

FIG. 2. Plot of the separation of the $(\frac{1}{2} - \frac{1}{2})$ transition of ^{209}Bi at 4.2°K . The range of resonant frequencies is 9.85 Mc/sec to 16.00 Mc/sec .

maxima of the powder pattern occurring when the principal axis of the field gradient tensor and the magnetic field are perpendicular, through the third order of perturbation theory, is given by

$$\nu(m \leftrightarrow m - 1)$$

$$= \nu_R \left\{ 1 - \left[a + \frac{1}{2} (\nu_Q / \nu_R) (m - \frac{1}{2}) \right] - \frac{1}{16} (\nu_Q / \nu_R)^2 \left[3m(m - 1) - I(I + 1) + \frac{3}{2} \right] \right\}, \quad (4)$$

and the separation of pairs of corresponding satellites is given by $\frac{1}{2} \nu_Q (2m - 1)$. The value of ν_Q was determined from different pairs of satellites over the same frequency range with the result

$\nu_Q = 2.44 \pm 0.07 \text{ Mc/sec}$ in agreement with the value from the $(\frac{1}{2} - \frac{1}{2})$ width.

The value of the quadrupole coupling e^2qQ/h of $58.5 \pm 2 \text{ Mc/sec}$ is higher by about a factor of two than the values estimated from the T^{-2} term of the specific heat. A correction for this term in the bismuth specific-heat data will make an appreciable change in the electronic specific-heat values. This quadrupole coupling will also be useful in checking any model of the electronic structure as has been done for antimony.⁷

The anisotropic Knight shift is an order of magnitude larger than any that have been reported to date. The isotropic Knight shift is the only reported shift that is appreciably different in the solid from that in the liquid state. In order to check these anomalies and because of the need for a temperature dependence study of these shifts, we are initiating a measurement of nmr in a single crystal of bismuth.

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TRANSMITTED PHONON DRAG EFFECT IN GERMANIUM AT VERY LOW TEMPERATURES

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Extensive measurements of the phonon drag contribution to the thermoelectric power in relatively pure germanium have been carried out by Geballe and Hull.¹ It is rather difficult to extend such measurements to degenerately doped germanium because the normal electronic thermoelectric power obscures the phonon drag effect in heavily doped material. We have been able to extend phonon drag measurements to degenerate germanium at very low temperatures by using the

transmitted phonon drag effect originated by Hubner and Shockley.² In the transmitted phonon drag effect the phonon part of the Peltier coefficient can be measured independently of any electronic contribution.

We prepared germanium n - p - n structures by depositing layers of heavily doped n -type germanium on both sides of p -type wafers by the iodide vapor growth method.³ The carrier concentration in the n -type layers was $6.5 \times 10^{18} \text{ cm}^{-3}$ and

the *p*-type layer was only very lightly doped. The *n*-type layers were 20 microns thick, and the *p*-type layer which separated them varied in thickness from 0.1 to 1 millimeter. At low temperatures, phonons could pass from one *n*-type layer to the other without scattering in the center layer. The evidence for this fact is that the phonon drag effect observed in the output layer was independent of the thickness of the center layer over a range of thicknesses which depended on the temperature in question. Thus, the magnitude of the transmitted phonon drag effect in these structures was a simple measure of the amount of momentum carried by the phonons in degenerate *n*-type germanium.

The results of the experiment are shown in Fig. 1, which is a plot of the ratio of the output voltage to the input voltage for structures with central layers of two different thicknesses. The

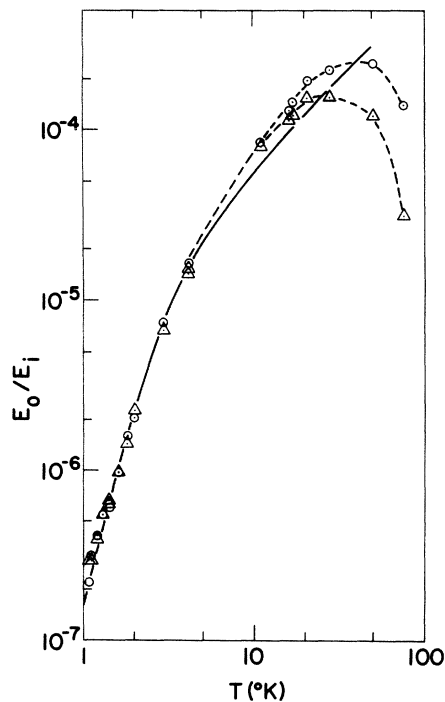


FIG. 1. The results of a transmitted phonon drag experiment expressed as the ratio of output to input voltage as a function of temperature. The thickness of the *p*-type center layer is 0.087 cm for the data represented by circles and 0.026 cm for the data represented by triangles. The Peltier coefficient of the degenerate germanium which formed the input and output layers is 7 volts times the ordinate. The fit of the data to the product of temperature and a Debye specific-heat function [see Eq. (3)] is shown by the solid line.

measurements were made at 20 cps using a lock-in amplifier whose input impedance was high compared to the sample output layer resistances. The observed output voltages were out of phase with the input voltage and linearly dependent upon them. It is seen that the ratio is independent of thickness below 25°K.

From the following method of analysis we can deduce that Peltier coefficient of the *n*-type germanium. Let P denote the phonon crystal momentum density in the input layer when an electric field E_i is applied. The rate of flow of phonon momentum out of the input layer, through the *p* layer, and into the output layer is $(\frac{1}{3}v)P$, where v is the phonon velocity.² This momentum is absorbed by the electrons in the output layer, where, under open-circuit conditions, it is balanced by a field E_0 which gives an equal and opposite momentum to the electrons. Thus

$$(\frac{1}{3}v)P = neE_0t. \quad (1)$$

Here n is the carrier concentration in the *n* layer and t is its thickness. The Peltier coefficient is the ratio of the energy current density in the input layer, Pv^2 , to the electric current density, $ne\mu E_i$. Thus, from Eq. (1), the phonon Peltier coefficient is

$$\pi_\phi = (6vt/\mu)(E_0/E_i).$$

The value of $6vt/\mu$ for the structures for which (E_0/E_i) is given in Fig. 1 is 7 volts.

It is of interest to interpret the magnitude of π_ϕ . Let the phonons be divided into two classes. The first class includes those phonons which interact strongly with the electrons, assumed to be all of the phonons with wave vector less than some maximum, q_m . The second class includes those phonons which only interact with the electrons to a negligible degree and play no role in the phonon drag effects, assumed to be the phonons with $q > q_m$. The system of interacting phonons in the input layer acquires a drift velocity equal to the velocity of the electrons, μE_i , and, therefore, a momentum density⁴

$$P = (C_v T/3v^2)\mu E_i. \quad (2)$$

Here C_v is the specific heat of the interacting phonons. C_v has the form of a Debye specific heat with maximum phonon energy $\hbar v q_m$. Combining Eqs. (1) and (2) allows (E_0/E_i) to be expressed in terms of C_v :

$$(E_0/E_i) = C_v T \mu / 18 n e t v. \quad (3)$$

The temperature dependence of E_0/E_i comes from the factor $C_v T$. The line in Fig. 1 represents the fit of (E_0/E_i) to the product of T and a Debye specific-heat function. This is a two-parameter fit; the Debye temperature is found from the fit to be 14.8°K and the low-temperature limit of (E_0/E_i) to be $E_0/E_i = 1.6 \times 10^{-7} T^4$. At very low temperatures all of the thermally excited phonons interact with the electrons and C_v is the total lattice specific heat of the crystal. Thus, the right-hand side of Eq. (3) involves no adjustable parameters in the low-temperature limit. Using values measured on our specimen for μ , n , and t , and Bryant and Keesom's⁵ value for C_v , in Eq. (3) gives $E_0/E_i = 1.1 \times 10^{-7} T^4$, about two thirds of the measured value.

The value of θ , the effective Debye temperature of the interacting phonons, is a measure of q_m , the maximum wave vector of the interacting phonons, or of the number of phonon modes which interact with the electrons. The observed value, $\theta = 14.8^\circ\text{K}$, corresponds to 8×10^{18} modes per cm^3 , in other words, about one mode per electron. Although calculation of the number of phonons which drift with the electrons is difficult because of the anisotropy of the electronic energy bands, we would have expected that the electronic motion would impart the drift velocity to a large

number of phonons. In the case of a spherical Fermi surface, phonons with wave vector less than twice the Fermi wave vector can scatter electrons directly. The sphere of such directly interacting phonons in wave-number space contains 12 phonon states (including the three polarization branches) per electron. Since each of the electron valleys contain $1/4$ of the electrons, the number of directly interacting phonons might be reduced to three per electron. However, because not all valleys interact with the same phonons, the number should be somewhat larger than three. The number might be reduced if the higher energy scattered phonons had short momentum-loss relaxation times, so that they remained in equilibrium even though scattered by the electrons.

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EXPERIMENTAL EVIDENCE FOR DELAYED ENTRY OF FLUX INTO A TYPE-II SUPERCONDUCTOR*

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Null-deflection torque magnetometer studies^{1,2} have been carried out on annealed homogeneous Pb-Tl alloys (4.3-12.3 at. % Tl) in the form of thick films (1.4-4.2 μ) and foils (25-100 μ) over the interval $1.1 \leq T \leq 4.2^\circ\text{K}$, with the applied field H very nearly in the plane of the specimen. By this method, the measured field dependence of the torque τ defines a characteristic field H_τ at which τ/H departs from its initially linear dependence on H . The departure from linearity can be ascribed to flux penetration, in accord with our measurements on thick annealed pure Pb films which exhibit linear τ/H behavior until

nearly H_c (bulk), whereupon τ/H drops rapidly to zero. Furthermore, a residual value of τ/H (negative) at $H=0$ is interpreted as evidence of flux trapping. For alloy films (4.2°K), direct comparison can be made between H_τ and the bulk lower critical field H_{c1} (bulk), where the latter is deduced from the data of Bon Mardion, Goodman, and Lacaze.³ Foils exhibited H_τ values approximately equal to H_{c1} (bulk), and flux trapping was observed only when H_τ was exceeded. In addition, H_τ varied parabolically with reduced temperature,⁴ a behavior similar to that observed for H_{c1} in the In-Bi system by Kinsel, Lynton,