections have been similarly separated. The large triangles delineate the kinematical limits. (h) shows the projection of all points [each event taken twice on the $M(\pi^+\pi^-)$ axis]. It should be noted that the scale for mass projection is not the same in all figures. The small triangles shown as inserts to the figure as well as on the following figures indicate by the shaded region that part of the four-particle phase-space triangle which is displayed in the particular figure.

The production mechanism of the ρ^0 meson appears to be distinctly different in the two channels. In channel (1a), ρ^0 mesons are produced with small four-momentum transfer Δ_1^2 to the N^{*++}, which we interpret to mean that the reaction proceeds principally via a peripheral collision. The Δ_1^2 distribution has a full width at half-maximum of about $15m_\pi^2$. In channel (1b), on the other hand, the formation of ρ^0 mesons proceeds via much larger momentum transfers to the nucleon and pion system (full width at half-maximum about $50m_\pi^2$). It is in this latter channel that we observe a strong enhancement in the distribution of $M(\rho^0\pi^+)$ in the mass region 1.0 to 1.4 BeV over that predicted by phase-space considerations. For purposes of discussion we refer to this enhancement as the formation of a state A^+ according to

$$\pi^+ + p \rightarrow A^+ + p, \tag{4}$$

which breaks up according to

$$A^+ \rightarrow \pi^+ + \rho^0. \tag{5}$$

To investigate the possible π-ρ interaction further, we now consider channels (1a) and (1b) as a three-"particle" final state, viz. $\rho^0\pi^+p$, and construct the corresponding Dalitz plot. In Figs. 2(a) and 2(b) we have plotted $M^2(p\pi^+)$ versus $M^2(\rho^0\pi^+)$. Because of the width of the ρ^0 meson we have two boundaries for the Dalitz plot. We note the strong N^{*++} band as well as a distinct enhancement in the region $M^2(\rho^0\pi^+)$, between 1.0 and 2.0 $(\mathrm{BeV})^2$. In Fig. 2(a) all $\pi^+\pi^-$ mass doublets falling inside the ρ^0 mass band are shown. Events in which both mass doublets fall inside the ρ^0 band will thus contribute two points corresponding to different $M^2(p\pi^+)$ but equal $M^2(\rho^0\pi^+)$ values. We call these "double-ρ events." In Fig. 2(b) and in all subsequent discussion we show only one point chosen arbitrarily for double-ρ events; furthermore, we have left out the $\pi_1^+\pi^-$ type-1 combinations [see Fig. 1(g)] to avoid excessive background in our ρ^0 sample. The projection on the $M^2(p\pi^+)$ axis is given in Fig. 2(c). The N^{*++} band is clearly seen as well as a long tail towards higher π^+p mass values. The latter events belong exclusively to type 3. In Fig. 2(d), we show the projection of the type-3 events on the $M^2(\rho^0\pi^+)$ axis (i.e., excluding the N^{*++} band). It is this projection as well as in the Dalitz plot itself that we note the A^+ enhancement effect over phase-space predictions. The 110 double-ρ^0 events (out of 428 type-3 events) are indicated by the

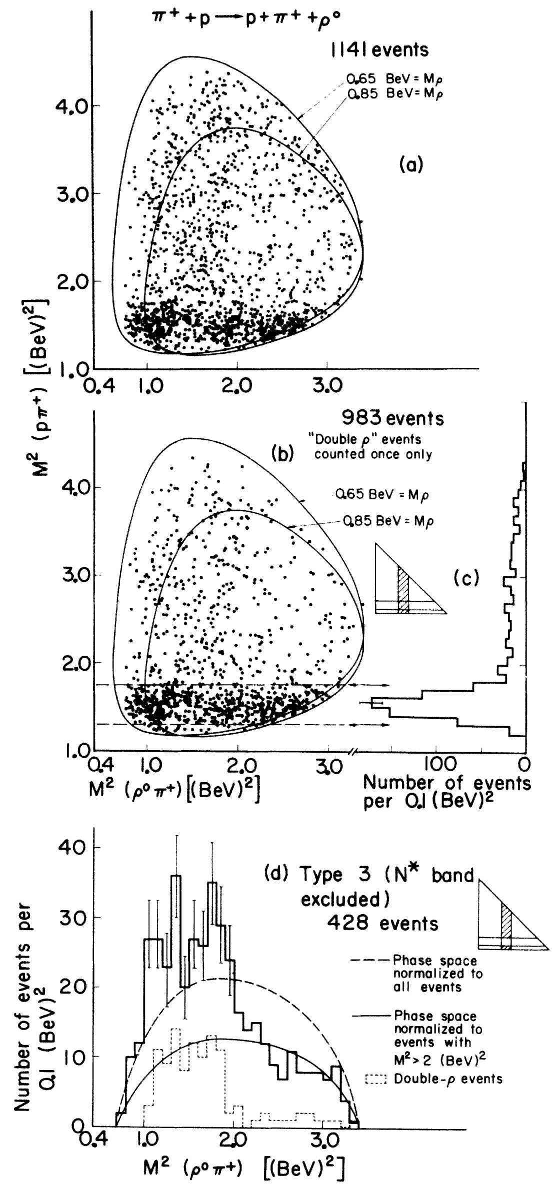

FIG. 2. (a) A Dalitz plot for the three "particles" $\rho^0\pi^+p$. The two curves correspond to the two mass limits used to define the ρ^0 meson. (b) A Dalitz plot of the same variables as in (a) is shown—excluding, however, the points due to the reflection of the N^{*++} and showing one point only per event for double-ρ^0 events. (c) and (d) show the projection of (b) on the M^2 axes as indicated.

dashed histogram in Fig. 2(d). It is noteworthy that the double-ρ events contribute considerably to the A^+ enhancement. This raises the question whether the A^+ enhancement might be related to Bose symmetrization or possibly be a dynamical effect favoring "double-ρ" formation. However, on closer examination of the problem, double-ρ

formation does not appear to be unduly enhanced. This can be appreciated by making a two-dimensional array in which the mass of the $\pi_1^+\pi^-$ pair is plotted against that of the $\pi_2^+\pi^-$ pair. Such an array for type-3 events clearly shows the two ρ^0 bands. The crossing of these ρ^0-meson bands corresponds to a particular symmetry in the $\pi^+\pi^-\pi^+$ system, i.e., double-ρ^0 events. We find that the number of double-ρ^0 events agrees with a simple superposition of the two ρ^0 bands.

We note that the "A^+ state" is produced with small four-momentum transfer, Δ^2, to the proton, while this is not the case for Δ_1^2, the four-momentum transfer to the π^+p in the corresponding ρ^0 formation. This is illustrated in Fig. 3 where we present a Chew-Low plot for the type-3 events. The projection of Δ^2 is given on the ordinate for the $M^2(\rho^0\pi^+)$ region 1 (BeV)^2 to 2 (BeV)^2 and separately for the remaining mass regions. The peaking of the small values of Δ^2 for the region of the A^+ enhancement is self-evident. The distribution falls to half its intensity at about $25m_\pi^2$. On the abscissa we show mass projections for $\Delta^2 \leq 50m_\pi^2$ and those for higher four-momentum transfers. The enhancement in the A^+ region for low momentum transfer is clearly seen.

A possible interpretation of our observation is that we are dealing with a resonance in the $\pi^+\rho^0$

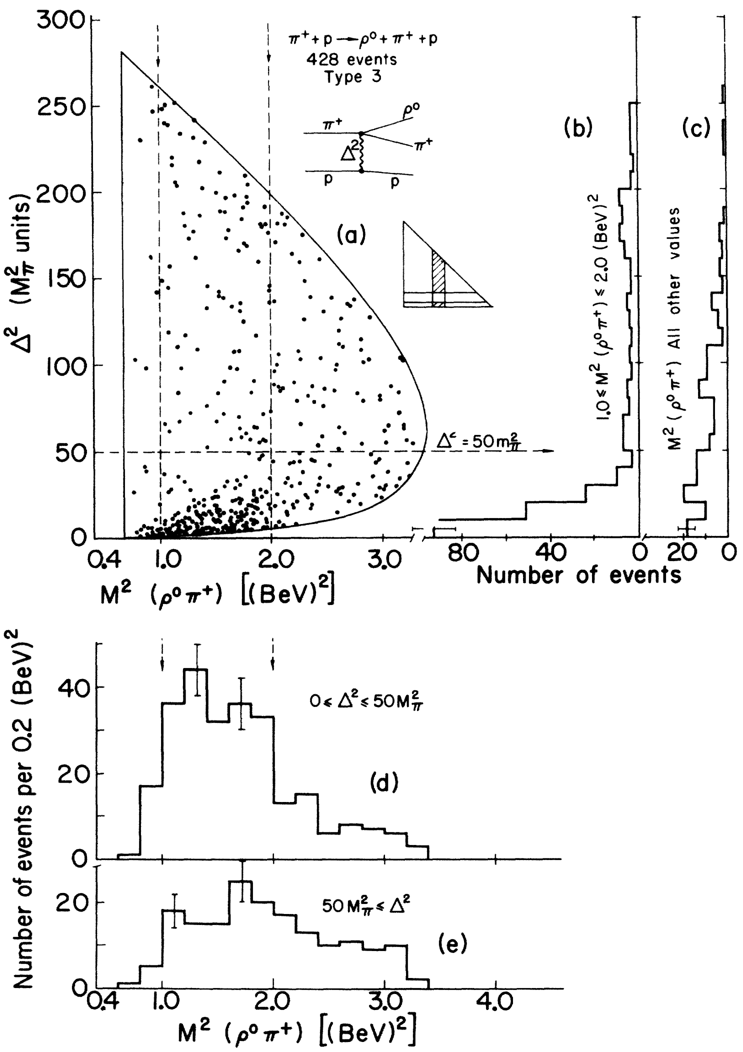

FIG. 3. A Chew-Low plot for the events of type 3. The four-momentum transfer to the proton Δ^2 is plotted against the M^2 of the $\rho^0\pi^+$ system. (b) and (c) show the projections on the Δ^2 axis for the events, respectively, inside and outside the A^+ band. The projections on the $M^2(\rho^0\pi^+)$ axis are shown in (d) and (e) for Δ^2 values less than and greater than $50m_\pi{}^2$, respectively.

system.[11] In this case the relevant parameters known at present are $E_{A^+} = 1.2$ BeV, $\Gamma_{A^+} = 0.35$ BeV, the G-parity is odd, and the isotopic spin $T = 1$ or 2. Alternatively, there may be some structure to the A^+ enhancement and our observations could arise from two unresolved peaks. Furthermore, the enhancement effects considered by Nauenberg and Pais,[12] which would not correspond to a unique isotopic spin and angular momentum state, could play a role in accounting for the lower half of the observed A^+ enhancement, i.e., in the region near 1 BeV.

We wish to thank the many members of the staff of the Brookhaven National Laboratory for their great helpfulness in making this experiment possible. In particular, we would like to express our appreciation to Dr. Hildred Blewett, Dr. Hugh Brown, Dr. Ralph Shutt, and the AGS operating crew. We also wish to thank Dr. Nicola Cabibbo, Dr. Geoffrey Chew, and Dr. Charles Zemach, for a number of helpful discussions, as well as Miss Ling-Lie Chau, Mr. Allan Hirata, Dr. Thomas O'Halloran, and Mr. Victor Seeger, who have participated in various aspects of this work. Finally, this work would not have been possible without the active help and interest of the Lawrence Radiation Laboratory scanning, measuring, and computing personnel.

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[4]Identification was made by χ^2 criteria for the various hypotheses and by visual inspection of the ionization for each event. This left 6.5 % of the events as ambiguous. These events are not included here.

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[6]For type 1 the two π^+ mesons are labeled π_1^+ and π_2^+; the π_1^+ is associated with the N^{*++}. For types 2 and 3 the subscripts are assigned randomly. In general, when we wish to distinguish between the two π^+ mesons without reference to possible resonance formation, we use subscripts "a" and "b."

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[9]In the reaction observed here, resonance parameters for the ρ^0 meson differ somewhat from the ones quoted in the literature. We find the central value of the experimental ρ peak to lie at $E_{\rho^0} = 770 \pm 10$ MeV with a full width at half-maximum of approximately 130 MeV. For a compilation of ρ^0-resonance parameters together with references, see M. Roos, Rev. Mod. Phys. 35, 314 (1963).

[10]We have observed f^0 production (60 ± 20 events). This process is clearly present and occurs almost exclusively with N^{*++} formation. The f^0 was originally observed in a three-particle final state: W. Selove, V. Hagopian, H. Brody, A. Baker, and E. Leboy, Phys. Rev. Letters 9, 272 (1962).

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EVIDENCE FOR A P_{11} PION-NUCLEON RESONANCE AT 556 MeV†

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The purpose of this note is to report strong evidence for the existence of a resonance in the P_{11} state of the pion-nucleon system. Previous pion-nucleon resonances were discovered from observations on the qualitative behavior of experimental observables. The resonance suggested in this paper, however, is not associated with conspicuous features in the observables measured so far and has been inferred from a more quantitative analysis.