HIGH-ENERGY p-p ELASTIC SCATTERING*

Toichiro Kinoshita
Laboratory of Nuclear Studies, Cornell University, Ithaca, New York
(Received 9 January 1964; revised manuscript received 12 February 1964)

Recently the p-p elastic-scattering cross section was measured at several energies in the 10- to 30-BeV range and at various center-of-mass angles θ up to 90°.[1,2] For angles outside the diffraction peak, the differential cross section is found to have a very strong dependence on both energy and momentum transfer. According to reference 1, these data may be fit approximately by

$$(\log_{10}X)/P_0^{1/2} = -2(1-0.9\cos^4\theta), \tag{1}$$

where $X=(d\sigma/dt)/(d\sigma/dt)_{t=0}$ and P_0 is the incoming beam momentum in BeV/c in the laboratory system. More recently, Orear[3] found that all these data can be fit within two standard deviations by the simple exponential formula

$$d\sigma/d\Omega = A\exp(-k_\perp/k_0), \tag{2}$$

where $A=34$ mb/sr, $k_0=0.151$ BeV/c, and $k_\perp = k\sin\theta$, k being the momentum of the proton in the center-of-mass system. The remarkable feature of the formula (2) is that it depends only on the transverse momentum $k_\perp$ and not on s or t in any significant way, where $s=4(k^2+m^2)$, $t=-2k^2(1-z)$, $z=\cos\theta$. Actually experimental points seem to show small but systematic s-dependent deviation from the formula (2). A better fit may be obtained using a formula of the type $\exp[-\alpha k_\perp \ln(s/s_0)]$ (α and s_0 are constants), although the present data are perhaps not accurate enough for such a detailed argument.[4]

According to Cerulus and Martin,[5] the upper bound of the scattering amplitude in any interval $(-z,z)$, where $0<z<1$, cannot decrease, under rather broad assumptions, faster than $\exp[-c'k \times \ln s]$ (c' is a constant) as s goes to $+\infty$. As we have seen, on the other hand, the observed elastic-scattering amplitude does not appear to be much larger than this theoretical lower limit. It may thus be worthwhile to explore the possibility that, at very high energies, the elastic p-p scattering takes place with minimum amplitude consistent with the lower limit of Cerulus and Martin.[6]

The purpose of this note is to point out that such an assumption leads us naturally to the energy and angular distribution of the elastic cross section which is very close to Formula (2) at all nonforward and nonbackward angles except near 90° where it is too small.[7] Thus the main feature of the experimental data seems to be consistent with the postulate of minimum amplitude. At present we do not know how this is related to the structure of the nucleon or the properties of the strong interaction.[8] But it might be an indication that the nucleon lacks any outstanding structure at very large s and t and the scattering amplitude is determined there primarily by analyticity. From this viewpoint, other processes such as the π^-p scattering will have a cross section similar to (2) at high energy. The fact that the Cerulus-Martin bound is too low in the neighborhood of 90° seems to be due to the fact that their method loses its effectiveness at 90° because of the singular behavior of their transformation. For a crucial test of the postulate of minimum amplitude, it is therefore necessary to improve the theoretical lower limit in the neighborhood of 90°. Of course, if the discrepancy at 90° persists even when the theoretical lower limit is improved, this postulate will be physically untenable.

Let us first outline the work of Cerulus and Martin. They assume that the scattering amplitude $f(s,z)$ has the following properties:

(a) $f(s,z)$ is analytic and bounded by s^N (N is independent of s and z) in a certain domain D of the cut z plane, where the cuts run from $-\infty$ to $+\rho$ and from $+\rho$ to $+\infty$ ($\rho = 1+t_0/2k^2$ and t_0 is a positive constant).[9,10]

(b) $f(s,z)$ satisifies the inequality

$$|f(s,z)| \leq \exp[-\varphi_a(s)] \tag{3}$$

on the segment of real axis $-a\leq z\leq a$, where $\varphi_a(s)$ is a positive function of s and $0<a<1$. Then, making use of the maximum modulus principle, they show that the inequality

$$|f(s,z)| \leq \exp\left[-\varphi_a(s)\left(1-\frac{\ln r}{\ln R}\right)+N\ln s\frac{\ln r}{\ln R}\right] \tag{4}$$

holds for $a\leq z\leq 1$ (and $-1\leq z\leq -a$), where

$$r=b^{-1}[w+(w^2-b^2)^{1/2}],\quad R=b^{-1}[\rho+(\rho^2-b^2)^{1/2}],$$

$$w=(\rho/z)[\rho-(\rho^2-z^2)^{1/2}],$$

$$b=(\rho/a)[\rho-(\rho^2-a^2)^{1/2}]. \tag{5}$$

At $z=1$, $\ln r/\ln R$ can be reduced to the form[11]

$$\ln r/\ln R = 1 - C t_0^{1/2}/k + O(k^{-2}) \quad (6)$$

as far as $t_0 \ll k^2(1-b^2)$, where

$$C^{-1} = \tfrac{1}{2}\lambda \ln\frac{1+\lambda}{1-\lambda},$$

$$\lambda = (1-b^2)^{1/2} = \left(\frac{2\sin\theta_a}{1+\sin\theta_a}\right)^{1/2}, \quad (7)$$

and $\theta_a = \cos^{-1}a$. Thus (4) becomes

$$|f(s,1)| \le \exp\left[-\varphi_a(s)C\frac{\sqrt{t_0}}{k} + N\ln s\left(1 - C\frac{\sqrt{t_0}}{k}\right)\right]. \quad (8)$$

If one adds the physical requirement that[12] (c) the total cross section approaches a constant for $s \to +\infty$, one derives from (8) the inequality

$$\varphi_a(s) \le \varphi_a{}^0(s) \equiv \frac{(N-1)}{C\sqrt{t_0}}k \ln s - N\ln s. \quad (9)$$

This means that, under the assumptions (a) and (c), the upper bound of the scattering amplitude for $-a \le z \le a$ cannot decrease faster than $\exp[-\varphi_a{}^0(s)]$ or $\exp[-c'k\ln s]$ as s goes to $+\infty$. This is the lower limit of Cerulus and Martin.

Let us now discuss the assumption that the elastic scattering at very high energy takes place with the minimum amplitude consistent with assumptions (a) and (c). By this we mean that $|f(s,a)|$ behaves as

$$\exp\{-[(N-1)/C\sqrt{t_0}]k\ln s + N\ln s\} \quad (10)$$

as a function of $\cos\theta_a$.[13] Since the $\cos\theta_a$ dependence of $\varphi_a{}^0(s)$ is contained in C, we have only to examine (7). For small θ_a we find from (7) that $C^{-1} \simeq 2\sin\theta_a$.[11] Numerical calculation shows that this equality holds within the accuracy of 10% up to $\theta_a = 60°$. In this manner we find that the relation

$$|f(s,a)| \simeq \exp\left[-\frac{2(N-1)}{\sqrt{t_0}}k\sin\theta_a \ln s + N\ln s\right] \quad (11)$$

holds fairly well for $0° < \theta_a \lesssim 60°$ and $120° \lesssim \theta_a < 180°$. As θ_a approaches 90°, this approximation becomes poorer logarithmically $\{[\sin\theta_a - (2C)^{-1}]/\sin\theta_a = -0.4, -0.9, -1.4$ for $\theta_a = 75°$, 85°, and 88°, respectively$\}$.

We want to emphasize here that our theoretical prediction is expressed not by (11), but by Formula (10) itself, which is a definite function of s and θ_a. Thus we are not able to give theoretical justification of the exponential transverse momentum distribution (2) even though (11) has such a form.[14] As was discussed already, the Cerulus-Martin limit is perhaps much lower than the best possible theoretical lower limit in the neighborhood of 90°. If we can find such an improved limit, its angular distribution will be closer to (2) than that of (10). However, it is quite possible that its analytical form is not as simple as (2).

Finally, a remark about the diffraction region: As was noted by Orear,[3] Formula (2) does not apply to angles inside the forward peak. The postulate of minimum amplitude does not seem to be successful there either, since it gives an amplitude of the form $\exp\{[1-\alpha(-t)^{1/2}]\ln s\}$ approximately,[11] which decreases more steeply than the observed behavior $\exp[(1+\beta t)\ln s]$ $(\alpha,\beta > 0)$ in the range $0 > t > -(\alpha/\beta)^2$.[15,16] This might be regarded as an indication of failure of the minimum amplitude postulate. However, it is also possible to regard it as evidence that the Cerulus-Martin limit is not the best possible lower limit in this region. From the latter point of view, we may try to resolve the difficulty by first proving that the scattering amplitude has a certain lower limit of diffraction type at small negative t,[17] and then using it as an input assumption that supersedes assumption (c).

I should like to thank Dr. A. Martin for a stimulating discussion of his work. I should also like to thank Professor H. A. Bethe, Professor J. Orear, and Dr. A. D. Krisch for useful discussions.

---

*Work supported in part by the U. S. Office of Naval Research.

[1]G. Cocconi et al., Phys. Rev. Letters 11, 499 (1963).

[2]W. F. Baker et al., Phys. Rev. Letters 12, 132 (1964).

[3]J. Orear, Phys. Rev. Letters 12, 112 (1964).

[4]A. D. Krisch obtained a good fit of data making use of a Gaussian-type function of $k_\perp$ [Phys. Rev. Letters 11, 217 (1963)]. In our approach, however, it seems to be difficult to obtain a Gaussian function. It should also be noted that if Krisch's formula holds for $s \to +\infty$ the temperedness assumption $|f(s,z)| < s^N$ (N independent of s, z) of Mandelstam representation must be abandoned according to the argument of reference 5.

[5]F. Cerulus and A. Martin, Phys. Letters 8, 80 (1964).

[6]Although Cerulus and Martin treated the scattering of spinless particles, the essential feature of their result will not be affected by the consideration of spin.

[7]The Cerulus-Martin limit is too low at forward angles corresponding to small negative t. This is discussed

at the end of this Letter.

[8]It should be noted that the postulate of minimum amplitude does not necessarily mean that the interaction is weak. The interaction may be as strong as it can be, but it may still give very small amplitude because of destructive interference of different partial waves. This may happen, for instance, if the partial-wave amplitude is a very smooth function of the angular momentum l. This problem is under investigation.

[9]See reference 5 for precise definition of the domain D.

[10]It is possible to replace the domain D by the entire cut z plane in assumption (a). However, this does not improve the result of reference 6 significantly. Details will be discussed elsewhere.

[11]If θ_a is the forward angle corresponding to the fixed momentum transfer t_a, we obtain

$$\ln r/\ln R = (1-y)^{1/2} + O(k^{-1/2})$$

instead of (6), where $y = [t_0/(t_0 - t_a)]^{1/2}$. This leads us to

$$\varphi_a^{\;0}(s) = [(N-1)/y](1-y)[1+(1-y)^{1/2}]\ln s - \ln s$$

instead of (9).

[12]Actually it is enough to assume that the total cross section is a slowly varying function of s.

[13]Here we assume that $|f(s,a)|$ is an even function of $\cos\theta_a$, which is practically the case for p-p scattering.

[14]The approximation (11) is written down simply to facilitate the comparison of our theoretical prediction (10) with the empirical formula (2), and it is useful only for $0° < \theta_a \lesssim 60°$ and $120° \lesssim \theta_a < 180°$. In this angular range, however, (10) may be approximated by various other formulas that work equally well but have no simple $k_\perp$ dependence. For instance, noting that $\sin\theta \simeq 2\sin\frac{1}{2}\theta$ for $0° < \theta < 60°$, we may approximate (10) by a function of the form $\exp[-c(-t)^{1/2}\ln s] + \exp[-c(-u)^{1/2}\ln s]$ instead of (11) in the same angular range, where $u = -2k^2(1+\cos\theta)$. I should like to thank Professor R. Serber and Professor N. Khuri for clarifying comments on this point.

[15]K. J. Foley et al., Phys. Rev. Letters 11, 425 (1963).

[16]This range may be smaller than the diffraction width.

[17]Considerations along the line of L. Van Hove's work [Nuovo Cimento 28, 798 (1963)] might be useful for this purpose.

EXCITED STATES OF BARYONS*

P. Carruthers†
Laboratory of Nuclear Studies and Laboratory of Atomic and Solid State Physics, Cornell University, Ithaca, New York
(Received 16 January 1964)

The purpose of this Letter is to generalize to baryon-pseudoscalar meson scattering the model proposed[1,2] recently for the higher resonances in pion-nucleon scattering. The most complete and promising results are found for the case of unitary symmetry[3] wherein the baryons and mesons are members of octets, interacting through forces invariant under the transformations of the group SU(3). The results predict and give a physical mechanism for the "Regge recurrences" of the baryon octet and a $P_{3/2}$ decuplet.[4,5] In analogy to the πN "superbootstrap," the exchange (see Fig. 1) of the particles and resonances on the baryon

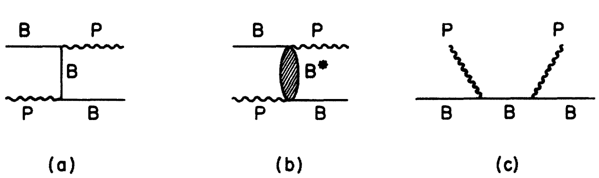

FIG. 1. (a) The forces due to the exchange of baryons B and (b) baryon resonances B^* in pseudoscalar meson (P) baryon scattering are shown, along with (c) the direct pole terms.

trajectory ($P_{1/2}, F_{5/2}, \cdots$) give rise to a decuplet trajectory ($P_{3/2}, F_{7/2}, \cdots$) whose exchange generates the states of the octet trajectory.

To begin we summarize the results of reference 2, carefully distinguishing between the isospin and angular-momentum factors in order to facilitate generalization. For clarity we make two inessential approximations: (1) When considering states of orbital momentum l, we neglect the exchange of states with $l' > l$; (2) the "static" crossing matrix is used. By (2) we do not mean that only states of the same l are coupled, but rather that reasonable kinematical approximations are made,[6] so that the contribution to the amplitude $h_{l\pm} = \exp(i\delta_{l\pm})\sin\delta_{l\pm}/k^{2l+1}$ belonging to an irreducible representation (labeled m) of some group due to crossed scattering in (i.e., exchange of) a state of orbital momentum l' and representation n is[7]

$$\frac{C_{mn}\Gamma_{ll'}}{2\pi k^2}\int_{(M+\mu)^2}^{\infty} ds' k^{2(l'-l)} Q_{l-l'}\left[1+\frac{s'-u_0(s)}{2k^2}\right] \times \operatorname{Im} h_{l'\pm}(s'), \qquad (1)$$