

that it be small enough for electromagnetic radiative corrections to be negligible. Then Z_i is replaced by $Z_i^{(s)}$ and the condition for the matrix element of the current density between nucleon states of four-momentum p and p' reads

$$\langle p' | j_\mu(0) | p \rangle = Z_1^{(s)} \bar{Q} u(p') \gamma_\mu u(p), \quad (8)$$

where $u(p)$ is the usual spinor. Since Eq. (8) is a four-vector condition it clearly contains two statements about the form factors, one for the spacelike part of the vector and another for the timelike part. This can be seen most easily in the Breit frame, $\vec{p}' = -\vec{p}$. There, Eq. (8) reads

$$\langle p' | j_4(0) | p \rangle = i Z_1^{(s)} Q, \quad (9a)$$

$$\langle p' | \vec{j}(0) | p \rangle = i(\vec{\sigma} \times \vec{q}) Z_1^{(s)} Q / 2M. \quad (9b)$$

On the other hand, the general expression for the matrix element in terms of the form factors in the Breit frame is⁴

$$\langle p' | j_4(0) | p \rangle = i G_E(q^2), \quad (10a)$$

$$\langle p' | \vec{j}(0) | p \rangle = i(\vec{\sigma} \times \vec{q}) G_M(q^2) / 2M. \quad (10b)$$

Comparison of Eqs. (9) and (10) leads directly to Eqs. (3).

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⁵We note that in a direction calculation of F_2 treating the pion interaction by perturbation theory, I. K. Hiida, N. Nakanishi, Y. Nogami, and M. Uehara, Progr. Theoret. Phys. (Kyoto) **22**, 247 (1959), find that F_2 does not satisfy Eq. (8b). Although F_2 does not require renormalization, it is essential that it be calculated in a properly cut-off theory if the conditions of reference 2 are to be met. The essential condition is that $q^2 \gg \Lambda^2$. There is some indication from a preliminary calculation of L. E. Evans and K. C. Wali (private communication) that a properly cut-off perturbation theory will lead to the result $q^2 F_2(q^2, \Lambda^2) \rightarrow 0$ for $q^2 \gg \Lambda^2$.

⁶G. Källén explicitly stated this result for quantum electrodynamics, in *Handbuch der Physik* (Springer-Verlag, Berlin, 1958), Vol. V, Pt. 1, pp. 362-363. He also generalized the suggestion to other field theories in unpublished lectures. I am indebted to Dr. W. Theis for calling to my attention the connection with Källén's results that is discussed here.

MUON POLARIZATION IN $K_{\mu 3}^+$ DECAY

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Investigation of $K_{\mu 3}$ decay permits verification of the applicability of the universal $V-A$ interaction to lepton decays of strange particles and also determination of the characteristics of the interaction form factors. For a two-component neutrino, the μ mesons generated in a $\pi \rightarrow \mu + \nu$ or $K_{\mu 2} \rightarrow \mu + \nu$ decay are known to be completely polarized longitudinally, whereas in a three-particle decay the μ -meson polarization depends on space correlation of the decay particles, i. e., on the mode of interaction realized in the decay.

As shown by Pais and Treiman,¹ the matrix element for a K_{l3} decay depends on two scalar form factors and can be written as follows (see, e. g., Brene, Egardt, and Qvist²):

$$M = [\frac{1}{2} f_+(P_K + P_\pi) + \frac{1}{2} f_-(P_K - P_\pi)] \bar{\mu}(P_\nu) \gamma_\lambda (1 + \gamma_5) \nu(P_l), \quad (1)$$

where f_+ and f_- are form factors due to strong interactions, and P is the particle momentum.

According to the theory of the universal $V-A$ interaction, the matrix element describes both the K_{e3} and $K_{\mu 3}$ decays. Assuming the form factors to be constant, Gatto,³ Zachariasen,⁴ and Fujii and Kawaguchi⁵ estimated the ratio $\xi = f_-/f_+$. The decay-branching ratio, known from experiment,⁶ can likewise be determined from the expression (1). It is found to be a quadratic function of the parameter ξ :

$$W(K_{\mu 3})/W(K_{e3}) = 0.651 + 0.126\xi + 0.0189\xi^2.$$

A solution of this equation gives two values of ξ , to which correspond different shapes of the μ -meson spectrum in $K_{\mu 3}$ decay. This difference, however, is large only in the low-energy region, where measurements are difficult to carry out. It would therefore be more convenient to measure

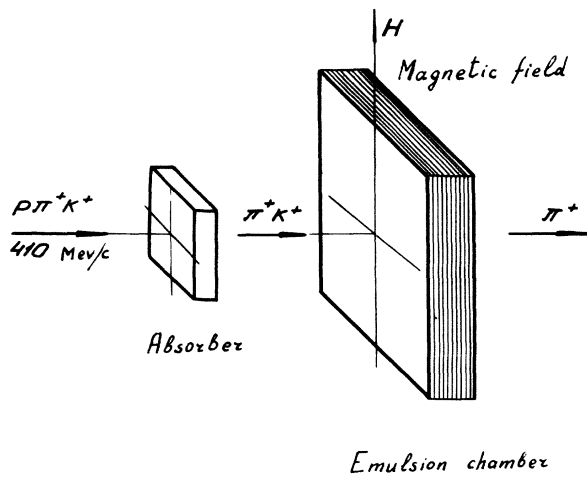


FIG. 1. Schematic drawing of emulsion chamber placement in K^+ beam.

the μ -meson longitudinal polarization, in order to select an appropriate value of ξ .

It was shown by Matinyan and Okun⁷ that the sign of the longitudinal polarization depends on the parameter ξ . Thus, measurements of longitudinal polarization of μ mesons from $K_{\mu 3}^+$ decay make it possible to determine the form factors and to verify the applicability of the $V-A$ interaction scheme to three-particle K -meson decays.

To this end an emulsion chamber of R emulsion, of volume of about 1 liter, was exposed to a non-separated beam from the proton synchrotron of the Institute for Experimental and Theoretical Physics, having a momentum of 410 MeV/c. The beam was extracted at an angle of 60° to the di-

rection of the primary protons and was focused at the center of the emulsion chamber by means of two quadruple lenses and a deflecting magnet. The length of the beam path was close to 5 meters and the intensity was about $30 K^+$ mesons per 10^{10} protons, with a beam diameter of about 50 mm.

To avoid overloading the volume of the chamber by K^+ -meson and pion tracks, the emulsion plane was placed normal to the beam. The diagram of the experimental arrangement is shown in Fig. 1. The protons contained in the beam were slowed down to a carbon absorber and the K^+ mesons stopped at the center of the emulsion chamber. For pions of the same momentum the relatively thin chamber was practically transparent. The method of normal incidence permitted about $5 \times 10^5 K^+$ mesons to be stopped in a small volume of the emulsion. To prevent muon depolarization, the chamber was placed in a 6-kG magnetic field normal to the beam and directed vertically parallel to the emulsion plane.

The search for $\mu^+ \rightarrow e^+$ decays was carried out by area scanning. Subsequent following of the muon track makes it possible to establish from the muon path and direction of motion whether it was generated in a $\pi^+\mu^+$ or a $K_{\mu 3}^+$ decay. Scanning was carried out only over a relatively small part of the emulsion, in which most of the $K_{\mu 3}^+$ -decay μ^+ mesons stopped ($T_\mu = 40-100$ MeV). π^+ mesons from $K_{\pi 2}^+$ and partly from $K_{\pi 3}^+$ decay also stop in this region. π^+ -meson $\mu^+ \rightarrow e^+$ decays were used for checking the experimental results. To exclude cosmic-ray μ mesons from polarization measurement data, only that part of

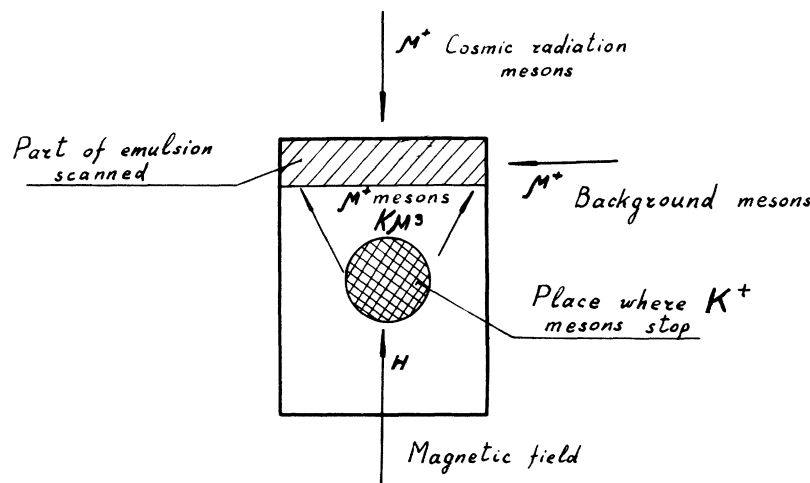


FIG. 2. Location of the region where K^+ mesons stop and of scanned part of emulsion.

Table I. Measured values of space asymmetry coefficient for $\mu^+ \rightarrow e^+$ -decay positrons.

Decay mode	Asymmetry	Expected value a	Measured value $a \pm \Delta a$	Number of events
$\pi^+ \mu^+ e^+$	Forward-backward	-0.25	-0.22 ± 0.04	3354
	Right-left	0	0.02 ± 0.04	
$\mu^+ e^+ (K_{\mu 3}^+)$	Forward-backward	+0.10 (for $\xi = +2$) or -0.12 (for $\xi = -9$)	$+0.18 \pm 0.11$	309
$\mu^+ e^+$ (Background)	Right-left	0	0.04 ± 0.09	507

emulsion was used in which $K_{\mu 3}^+$ -decay muons were directed upwards along the magnetic field (Fig. 2).

The μ^+ -meson polarization was measured from the asymmetry of $\mu^+ \rightarrow e^+$ decay angular distribution. The probability of μ^+ decay in a magnetic field is known⁸ to depend on the cosine of the angles θ_μ and θ_e between the magnetic field and the directions of the μ^+ -meson and positron momenta, respectively. It is expressed by the equation

$$dN \sim (1 + a \cos \theta_\mu \cos \theta_e) d\theta_\mu d\theta_e,$$

where a is the asymmetry coefficient. By measuring the forward-backward asymmetry (in the direction of the magnetic field and in the opposite direction), the sign and absolute value of μ^+ -meson polarization can be determined.^{9,10}

Scanning of the chamber revealed in the plates the presence of background μ^+ mesons, whose direction of motion coincides with the orbital plane of the accelerator and is practically normal to the magnetic field (Fig. 2). For these μ^+ mesons the $\mu^+ e^+$ -decay asymmetry along the magnetic field and in the opposite direction should be close to zero (right-left asymmetry). These decays were used for checking the results of measuring $K_{\mu 3}^+$ -decay muon polarization.

The experimental values of the space asymmetry coefficient are summarized in Table I. The asymmetry coefficient was determined from formula $a = K(N_f - N_b)/(N_f + N_b)$ in which $K = 1.94$ is a factor depending on the geometry of the experiment. N_f and N_b are the numbers of decay events in which the positron is directed forward or backward (right or left for background muons).

The statistical error is given by

$$\Delta a \cong 2/(N_f + N_b)^{1/2}.$$

From the data of Table I it follows that the observed asymmetry coefficients for μ^+ mesons generated in $\pi^+ \mu^+$ and in $K_{\mu 3}^+$ decays have different signs. The measured asymmetry coefficient listed for $K_{\mu 3}^+$ muons gives a longitudinal muon polarization of 0.70 ± 0.45 . The angular distribution of positrons resulting from $K_{\mu 3}^+$ -muon decay is shown in Fig. 3, where the abscissas represent values of $\cos \theta_\mu \cos \theta_e$. The dashed line corresponds to the measured value $a = +0.18$. Estimation of the agreement between the angular distribution and the dashed line by the use of the χ^2 criterion yields a value of χ^2

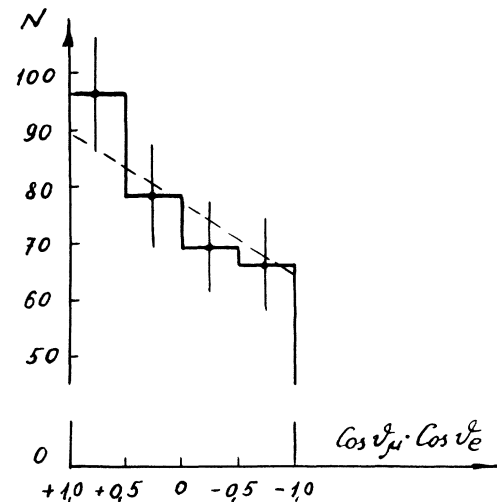


FIG. 3. Angular distribution of positrons originating in the decay of $K_{\mu 3}^+$ muons.

Table II. Measured values of the form-factor ratio in $K_{\mu 3}^+$ decay.

Authors	Method of measurement	Number of events	In accordance with ξ
J. M. Dobbs <i>et al.</i> ^a	Muon spectra measured by means of a filament scintillating chamber	127	-9
J. L. Brown <i>et al.</i> ^b	π^0 -meson spectra measured in a xenon bubble chamber	76	+2
A. M. Boyarski <i>et al.</i> ^c	Muon spectra and polarization by means of a scintillating hodoscope	122 (spectra) 50 (polarization)	-9 +2
Authors of this work	Measurement of muon polarization in nuclear emulsion	309	+2

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^bJ. L. Brown, J. A. Kadyk, G. H. Trilling, R. T. Van der Walle, B. P. Roe, and D. Sinclair, *International Conference on High-Energy Nuclear Physics, Geneva, 1962* (CERN Scientific Information Service, Geneva, Switzerland, 1962), p. 462; J. L. Brown, J. A. Kadyk, G. H. Trilling, R. T. Van der Walle, B. P. Roe, and D. Sinclair, *Phys. Rev. Letters* **8**, 450 (1963).

^cA. M. Boyarski, E. C. Loh, L. Q. Niemela, D. M. Ritson, R. Weinstein, and S. Ozaki, *Phys. Rev.* **128**, 2398 (1962).

= 1.4, the expected value being $\chi^2 = 3$.

Despite the low statistical accuracy of the experimental results, the value of polarization $P = -0.5$ corresponding to $\xi = -9$ could be excluded with a probability greater than 95%.

Of the two possible values of the form-factor ratio, the above results favor the choice $\xi = +2$. This ratio is known¹¹ to correspond to an "effective-vector" interaction, whereas the value $\xi = -9$ corresponds to an "effective-scalar" interaction.

In Table II are given the experimental results obtained by various authors for the form-factor ratio in $K_{\mu 3}^+$ decay.

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scanning team.

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¹¹L. Okun' (to be published).