# ASYMPTOTIC BEHAVIOR OF ELECTROMAGNETIC FORM FACTORS*

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In this Letter we want to explore the consequences of the strong asymptotic decrease of electromagnetic form factors observed in the high-energy electron-proton scattering experiment recently completed at Cambridge.[1] As we shall see below, this asymptotic behavior can be accounted for by the usual picture of resonance-dominated form factors. One is then unambiguously led to the existence of a new $I=1$ vector meson with mass between 750 and 1360 MeV that could be identified with the observed 1220-MeV $\omega\pi$ resonance.[2] The keystone of our analysis is the indicated absence of a core in both $G_{Ep}$ and $G_{Mp}$,[1] which permits the derivation of a nontrivial sum rule suggested earlier by Sachs.[3] We are also led to unique fits to all the nucleon form factors, which are in good agreement with the experimental data. Our model suggests that the new $I=1$ vector meson and the[4] $\varphi$ may belong to a second octet of vector mesons.

In reference 1 it has been shown that the proton's electromagnetic form factors $G_{Ep}$ and $G_{Mp}$ both fall off as $1/q^2$ at large momentum transfers Considering their difference, it follows that the Pauli form factor obeys the asymptotic condition

$$tF_{2p}(t)\to 0 \tag{1}$$

as $q^2\equiv -t\to\infty$. Since the function $F_{2p}(t)$ satisfies the dispersion relation

$$F_{2p}(t)=\pi^{-1}\int_{a_V}^{\infty}\frac{\mathrm{Im}F_{2p}(t')dt'}{t'-t-i\epsilon}, \tag{2}$$

where $a_V=4m_\pi^2$, Eq. (1) leads to the sum rule[5]

$$\int_{a_V}^{\infty}\mathrm{Im}F_{2p}(t')dt'=0. \tag{3}$$

It is readily verified that (3) is unaltered even if (2) is replaced by a subtracted dispersion relation. For (3) to be true it is sufficient that $G_{Ep}$ and $G_{Mp}$ approach the same constant at infinity.[3]

The corresponding high-energy experiments are not yet available for the neutron, but it would be surprising if $G_{En}$ and $G_{Mn}$ did not vanish as $q^2\to\infty$, since the neutron presumably has no core.[3] Therefore an analog of (3) should hold for the neutron and thus for the isotopic vector and scalar form factors separately.

Making the usual assumption[6] that the form factors are dominated by multipion resonances with the appropriate quantum numbers, we have

$$\mathrm{Im}F_{2i}(t)=\pi\sum_{j=1}^{N_i}\epsilon_{ij}m_{ij}^2\delta(t-m_{ij}^2), \tag{4}$$

where $i=S$ $(V)$ and the sum extends over all existing $I=0$ $(I=1)$ vector mesons with masses $m_{Sj}$ $(m_{Vj})$. The $\epsilon_{ij}$ are constants. It is apparent that a relationship of the form (3) requires that the sum of the residues coming from (4) must vanish and thus is incompatible with one-resonance fits to either of the $F_{2i}(t)$. Therefore,

$$N_i\geq 2. \tag{5}$$

Experimentally, two $I=0$ vector mesons have been found: the $\omega$ and the $\omega'$ (usually known as[4] $\varphi$). However, only one $I=1$ vector meson, the $\rho$, has so far been definitely established. In order to comply with (5), we are led to require the existence of at least one other $I=1$ vector meson which we shall label $\rho'$.

With (4) and these vector mesons, the sum rules obtained from (3) read

$$\begin{aligned}\epsilon_\rho m_\rho^2+\epsilon_{\rho'}m_{\rho'}^2&=0,\\ \epsilon_\omega m_\omega^2+\epsilon_{\omega'}m_{\omega'}^2&=0.\end{aligned} \tag{6}$$

Inserting (4) into (2), we have

$$\begin{aligned}F_{2V}(t)&=\epsilon_\rho m_\rho^2/(m_\rho^2-t)+\epsilon_{\rho'}m_{\rho'}^2/(m_{\rho'}^2-t),\\ F_{2S}(t)&=\epsilon_\omega m_\omega^2/(m_\omega^2-t)+\epsilon_{\omega'}m_{\omega'}^2/(m_{\omega'}^2-t).\end{aligned} \tag{7}$$

Hence from the experimental data[7] at $t=0$,

$$\begin{aligned}\epsilon_\rho+\epsilon_{\rho'}&=F_{2V}(0)=1.85,\\ \epsilon_\rho m_\rho^{-2}+\epsilon_{\rho'}m_{\rho'}^{-2}&=F_{2V}'(0)=(0.216\pm 0.05)\ \mathrm{F}^{-2},\\ \epsilon_\omega+\epsilon_{\omega'}&=F_{2S}(0)=-0.06,\\ \epsilon_\omega m_\omega^{-2}+\epsilon_{\omega'}m_{\omega'}^{-2}&=F_{2S}'(0)\\ &=(0.005\pm 0.05)\ \mathrm{F}^{-2}.\end{aligned} \tag{8}$$

Equations (6) and (8) allow us to determine the $\rho'$ and $\omega'$ masses and the "coupling constants" $\epsilon_\rho$, $\epsilon_{\rho'}$, $\epsilon_\omega$, and $\epsilon_{\omega'}$ in terms of $m_\rho$ (= 750 MeV) and $m_\omega$ (= 780 MeV). We find

$$750 \text{ MeV} \leq m_{\rho'} \leq 1360 \text{ MeV} \tag{9}$$

and

$$\epsilon_\rho = \frac{1.85 m_{\rho'}^2}{m_{\rho'}^2 - m_\rho^2}, \quad \epsilon_{\rho'} = \frac{1.85 m_\rho^2}{m_\rho^2 - m_{\rho'}^2},$$
$$\epsilon_\omega = \frac{-0.06 m_{\omega'}^2}{m_{\omega'}^2 - m_\omega^2}, \quad \epsilon_{\omega'} = \frac{-0.06 m_\omega^2}{m_\omega^2 - m_{\omega'}^2}. \tag{10}$$

The large uncetainties connected with $F_{2S}'(0)$ prevent us from deriving any useful limits on $m_{\omega'}$. However, it is natural to identify $\omega'$ with $\varphi$ ($m_\varphi$ = 1020 MeV). The sufficiently strong limits (9) make it tempting to identify $\rho'$ with the recently discovered $\omega\pi$ resonance at 1220 MeV.[2] This agreement between (9) and the observed resonance mass seems significant and suggests the assignment $1^{-+}$ for the resonance. The same assignment has been favored by Frazer, Patil, and Watson on different theoretical grounds.[8] We note that use of the experimental $\rho'$ and $\omega'$ masses will overdetermine the parameters in (7). The resultant $F_{2i}(t)$'s given by the above parameters provide a good fit to the existing data.

The Sachs charge and magnetic form factors can also be written in a form similar to (7). Although we have no sum rules analogous to (6) in this case, we may still obtain a unique fit merely from the low-energy limits $G_{ki}(0)$ and $(d/dt)G_{ki}(0)$ ($k = E, M$; $i = V, S$) and the $\rho$, $\rho'$, $\omega$, and $\omega'$ masses, all of which we take from experiment,[7] ($m_\rho^2$ = 14.5 F$^{-2}$; $m_\omega^2$ = 15.8 F$^{-2}$; $m_\varphi^2$ = 26.7 F$^{-2}$; $m_{\rho'}^2$ = 38.2 F$^{-2}$). This yields

$$G_{EV}(t) = 0.5\left[\frac{1.91 \pm 0.08}{1 - t/14.5} - \frac{0.91 \pm 0.08}{1 - t/38.2}\right],$$
$$G_{MV}(t) = 2.35\left[\frac{1.9 \pm 0.5}{1 - t/14.5} - \frac{0.9 \pm 0.5}{1 - t/38.2}\right],$$
$$G_{ES}(t) = 0.5\left[\frac{2.73 \pm 0.13}{1 - t/15.8} - \frac{1.73 \pm 0.13}{1 - t/26.7}\right],$$
$$G_{MS}(t) = 0.44\left[\frac{3.0 \pm 4.4}{1 - t/15.8} - \frac{2.0 \pm 4.4}{1 - t/26.7}\right]. \tag{11}$$

The expressions (11) are in good agreement with all existing experimental data.[1,7] We also find that the asymptotic coefficients of $1/t^2$ in $F_2^V$ and $F_2^S$ as predicted from (7) and (10) agree with those given by (11).

Our fit (11) has a very interesting peculiarity. Within errors

$$G_{Mi}(t) \cong (1 + 2\mu_i) G_{Ei}(t)$$
$$(\mu_V = 1.85,\ \mu_S = -0.06),$$

which can be also written as

$$G_{Mp}(t) \cong \mu_p G_{Ep}(t) + \mu_n G_{En}(t),$$
$$G_{Mn}(t) \cong \mu_n G_{Ep}(t) + \mu_p G_{En}(t)$$
$$(\mu_p = 2.79,\ \mu_n = -1.91). \tag{12}$$

Since experimentally $G_{En}(t) \cong 0$, (12) can be approximated by

$$G_{Mp}(t)/\mu_p \cong G_{Ep}(t) \cong G_{Mn}(t)/\mu_n. \tag{13}$$

The first of these equalities is known,[7] whereas the last appears to be new. The approximate picture of the nucleons suggested by (13) is one in which the magnetic moments of the neutron and proton and the charge of the proton all have similar spatial distributions, whereas the neutron has a nearly vanishing charge distribution. The first equality (13), of course, does not contradict Sachs' conjecture

$$G_{Ep}(t) - G_{Mp}(t) \xrightarrow[-t \to \infty]{} 0,$$

which is trivially satisfied by the expressions (11) according to which both $G_{Ep}$ and $G_{Mp}$ vanish asymptotically. Note in particular that for $Z_2 = 0$ (i.e., vanishing unrenormalized proton field), Sachs' conjecture does not imply

$$G_{Ep}/G_{Mp} \xrightarrow[-t \to \infty]{} 1.$$

Our preceding discussions strongly suggest that the $\rho'$ and $\omega'$ together with a yet-to-be-assigned $K^{*\prime}$ constitute a second SU(3) octet of vector mesons analogous to that formed by the $\rho$, $\omega$, and $K^*$. This, of course, is inconsistent with the customary (though not compelling) assignment of the $\omega'$ ($\equiv \varphi$) to a unitary singlet. The two vector-meson octets would be mixed by SU(3)-breaking interactions, and in addition to $\omega$-$\varphi$ mixing we would have $\rho$-$\rho'$ and $K^*$-$K^{*\prime}$ mixing. The usual mass formulas would not be expected to hold in the presence of octet-octet mixing, but to lowest order in the standard SU(3)-breaking interaction,[9]

$$m_{K^*}^2 + m_{K^{*\prime}}^2 = \tfrac{3}{4}(m_\omega^2 + m_{\omega'}^2) + \tfrac{1}{4}(m_\rho^2 + m_{\rho'}^2)$$
$$= 1.75\ (\text{BeV})^2. \tag{14}$$

Thus for $m_{K^*}$ = 880 MeV we would predict $m_{K^{*\prime}}$ = 990 MeV, whereas for $m_{K^*}$ = 725 MeV we obtain $m_{K^{*\prime}} = 1115$ MeV. Of course, these predictions could be appreciably altered if a unitary singlet vector meson were to exist in addition to the two octets and mix with these. We would expect the $K^*$ and $K^{*\prime}$ to dominate the strangeness-changing weak magnetic form factor, which would also satisfy an asymptotic condition of the type (1). The existence of two vector-meson octets is actually suggested if one thinks of them as baryon-antibaryon bound states, since $8\otimes 8$ contains two 8 representations.[10]

The present experimental evidence does not exclude stronger asymptotic conditions on $F_{2i}(t)$ or even conditions of the type (1) for $F_{1i}$ (the Dirac form factors). Should any additional asymptotic conditions be found experimentally, they would lead to new sum rules and possibly to very strong and interesting implications. We do not know, however, of any satisfactory theoretical derivation even of the observed asymptotic behavior of $F_{1p}(t)$. We would like to conclude by emphasizing that if no $\rho'$ is found, the resonance model of form factors will have to be seriously revised.

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[5]This sum rule and the one for the neutron have actually been written down by Sachs[3] on the basis of his physical interpretation of $G_{E,M}(\infty)$. Note, however, that here the results for the proton follow directly from experiment. Sachs has also emphasized that the usual fits to the form factors do not satisfy this sum rule.

[6]We do not consider here the unattractive alternative of a large nonresonant background term in the form factors.

[7]The numerical values are taken from L. N. Hand, D. G. Miller, and Richard Wilson, Rev. Mod. Phys. 35, 335 (1963), with the exception that we use the more precise numbers for $G_{En}$ given by D. L. Drickey and L. N. Hand, Phys. Rev. Letters 9, 521 (1962). Notice the difference in normalization and metric between these references and the present paper.

[8]W. R. Frazer, S. Patil, and H. L. Watson, Phys. Rev. Letters 11, 231 (1963).

[9]This relation is due to P. P. Divakaran, and we wish to thank him for permission to use it.

[10]In the unphysical case of exact $R$ invariance one of these meson octets would have odd $R$ parity, whereas the other would have even $R$ parity. The particles of the odd $R$-parity octet could then be identified as "gauge bosons" [J. J. Sakurai, Ann. Phys. (N.Y.) 11, 1 (1960)]. The $I=1$, $Y=0$ member of this octet will be able to decay into two pions. The $I=1$, $Y=0$ member of the $R$-even octet will evidently not decay into two pions. Since the $\pi\pi$ mode does not seem to be dominant for the $\rho'$,[2] it might well be that the $\rho$ is essentially the gauge boson of isospin, whereas the $\rho'$ is an approximately $R$-even meson.

## ERRATUM

VARIATIONAL PROPERTY OF FREE-ENERGY PERTURBATION THEORY. Harold Falk [Phys. Rev. Letters 12, 93 (1964)].

Due to errors in the proof, pointed out by R. B. Griffiths and G. Baym, the theorem stated is invalid.