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SECONDARY DIFFRACTION PEAK IN PION-NUCLEON SCATTERING*

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It is commonly believed that the forward peaking of the elastic-scattering differential cross section at high energies is due principally, if not exclusively, to the absorption of the incident wave into the many available inelastic channels. In a recent Letter,[1] Damouth, Jones, and Perl have reported their measurement of π^-p and π^+p differential cross sections at 2.01 GeV/c and 2.02 GeV/c, respectively. These data exhibit not only the well-known forward diffraction peak, but also a secondary peak at about $\cos\theta = 0.2$ (center-of-mass angle). Damouth, Jones, and Perl draw the tentative conclusion that this second peak is related to the 2.08-GeV/c π^-p total cross-section maximum,[2] even though the π^+p data exhibit a secondary peak which is only slightly less pronounced.

It is the purpose of this note to point out that the second maximum can be interpreted quite naturally as a secondary diffraction peak, according to a most elementary optical model. The idea of considering the effects of diffraction scattering at angles away from the forward direction is certainly not new.[3,4] In particular, Serber[4] has pointed out that an optical model, applied for large momentum transfer, can explain some of the features of the p-p differential cross section.

For simplicity, let us assume that all partial waves for $l \leq L$ are equally absorbed, that those for $l > L$ do not interact at all, and that the partial-wave amplitudes are purely imaginary. The scattering amplitude is then[5]

$$f(\theta) = \sum_{l=0}^{L} (2l+1) f_l P_l(\cos\theta), \tag{1}$$

where

$$f_l = i[1 - \eta_l \exp(2i\delta_l)]/2k = i(1-\eta)/2k, \quad l \leq L,$$
$$f_l = 0, \quad l > L; \tag{2}$$

$$f = i(1-\eta)[P_L'(\cos\theta) + P_{L+1}'(\cos\theta)]/2k \tag{3}$$

and

$$d\sigma/d\Omega = |f(\theta)|^2. \tag{4}$$

It should be emphasized that the above choices of the values of η_l and δ_l are made purely for convenience (or, perhaps more properly, because we have no theory upon which to base a more accurate choice). In particular, the prediction which this model makes of secondary diffraction maxima does not depend on the choice of a sharp cutoff in l. A "diffuse edge" can be added to the nucleon by adding to (3) small amounts of $P_l(\cos\theta)$ for some values of $l > L$. Such additions will, in general, not alter the qualitative fact that secondary maxima are predicted. They will, of course, affect the quantitative details such as position and amplitude, just as would other choices of the values of η_l for $l \leq L$. Note that the expression (3) can only be accurate for those values of θ at which one can neglect dynamical elastic scattering compared to diffraction scattering. Thus, one must discount the prediction that $d\sigma/d\Omega$ vanishes at certain angles.

From (3) one can calculate the slope of $d\sigma/d\Omega$ as a function of $\cos\theta$, and compare the result with the data, at small angles, where the model should be most accurate. The result is quite sensitive to L, and requires, for the data of Damouth, Jones, and Perl, $3 \leq L < 4$, a very sensible result at this energy.

The over-all scale is set by

$$|f(0)|^2 = [(1-\eta)(L+1)^2/2k]^2 = (d\sigma/d\Omega)_0,$$

so

$$\frac{d\sigma}{d\Omega} = \left(\frac{d\sigma}{d\Omega}\right)_0 \left[\frac{P_L'(\cos\theta) + P_{L+1}'(\cos\theta)}{(L+1)^2}\right]^2. \quad (5)$$

We can determine the scale factor $(d\sigma/d\Omega)_0$ in one of several ways, namely, by extrapolating the experimental differential cross section to $\cos\theta = 1$, by using the optical theorem, or by normalizing (5) to agree with the data at some convenient value of $\cos\theta$ (in the forward peak). This last course seems to be the most reasonable, especially in view of the fact that the data contain an over-all normalization uncertainty.

The predictions of (5) with $L = 3$ are compared with the data of Damouth, Jones, and Perl in Figs. 1 and 2. The agreement for $\cos\theta > 0$ is rather striking, except for the zero near $\cos\theta = 0.6$, which is of no consequence. At backward angles, we must postulate the existence of other contributions to $d\sigma/d\Omega$, which modify the predictions of (5). These contributions are, in fact, expected to be present since (5) ignores all dynamical elastic scattering, and results from a very simple model of diffraction scattering. If we suppose that the model is approximately correct, then modifications can be expected to be most important in the backward hemisphere, where (5) predicts a relatively small cross section.

From a phenomenological point of view, it should not be difficult to improve the fit, but this is not our purpose. The point to be emphasized is that this simple and familiar model can account, in a thoroughly reasonable way, for the

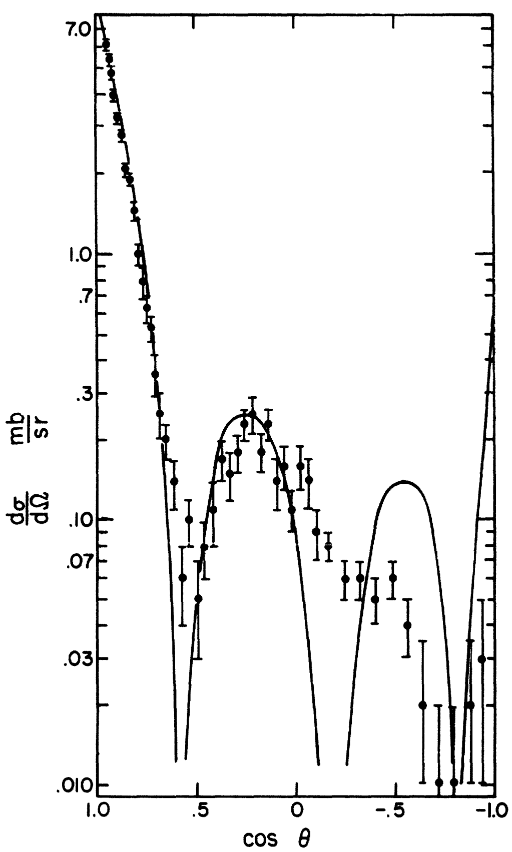

FIG. 1. π^-p elastic-scattering differential cross section in the center-of-mass system. The data points are from Damouth, Jones, and Perl (reference 1). The solid curve is the prediction of Eq. (5) with $L = 3$, normalized to agree with the data at $\cos\theta = 0.8$.

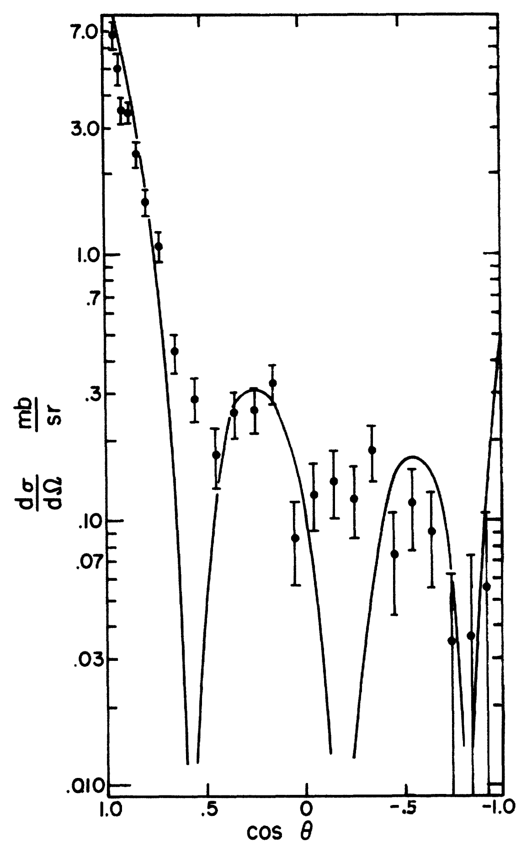

FIG. 2. π^+p elastic-scattering differential cross section in the center-of-mass system. The data points are from Damouth, Jones, and Perl (reference 1). The solid curve is the prediction of Eq. (5) with $L = 3$, normalized to agree with the data at $\cos\theta = 0.8$.

position, height, and width of the secondary peak, in addition to the expected agreement with the forward peak. In the face of such a simple explanation, it seems unwarranted to search for a more complex mechanism. In particular, the forward peak is not expected to be nearly so informative about possible resonant contributions as the backward scattering. The diffraction contribution to $f(\theta)$ in the forward direction has a weight $(L+1)^2$, where L is the highest partial wave which is appreciably absorbed. Therefore, it will be comparatively difficult for a dynamical contribution, whose weight will be $\lesssim(2l+1)$, to significantly alter the shape of the differential cross section at very forward angles. Outside the forward peak such alteration is to be expected, but it can only be interpreted reliably if the diffraction scattering is understood in detail.

It is to be expected that the secondary maxima will change in character (that is, strength and position) at other energies, since both η_l and L are functions of energy. It is obviously of interest to know whether such maxima exist at other energies, but the data appear to be inconclusive. It is clear from the results of Perl, Jones, and Ting[3] that the rapid decrease of $d\sigma/d\Omega$ with increasing θ does not persist at angles outside the extreme forward peak. Whether there is a second maximum or merely a leveling off is difficult to say. The data of Cook et al.[6] seem to show some secondary bumps, but the statistical errors are rather large. The results of Helland[7] for π^+p scattering at 1555 MeV show a definite bump near $\cos\theta=0.3$, which would be consistent with the present model. The statement that a secondary maximum is known not to exist at other energies would appear to be unjustified, at present. In any case, the attempt to associate the secondary maximum observed by Damouth, Jones, and Perl with the 2.08-GeV/c π^-p resonance entails considerable difficulty, since this second maximum appears to occur in both π^-p and π^+p scattering.

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NUCLEON ELECTROMAGNETIC FORM FACTORS AT HIGH MOMENTUM TRANSFER*

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The recently reported experimental results of Chen et al.[1] on the electromagnetic form factors of the proton at large values of the invariant momentum transfer, q^2, are consistent with the relationship

$$\lim_{q^2\to\infty} G_E^{\,p}(q^2)/G_M^{\,p}(q^2)=1 \tag{1}$$

suggested[2] earlier on the basis of physical arguments concerning the structure of the nucleon. If further experimental work confirms this result, we may draw the conclusion that the nucleon is indeed a fundamental particle rather than a composite generated by some sort of bootstrap mechanism as suggested, for example, by Chew and Frautschi.[3]

On the other hand, the claim by Chen et al.[1] that their data are consistent with

$$\lim_{q^2\to\infty} G_E^{\,p}(q^2)=\lim_{q^2\to\infty} G_M^{\,p}(q^2)=0, \tag{2}$$

namely, that there is no "core contribution" to the proton form factors, might be interpreted in just the opposite way. It is the purpose of this note to show that there need be no contradiction between the two results and that, in fact,