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PROPERTIES OF HIGH-SPIN ROTATIONAL STATES IN NUCLEI*

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It is well established that nuclei in the region $150 < A < 190$ have sizable prolate-spheroidal deformations, and consequently have well-defined rotational bands built on the various intrinsic levels. In the present work rather high-lying members of the ground-state rotational bands of a number of even-even nuclei in this region have been studied following heavy ion reactions of the type ^{165}Ho(^{11}B, $4n$)^{172}Hf. Odd-proton-number targets (^{159}Tb, ^{165}Ho, and ^{169}Tm) and projectiles (^{11}B, ^{14}N, and ^{19}F) have been used to produce ^{166}Yb, ^{166,168,170,172}Hf, and ^{172,174,176}W. The bombarding energy has been adjusted in each case to give predominantly the particular even-even nucleus desired. We have studied the conversion electron and gamma-ray spectra from such targets during the 3-msec beam burst of the Hilac in order to observe the de-excitation of the final nucleus to its ground state.[1]

The rotational transitions connecting states having spins up to about 12 were normally very well defined, and could be identified without question. Some of the transitions between higher spin states were equally well defined, but others were included among several unassigned transitions of comparable energy and intensity. Obviously, these latter assignments are not completely certain, and for the present discussion we have included only transitions whose association with the ground-state rotational band is highly probable. A thorough discussion of assignments will be presented, together with the detailed experimental data, in a more complete publication.

From the transition energies, $E(I \to I-2)$, we define the rotational constant, A_I, as follows:

$$A_I = \hbar^2/2\mathfrak{J}_I = E(I \to I-2)/(4I-2), \tag{1}$$

where $\mathfrak{J}_I$ represents the moment of inertia appropriate to the transition. In Fig. 1 $\log A_I$ has been plotted versus I for the nuclei studied. At low spins, the general features of this plot are well known: (1) a regular decrease in A_I with increasing spin, and (2) smaller slopes (more perfect rotors) associated with lower A_I values. The similarity in rotational properties of all these isotopes at higher spins is very pronounced. The points in all cases (except possibly ^{166}Hf for which there is no information above spin 12) are, or become with increasing spin, quite linear with a common limiting slope of 8 or 9% decrease in A_I per state Furthermore, the absolute A_I values are converging into two groups. Thus, for the $14 \to 12$ transition, five of the seven cases for which there is information have A_{14} values within 2% of 11.05 keV and the other two cases both have A_{14} values of 10.17 keV, within our limits of error (0.3%). It cannot be ruled out that at still higher spins these groups will diverge again; however, our most tentative data at the highest spins rather suggests that they may converge to a single group.

The two nuclei with low A_I values (^{172}W and ^{170}Hf) both have 98 neutrons, and a cursory exam-

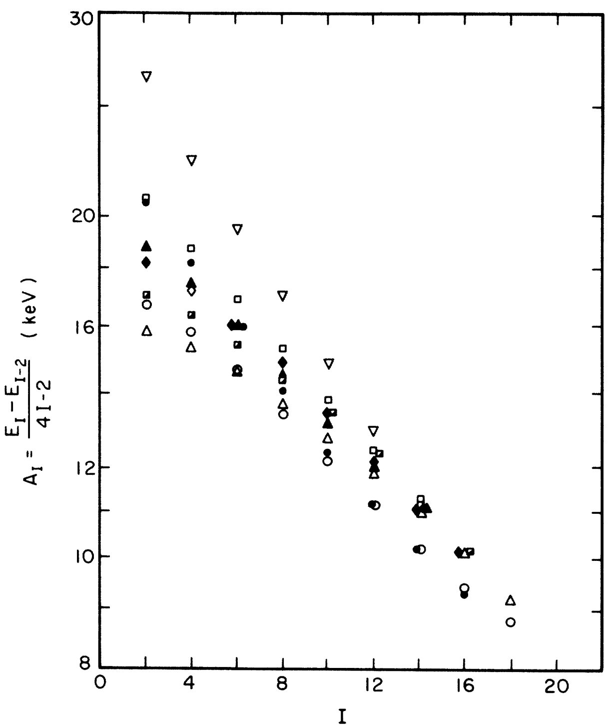

FIG. 1. Rotational constants (appropriate to the transitions) plotted against spin, I. The points are coded as follows: closed symbols, tungsten; open symbols, hafnium; half-closed symbols, ytterbium: neutron number 94, inverted triangles; 96, squares; 98, circles; 100, triangles; 102, diamonds.

ination of the (α, xn) data taken by the Amsterdam group[2] suggests that ^{168}Yb and ^{166}Er, with 98 neutrons, behave similarly. Furthermore, from a plot of first excited-state energies against neutron number in the rare-earth region, it appears that the energies for nuclei having 98 neutrons are low. Thus from the lowest to the highest spins observed, such nuclei seem to have rotational constants 5-10% lower than others in the region studied here (94-102 neutrons inclusive). A possible explanation is that this is due to a reduction of the pairing correlations due to the energy gap in the Nilsson diagram between the levels $\frac{5}{2}^-$ [523] (98 neutrons) and $\frac{7}{2}^+$ [633]. This effect seems to be reproduced in Nilsson and Prior's calculations of the moments of inertia based on the pairing model.[3]

There are a number of models with which these rotational energies can be compared. We have not attempted a thorough examination of all such models partly because sizable computations are required in some cases, and partly because some general observations have suggested to us certain essential features required in such a model. These observations are: (1) The very nearly identical behavior of the moments of inertia observed at high spin values for the nuclei studied (with the notable exception discussed in the preceding paragraph) suggests that a very general property of rotating nuclei must be involved; (2) the evidence accumulating from the study of vibrational states in deformed nuclei indicates that the β band is admixed into the ground-state rotational band some 10 times more heavily than is the γ band, and the deviations of the ground-state band from a perfect rotor can be largely accounted for at low spins by a perturbation treatment of such β-ground mixing[4,5]; and (3) the average change in moment of inertia with spin observed in this study is about a factor of two, which indicates that an attempt to explain this should avoid using perturbation theory. In a model, therefore, we must look primarily for a nonadiabatic treatment of the coupling between β vibrations and rotation. Within the framework of the hydrodynamic model developed by Bohr and Mottelson,[6] the solution of this problem has been given by Davydov and Chaban (DC) in conjunction with their treatment of asymmetric rotors.[7] We have used their treatment, but in accordance with observation (2) above we have set γ (asymmetry parameter) equal to zero and, therefore, have looked only at the effects calculated due to nonrigidity with respect to β deformation. As mentioned by DC this amounts in the ground-state band to accounting for the centrifugal expansion of the nucleus, and hence it seems a priori most consistent with observation (1) above.

A comparison of the ^{170}Hf data with the DC solution is shown in Fig. 2. The ordinate here is the ratio A_{I+2}/A_I, which is related to the slope of Fig. 1, and is used primarily because it gives a plot which is very sensitive to the transition energies.[8] Our error of ±0.25% is indicated on one of the points (the ratio of two transition energies can be measured more accurately than either one absolutely). For comparison, the effect of the usual $I^2(I+1)^2$ correction term is shown, fitted to the 2^+ and 4^+ energies, and also that of a three-parameter expression involving the third term in the series, $I^3(I+1)^3$, fitted to the 2^+, 4^+, and 6^+ energies. The DC calculation as used here $(\gamma \equiv 0)$ has two parameters, one of which $(\hbar\omega_0)$ cancels out in the ratio of two energies as plotted, so that there is only one adjustable parameter (μ). Similarly, the plots

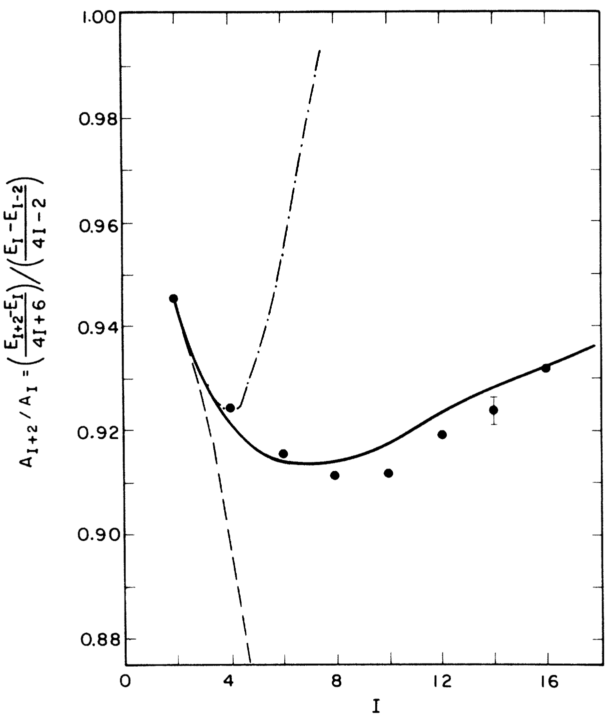

FIG. 2. Ratios of successive rotational constants, A_{I+2}/A_I, plotted against spin, I. The circles are the experimental data for ^{170}Hf, and the curves are as follows: solid line, the nonadiabatic Davydov-Chaban calculation with $\gamma=0$; dashed line, $AI(I+1)+BI^2(I+1)^2$ fitted to the 2^+ and 4^+ energies; and dot-dash line, the previous expression plus the term $CI^3(I+1)^3$ fitted to the 2^+, 4^+, and 6^+energies.

of the two comparison expressions have one and two adjustable parameters, respectively. This case, ^{170}Hf, represents one of the best fits of our eight nuclei to the DC model, but the others are not qualitatively different. The average deviation of the experimental points from the calculated curve for ^{170}Hf is 0.26%; for seven of the eight cases (49 points) it is 0.62%. The remaining case, ^{166}Hf, will be discussed later. We consider the agreement to be surprisingly good (and for level rather than transition energies the percent deviations would be much smaller). There are, however, systematic deviations. For all cases there is a tendency for the points to fall below the theoretical curves at the highest spins. Another way to express this is to say that the common limiting slope in Fig. 1 is 8 or 9% per state, whereas the DC common limiting slope is about 5-6% per state (the calculations should be extended above spin 20 to be sure of this number). The other systematic deviation only occurs when the first excited state is above roughly 105 keV. Then the first points fall below the calculation and progressively more so as the energy of the first excited state increases. Thus, for ^{166}Hf (first excited state 159 keV) the first point is experimentally about 5% low, and there is no real fit to the calculated curve. However, it does not seem to us reasonable to expect such a simple model to account for all deviations from the perfect rotor, particularly near the edge of the region of deformed nuclei (high first-excited-state energy).

Since our use of the DC calculation, with $\gamma\equiv0$, accounts for the rotational spacings quite impressively, we were interested to see whether the energy of the β-vibrational band could be correctly predicted using the same restriction. Table I summarizes the data for all cases we could find where both the β energy and the 2^+ and 4^+ ground-state band members are accurately known. The ratio of the β-band energy (0^+ state) to the first-excited-state energy is calculated from the ground-state band members and compared with the experimentally observed ratio. In all cases the DC ratios are high, and the last column gives the percentage error in the calculation. For the rare earths seven cases lie within the range $+8 \pm 5$%. In the one exception, ^{178}Hf, the 0^+ levels have been assigned as predominantly two-quasi-particle states,[9] and this, if correct, would mean the real β band might lie higher, as predicted. In the heavy elements, all cases fall within the range $+20\pm10$%. We feel the absolute agreement is not bad, and the higher internal consistency for each group is very encouraging.

In summary we can say that ground-state rotational band members for several nuclei have been identified to spin 14, and in a few cases, very probably to spin 16 or 18, and their energies measured to about ±0.3%. These energies provide a good test of various models for rotational bands, and impressive agreement is observed with a simplified ($\gamma\equiv0$) Davydov-Chaban calculation involving the centrifugal stretching of the nucleus along the symmetry axis. The closely related β-vibrational band energies are given to an accuracy of about 20%, with higher relative precision, within each of the two groups of deformed nuclei surveyed (rare earths and actinides). We consider this quite surprisingly good inasmuch as only two parameters are involved, and no corrections have yet been included for the γ (nonaxial) vibrations, or any other type of perturbation.

Table I. Beta-vibrational energies.

Nuclide	$E(2\to 0)$ (keV)	$E(4\to 2)$ (keV)	$E(0'\to 0)$ (keV)	$\left.\frac{E(0'\to 0)}{E(2\to 0)}\right\|_{\rm calc}$	$\left.\frac{E(0'\to 0)}{E(2\to 0)}\right\|_{\rm obs}$	Δ (%)
^{240}Pu	42.87	98.9	858[a]	24.9	20.0	+20
^{238}Pu	44.11	101.9	943.1[b]	26.5	21.38	+19
^{238}U	$\bar{\mu}=0.180$[c]		993[d]	29.0	22.2	+23
^{234}U	43.49	99.8	811.6[e]	20.6	18.66	+10
^{232}U	47.6	109.0	692.9[f]	19.5	14.6	+25
^{232}Th	$\bar{\mu}=0.232$[c]		730[d]	17.3	14.7	+15
^{230}Th	53.15	120.8	634[g]	16.3	11.9	+27
^{188}Os	155.03	322.94	{1086[h] 1766}	7.2	{7.01 11.4}	{+3 -58}
(^{178}Hf	93.17	213.42	{1197[j] 1440}	19.6	{12.9 15.5}	{+34 +21})[i]
^{166}Er	80.57	265.1	1460.3[h]	19.5	18.12	+7
(^{158}Gd	79.5	182.4	1427[k]	20.5	17.9	+13)[i]
^{156}Gd	88.97	199.19	1040[l]	13.0	11.7	+10
^{154}Gd	123.07	248.08	680.6[m]	6.0	5.53	+8
^{152}Sm	121.79	244.84	685.0[h]	6.0	5.62	+7
(^{150}Nd	131	259	687[l]	5.5	5.2	+5)[i]

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SECONDARY DIFFRACTION PEAK IN PION-NUCLEON SCATTERING*

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It is commonly believed that the forward peaking of the elastic-scattering differential cross section at high energies is due principally, if not exclusively, to the absorption of the incident wave into the many available inelastic channels. In a recent Letter,[1] Damouth, Jones, and Perl have reported their measurement of π^-p and π^+p differential cross sections at 2.01 GeV/c and 2.02 GeV/c, respectively. These data exhibit not only the well-known forward diffraction peak, but also a secondary peak at about $\cos\theta = 0.2$ (center-of-mass angle). Damouth, Jones, and Perl draw the tentative conclusion that this second peak is related to the 2.08-GeV/c π^-p total cross-section maximum,[2] even though the π^+p data exhibit a secondary peak which is only slightly less pronounced.

It is the purpose of this note to point out that the second maximum can be interpreted quite naturally as a secondary diffraction peak, according to a most elementary optical model. The idea of considering the effects of diffraction scattering at angles away from the forward direction is certainly not new.[3,4] In particular, Serber[4] has pointed out that an optical model, applied for large momentum transfer, can explain some of the features of the p-p differential cross section.

For simplicity, let us assume that all partial waves for $l \leq L$ are equally absorbed, that those for $l > L$ do not interact at all, and that the partial-wave amplitudes are purely imaginary. The scattering amplitude is then[5]

$$f(\theta) = \sum_{l=0}^{L} (2l+1) f_l P_l(\cos\theta), \tag{1}$$

where

$$f_l = i[1 - \eta_l \exp(2i\delta_l)]/2k = i(1-\eta)/2k, \quad l \leq L,$$
$$f_l = 0, \qquad l > L; \tag{2}$$

$$f = i(1-\eta)[P_L{}'(\cos\theta) + P_{L+1}{}'(\cos\theta)]/2k \tag{3}$$

and

$$d\sigma/d\Omega = |f(\theta)|^2. \tag{4}$$

It should be emphasized that the above choices of the values of η_l and δ_l are made purely for convenience (or, perhaps more properly, because we have no theory upon which to base a more accurate choice). In particular, the prediction which this model makes of secondary diffraction maxima does not depend on the choice of a sharp cutoff in l. A "diffuse edge" can be added to the nucleon by adding to (3) small amounts of $P_l(\cos\theta)$ for some values of $l > L$. Such additions will, in general, not alter the qualitative fact that secondary maxima are predicted. They will, of course, affect the quantitative details such as position and amplitude, just as would other choices of the values of η_l for $l \leq L$. Note that the expression (3) can only be accurate for those values of θ at which one can neglect dynamical elastic scattering compared to diffraction scattering. Thus, one must discount the prediction that $d\sigma/d\Omega$ vanishes at certain angles.

From (3) one can calculate the slope of $d\sigma/d\Omega$ as a function of $\cos\theta$, and compare the result with the data, at small angles, where the model should be most accurate. The result is quite sensitive to L, and requires, for the data of Damouth, Jones, and Perl, $3 \leq L < 4$, a very sensible result at this energy.