Løvseth, and Dr. M. Veltman for making available to the author a copy of their paper before publication. He is grateful to Professor J. R. Oppenheimer for the hospitality at the Institute for Advanced Study.

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STUDY OF $\pi^-\pi^+$ SYSTEM IN $\pi^- + p \to \pi^- + \pi^+ + n$ REACTION*

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We present some results on the study of the $\pi^-\pi^+$ system produced in the reaction $\pi^- + p \to \pi^- + \pi^+ + n$ in the BeV region. It has been pointed out by Bernstein and Feinberg[1] that the 2π decay mode of the ω meson can give rise to a sharp peak over the broad ρ peak in the $\pi^-\pi^+$ mass distribution. Peaks corresponding to ω production have been observed experimentally.[2,3] We investigate this point using a model calculation. We consider that the ω is produced through ρ exchange[4] and then decays electromagnetically into $\pi^- + \pi^+$; on the other hand, the ρ itself is produced through single-pion exchange and then decays strongly into $\pi^- + \pi^+$. Another point we investigate is the $\pi^-\pi^+$ center-of-mass distribution in the ρ-resonance region. This distribution is symmetric for a pure p-wave resonance, while a very asymmetric distribution has been observed.[3,5,6] We find that a strong $T=0$, s-wave π-π interaction can well explain such an asymmetric distribution.

Our considerations are based on Figs. 1(a) and 1(b). Due to electromagnetic mixing and the small difference between ρ and ω masses, the decay $\omega \to \pi^- + \pi^+$ is greatly enhanced.[7,1] This is reflected in our calculation through a large effective $\omega\pi\pi$ coupling constant. We notice that the interference term between the amplitudes of Figs. 1(a) and 1(b) vanishes. This is because of the single γ_5 that occurs in the evaluation of the trace of the interference term. This leads us to the interesting conclusion that the contribution of Fig. 1(a) adds incoherently to the contribution of Fig. 1(b).

Experimentally, it has been found that the two-pion c.m. angular distribution in the reaction $\pi^\mp + p \to \pi^\mp + \pi^0 + p$ is almost symmetric and cor-

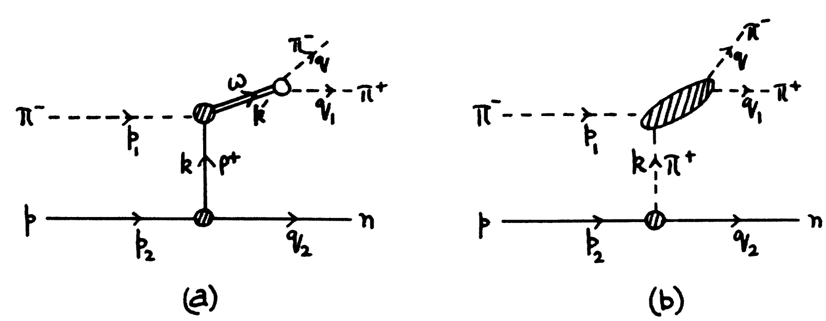

FIG. 1. (a) ρ exchange, ω production, and $\omega \to \pi^- + \pi^+$; (b) pion exchange and $\pi^-\pi^+$ scattering.

responds to a dominant p-wave resonance.[5,6,8,9] On the other hand, as we have mentioned earlier, the $\pi^-\pi^+$ c.m. angular distribution in the reaction $\pi^- + p \rightarrow \pi^- + \pi^+ + n$ is very asymmetric. Calculation of the $\pi^-\pi^+$ c.m. differential cross section from Fig. 1(a) shows that it is proportional to $(1 - \cos^2\theta)$, where θ is the c.m. angle between the incident and the outgoing π^-. Thus the 2π decay of ω cannot account for the observed asymmetry. Since the $T=0$ 2π state does not occur in $\pi^\mp\pi^0$ system, we are therefore led to conclude that the asymmetric distribution for $\pi^-\pi^+$ is due to $T=0$ state interaction.[10]

To investigate the $T=0$ interaction further, we approximated the off-shell $\pi^-\pi^+$ amplitude in Fig. 1(b), by keeping only the s and d waves in the $T=0$ state and the p wave in the $T=1$ state. Following a suggestion by Selleri,[11] we multiplied the partial-wave amplitudes by some kinematic factors to take into account the off-mass-shell nature of them. Now, the $T=1$ p wave is known from the mass and the width of the ρ meson. The $T=0$ d wave is also somewhat known from the analysis of Oades.[12] He concludes that to be consistent with πN scattering data, $\delta_2^0 < 13°$ at $M_{\pi\pi} = 650$ MeV and $\delta_2^0 < 18°$ at $M_{\pi\pi} = 840$ MeV. From Oades's result, we can say that $\delta_2^0 \approx 15°$ represents the maximum d-wave interaction in the ρ resonance region. We may now try to find out the unknown $T=0$ s-wave interaction, by calculating the $\pi^-\pi^+$ c.m. angular distribution from Fig. 1(b) and comparing with experimental results. We have considered two possibilities: (i) a constant s-wave phase shift, (ii) a resonant s-wave phase shift. The results of our calculations are shown in Figs. 2(a) and 2(b). The solid curves correspond to constant s-wave phase shift of $60°$ and the dashed curves to resonant s-wave phase shift, the position and the width of the resonance being $27\mu^2$ and $0.7\mu = 100$ MeV, respectively. The experimental results are of the Saclay-Orsay-Bari-Bologna collaboration at 1.59 BeV/c and of Guiragossián at 3.3-BeV/c incident momentum. In Fig. 2(c), we have plotted the asymmetry parameter and compared with the experimental results. The interesting feature is that while this parameter remains positive for $\pi^-\pi^+$ around the ρ resonance, it goes through zero for $\pi^-\pi^0$ at the resonance and is negative below the resonance.[8] We may add here that decreasing the d-wave phase shift decreases the asymmetry in the angular distribution. The same thing happens if the constant s-wave phase shift or the position of the s-wave resonance is decreased. Finally, the resonant s-wave distribution is not very sensitive to the width.

We now consider the $\pi^-\pi^+$ mass distribution. In Fig. 3(a) we have shown the mass distribution due to $\pi^-\pi^+$ scattering for incident π^- momentum 1.7 BeV/c and $\Delta^2 < 0.15$ (BeV/c)^2 and have compared it with the experimental results of Fickinger, Robinson, and Salant. The distribution due to $\omega \rightarrow \pi^- + \pi^+$ is just added to this distribution. However, we find that for a sharp ρ form factor,[13] this is one fifth of the $\pi^-\pi^+$ contribution. Thus, the process shown in Fig. 1(a) gives very little peak for small Δ^2. In Fig. 3(b), we have shown the mass distribution due to $\pi^-\pi^+$ scattering and $\omega \rightarrow \pi^- + \pi^+$ for $0.25 \leq \Delta^2 < 0.7$ (BeV/c)^2. As can be seen, the 2π decay of the ω, in this case,

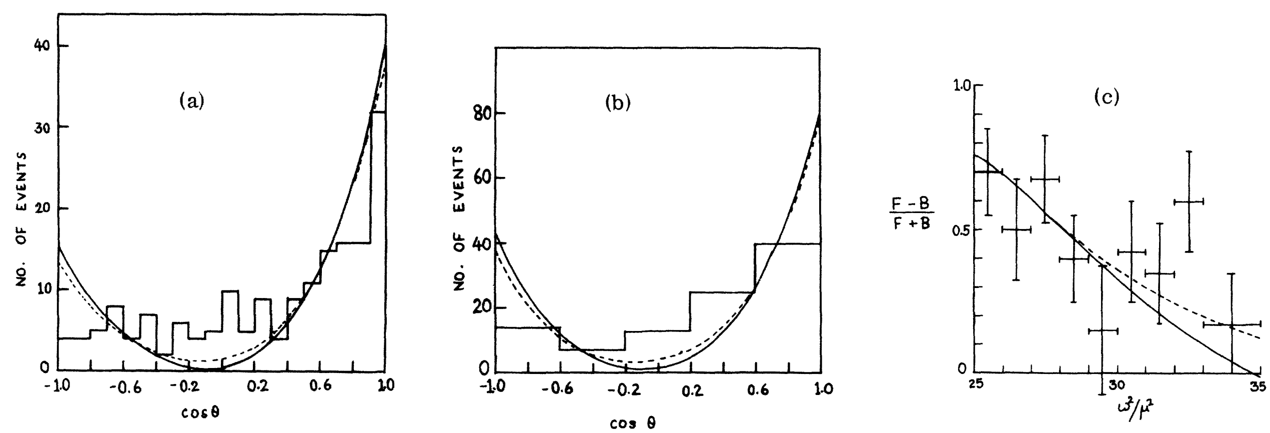

FIG. 2. (a) $\pi^-\pi^+$ c.m. angular distribution for π^- incident momentum 1.59 BeV/c. The experimental results are of Saclay-Orsay-Bari-Bologna Collaboration[3] ($25 \leq \omega^2/\mu^2 \leq 33$, $1.5 \leq \Delta^2/\mu^2 < 8$). (b) $\pi^-\pi^+$ c.m. angular distribution for π^- incident momentum 3.3 BeV/c. The experimental results are of Guiragossián[6] ($700 \leq \omega < 850$ MeV, $\Delta_{\min}^2 < \Delta^2 \leq 20\mu^2$). The curves in (a) and (b) are normalized to the number of events in the corresponding histograms. (c) Asymmetry parameter as a function of ω^2/μ^2 and the corresponding experimental results[3] ($1.5 \leq \Delta^2/\mu^2 < 8$).

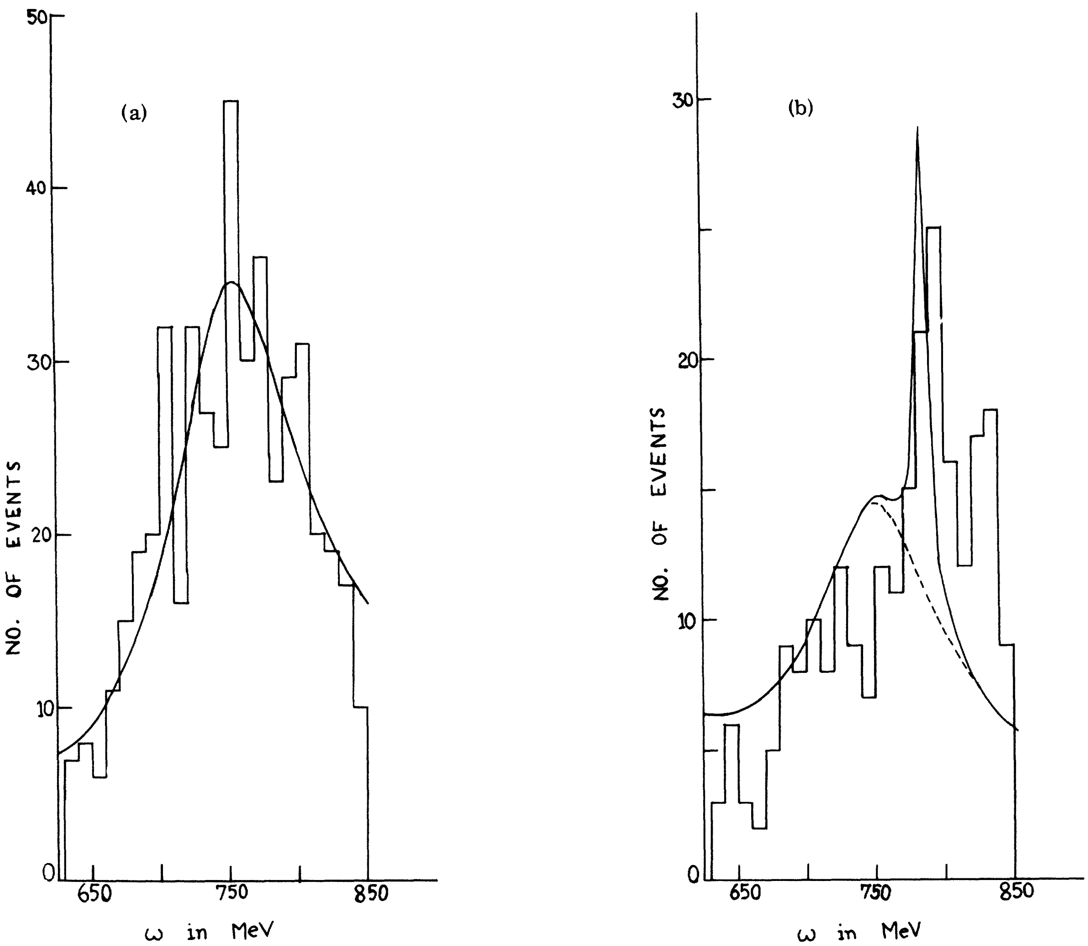

FIG. 3. $\pi^-\pi^+$ mass distribution: (a) $\Delta^2 < 0.15$ (BeV/c)^2 due to $\pi^-\pi^+$ scattering only; the 2π decay of ω gives very little peak in this case; (b) for $0.25 \leq \Delta^2 < 0.70$ (BeV/c)^2 due to $\pi^-\pi^+$ scattering as well as $\omega \to \pi^- + \pi^+$; the dashed curve is the $\pi^-\pi^+$ scattering contribution along in the ω-resonance region. The experimental results are those of Fickinger, Robinson, and Salant[2] for $630 \leq \omega \leq 850$ MeV at an incident momentum 1.7 BeV/c. The theoretical curves are normalized to the number of events in the corresponding histograms.

gives a prominent sharp peak sitting on top of the ρ peak. For the ρ form factor,[14] we have used an average value $\overline{F}_\rho = 3^{-1/2}$. If we do not use any form factor, then the peak is twice as large when normalized to the total number of events in the histogram.

From the consistency of our theoretical calculations with experimental results, we arrive at the following conclusions. The very asymmetric c.m. angular distribution of $\pi^-\pi^+$ in the ρ-resonance region indicates a strong s-wave interaction in the $T = 0$ state.[15] This can be approximately represented by a large s-wave phase shift (~60°) or by a resonant s wave with position very near the ρ mass.[16] The ω is mainly produced in the reaction $\pi^- + p \to n + \omega$ through ρ exchange (for $\Delta^2 \lesssim 35\mu^2$), as shown in Fig. 1(a), and there is no interference between the 2π decay of ω and the $\pi^-\pi^+$ scattering; i.e., the contributions of these two processes are incoherent. The process in Fig. 1(a) gives very little peak for small Δ^2 ($<10\mu^2$), but very prominent sharp peak for intermediate values of Δ^2 ($10\mu^2 \lesssim \Delta^2 \lesssim 35\mu^2$) in the $\pi^-\pi^+$ mass distribution.

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[16]These conclusions are in complete agreement with those of Hagopian and Selove (reference 5). We have also found $\pi^-\pi^+$ angular distributions very similar to Figs. 2(a) and 2(b) with $T=0$ s-wave phase shift 120°.

QUANTUM ELECTRODYNAMICS WITH ZERO BARE FERMION MASS

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It is the purpose of this Letter to show that the logarithmic divergence of the fermion self-mass in lowest order perturbation theory of quantum electrodynamics disappears if the approximation is sufficiently improved and the bare fermion mass is assumed to be zero.

We start from the Dyson equation for the fermion without the bare mass term; as in lowest order perturbation theory we neglect the corrections in the photon propagator and in the vertex and put $Z_2=1$:

$$S_F'^{-1}(p)=i\not{p}-\frac{i\alpha}{4\pi^3}\int\gamma_\mu S_F'(k)\gamma_\nu \times\left[g^{\mu\nu}-\frac{(p-k)^\mu(p-k)^\nu}{(p-k)^2}\right]\frac{1}{(p-k)^2-i\epsilon}d^4k. \quad (1)$$

The use of the Landau gauge[1] has the result that the $\not{p}$ part of the self-energy integral vanishes.[2] Defining the function $m(p^2)$ by[1]

$$S_F'^{-1}(p)=i\not{p}+m(p^2), \quad (2)$$

one finds the following nonlinear integral equation:

$$m(p^2)=-\frac{i3\alpha}{4\pi^3}\int\frac{m(k^2)}{m^2(k^2)+k^2-i\epsilon}\,\frac{1}{(p-k)^2-i\epsilon}d^4k. \quad (3)$$

This equation can be reduced to a nonlinear differential equation in momentum space[3] which, however, is difficult to solve.

Because the theory does not contain a constant with the dimension of a length, Eq. (3) is dilatationally invariant[3,5]: If $m(p^2)$ is a solution, the same is true of $\lambda^{-1}m(\lambda^2p^2)$. As we will see, the solutions of Eq. (3) are, for not too high momenta, closely approximated by the usual second-order perturbation result:

$$m(k^2)=m+\alpha f(k^2) \quad (4)$$

with $f(-m^2)=0$. The constant m corresponds to the mass of the particle and may be used as a label to characterize a certain member of the set of solutions. Concentrating on one of the solutions of Eq. (3), defined by $m=m_1$, we make for nearly all momenta only a small error if we replace in the denominator of the right-hand side of Eq. (3) $m(k^2)$ by m_1. For not too high values of k^2 this is true because of Eq. (4) and for ultrahigh momenta $m^2(k^2)$ is negligible compared with