assisted in the computations.

*Work supported in part by the U. S. Air Force Office of Scientific Research and the U. S. Office of Naval Research.

[1]A. M. Lane and R. G. Thomas, Rev. Mod. Phys. 30, 257 (1958).
[2]C. E. Watson, C. F. Moore, and R. K. Sheline (to be published).
[3]J. D. Anderson, C. Wong, and J. W. McClure, Phys. Rev. 126, 2170 (1962).

PHOTON-PROTON SCATTERING AT 16.7 BeV/c AND INTERACTION WITH THE NUCLEON CORE*

Shigeo Minami†
Department of Physics and Astronomy, Louisiana State University, Baton Rouge, Louisiana
(Received 17 January 1964)

The detailed experimental data for small-angle elastic scatterings in proton-proton collisions at 7-20 BeV/c were reported by Foley et al.[1] In addition, Cocconi et al.[2] examined recently the large-angle proton-proton elastic scatterings at high energy. On the basis of these experimental data we study in this paper the proton-proton interaction at[3] 16.7 BeV/c by taking into account the effect of Fermi statistics.

In a previous study[4] of the pion-proton interaction we adopted the following expression for the scattering amplitude[5]:

$$f(\theta)=i\{\exp\tfrac{1}{2}(A_0+A_1t)+C \pm \exp\tfrac{1}{2}[B_0+B_1(u-u_0)]\}\ (\mathrm{mb})^{1/2}, \quad (1)$$

where the first and third terms are responsible, respectively, for the forward peak and the backward peak. The second term iC depends strongly on the incident energy and may be interpreted as an effect due to inelastic scattering which might be described in terms of the statistical model. Since the experimental data[1] are given by the form $|d\sigma/dt|=(\pi/k^2)d\sigma/d\Omega$, it is better to reexpress the scattering amplitude as follows:

$$f(\theta)=(ik/\sqrt{\pi})[\exp\tfrac{1}{2}(A_0+A_1t)+C \pm \exp\tfrac{1}{2}(B_0+B_1u)]\ (\mathrm{mb})^{1/2}, \quad (1')$$

where $t=-2k^2(1-\cos\theta)$ and $u=-2k^2(1+\cos\theta)$.[6] An exchange $t\rightleftarrows u$ in Eq. (1′) leads to the expression of $f(\pi-\theta)$. Note that, if the upper (lower) sign in the double sign ($\pm$) is taken, one must take the upper (lower) sign in all the double signs which will appear throughout this paper.[7] By taking into account the effect of Fermi statistics, the differential cross section for p-p elastic scattering is given by

$$d\sigma/d\Omega=\tfrac{3}{4}|f(\theta)-f(\pi-\theta)|^2+\tfrac{1}{4}|f(\theta)+f(\pi-\theta)|^2. \quad (2)$$

The factors $\frac{3}{4}$ and $\frac{1}{4}$ indicate the statistical weight factors for the spin triplet and singlet states, respectively.

So far as elastic scattering in the forward direction (0°-90°) is concerned, $d\sigma/dt$ can be expressed approximately as follows:

$$\begin{aligned}|d\sigma/dt| &\cong \tfrac{3}{4}[\exp\tfrac{1}{2}(A_0+A_1t)\mp\exp\tfrac{1}{2}(B_0+B_1t)]^2 \\ &\quad +\tfrac{1}{4}[\exp\tfrac{1}{2}(A_0+A_1t)\pm\exp\tfrac{1}{2}(B_0+B_1t)+2C]^2, \\ &= \exp(A_0+A_1t)+\exp(B_0+B_1t) \\ &\quad \mp\exp\tfrac{1}{2}[A_0+B_0+(A_1+B_1)t] \\ &\quad +C[\exp\tfrac{1}{2}(A_0+A_1t)\pm\exp\tfrac{1}{2}(B_0+B_1t)] \\ &\quad +C^2\ \mathrm{mb}/(\mathrm{BeV}/c)^2. \end{aligned} \quad (3)$$

In our study of $d\sigma/dt$, the scattering angles from 0° to 90° are divided into the following three regions: (I) the region of the diffraction peak; $|t|=0-1$ (BeV/c)^2, (II) the intermediate region; $|t|=1-8$ (BeV/c)^2, and (III) the region in which a statistical model is available and in which there is no effect of diffraction scattering; $|t|=8-14.81$ (BeV/c)^2. In region (III), $d\sigma/dt$ is nearly equal to C^2 [see Eq. (3)].[8] Using the experimental data given by Cocconi et al.,[2] we estimate the value of C as

$$C=\pm 0.8\times10^{-3}\ (\mathrm{mb})^{1/2}/(\mathrm{BeV}/c). \quad (4)$$

The behavior of $d\sigma/dt$ in region (I) can be approximately described by the first, second, and third terms in Eq. (3), because the contribution from the C term can be neglected. We now try to determine the parameters A_0, A_1, B_0, and B_1 so that the experimental results given by Foley et al.[1] may be reproduced, although the results have been fitted by an expression of the form[1]

$$|d\sigma/dt|=\exp(a+bt+ct^2), \quad (5)$$

with

$$a=4.52,\ b=9.79\ (\mathrm{BeV}/c)^{-2},$$

and

$$c = 1.48 \text{ (BeV}/c)^{-4}. \quad (6)$$

As is seen from expression (1′), the first term, $\exp(A_0+A_1t)$, in Eq. (3) plays the most important role in the description of elastic scattering in the neighborhood of $t=0$. That is, $A_0 \gg B_0$ and $\exp(A_0+A_1t) \gg \exp(B_0+B_1t)$ in the region of very small $|t|$, for instance $|t| = 0-0.3$ (BeV/c)^2. In order to explain the experimental results for $d\sigma/dt$ in the region $|t| = 0-0.3$ (BeV/c)^2, the values of A_0 and A_1 must be within the regions 4-5 and 8-11 (BeV/c)$^{-2}$, respectively. This can easily be seen through the empirical formula (5) which has been given by Foley et al.[1] [see Eq. (6)]. Then the term $\exp\frac{1}{2}(A_0+A_1t)$ damps very rapidly as the value of $|t|$ increases. Because this term has no large effect on the $d\sigma/dt$ in region (II), the function $\exp\frac{1}{2}(B_0+B_1t)$ and the sign of C in Eq. (4) must be chosen so that the experimental data for $d\sigma/dt$ in region (II), particularly in region $|t| = 3-8$ (BeV/c)^2, may be reproduced. This leads to the conclusion that B_1 is much smaller than A_1. Hereafter we refer to as case (I) and case (II), respectively, when the upper and lower signs of the double signs in Eqs. (1′) and (3) are adopted. For cases (I) and (II), parameters A_0, A_1, B_0, and B_1 are here estimated as follows:

Case (I);

$$A_0 = 4.48, \quad A_1 = 8.8 \text{ (BeV}/c)^{-2}, \quad B_0 = -3.0,$$

$$B_1 = 1.74 \text{ (BeV}/c)^{-2}, \text{ and } C = 0.8\times10^{-3}. \quad (7)$$

Case (II);

$$A_0 = 4.44, \quad A_1 = 9.34 \text{ (BeV}/c)^{-2}, \quad B_0 = -2.30,$$

$$B_1 = 1.74 \text{ (BeV}/c)^{-2}, \text{ and } C = -0.8\times10^{-3}. \quad (8)$$

As is shown in Table I, the values of $d\sigma/dt$ thus estimated, particularly those in case (II), agree very well with the experimental values.[1] For comparison we have added in the last column the values of $|d\sigma/dt| = \exp(a+bt+ct^2)$ estimated by Foley et al.[1]

The $|d\sigma/dt|$ in regions (II) and (III) can be estimated by employing Eq. (3) with the parameters mentioned in Eq. (7) or (8). We show the results in Fig. 1. It can be said that the experimental angular distribution for elastic scattering at 16.7 BeV/c can be well described by our empirical formula.

Next let us perform the partial-wave analysis. When the spin dependence of the S matrix is neglected and the assumption that $\delta_l = 0$ is introduced, the elastic-scattering amplitude can be expressed as

$$f(\theta) = \frac{i}{2K}\sum_l (2l+1)(1-\eta_l)P_l(\cos\theta)$$

$$= \tfrac{1}{2}i\sum_l (2l+1)\xi_l P_l(\cos\theta), \quad (9)$$

Table I. $|d\sigma/dt|$ in the region $|t| = 0-0.7$ (BeV/c)^2. The values of $d\sigma/dt$ in case (I) are estimated by $|d\sigma/dt| = \exp(A_0+A_1t)+\exp(B_0+B_1t)-\exp\frac{1}{2}[A_0+B_0+(A_1+B_1)t]$, with $A_0=4.48$, $A_1=8.8$ (BeV/c)$^{-2}$, $B_0=-3.0$, $B_1=1.74$ (BeV/c)$^{-2}$, and $C=0.8\times10^{-3}$. The values of $d\sigma/dt$ in case (II) are estimated by $|d\sigma/dt| = \exp(A_0+A_1t)+\exp(B_0+B_1t)+\exp\frac{1}{2}[A_0+B_0+(A_1+B_1)t]$, with $A_0=4.44$, $A_1=9.34$ (BeV/c)$^{-2}$, $B_0=-2.30$, $B_1=1.74$ (BeV/c)$^{-2}$, and $C=-0.8\times10^{-3}$. The expression $|d\sigma/dt| = \exp(a+bt+ct^2)$, with $a=4.52$, $b=9.79$ (BeV/c)$^{-2}$, and $c=1.48$ (BeV/c)$^{-4}$ was given by Foley et al.[a]

$-t$ (BeV/c)^2	$\lvert d\sigma/dt\rvert$ mb/(BeV/c)^2 Case (I)	Case (II)	Experimental values[a]	$\exp(a+bt+ct^2)$
0	86.2	87.8	92.2 ± 5.5	91.8
0.042	59.3	59.6	56.0 ± 4.3	61.0
0.084	40.8	40.6	42.4 ± 2.9	40.8
0.141	24.6	24.1	24.42 ± 0.98	23.8
0.204	13.97	13.62	13.24 ± 0.68	13.26
0.281	7.00	6.82	7.03 ± 0.43	6.59
0.308	5.48	5.36	5.06 ± 0.22	5.18
0.380	2.86	2.84	2.81 ± 0.26	2.76
0.393	2.54	2.54	2.43 ± 0.13	2.46
0.488	1.065	1.126	1.114 ± 0.072	1.100
0.491	1.036	1.099	1.090 ± 0.170	1.072
0.589	0.419	0.493	0.471 ± 0.038	0.481
0.698	0.152	0.216	0.214 ± 0.023	0.203

[a]See reference 1.

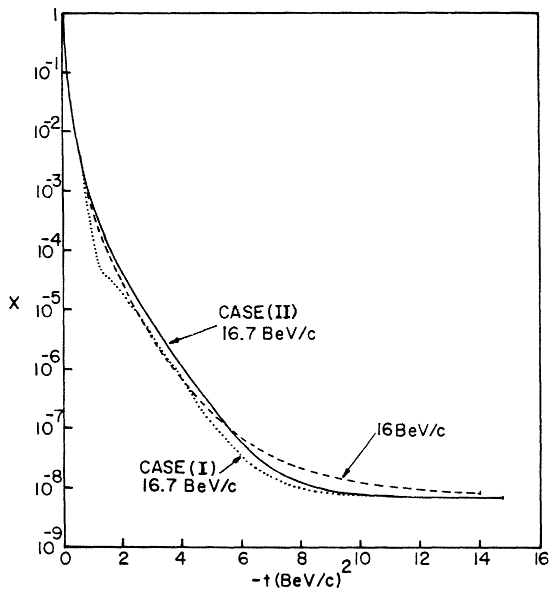

FIG. 1. Differential cross section for elastic p-p scattering, normalized to the forward scattering cross section, as a function of the squared four-momentum transfer $-t$, where $X = (d\sigma/dt)/(d\sigma/dt)_{t=0}$. The dotted and solid curves indicate our results in case (I) and case (II), respectively. The dashed curve shows the differential cross section estimated by Cocconi et al.[2]

where $\eta_l \exp(2i\delta_l) - 1$ is the R matrix for the lth partial wave.[9] Since we have

$$f(\theta) - f(\pi - \theta) = i \sum_{l=\text{odd}} (2l+1)\xi_l P_l(\cos\theta)$$

and

$$f(\theta) + f(\pi - \theta) = i \sum_{l=\text{even}} (2l+1)\xi_l P_l(\cos\theta), \quad (10)$$

the following relations are obtained by making use of Eq. (1′).

$$\sum_{l=\text{odd}} (2l+1)\xi_l P_l(x)$$
$$= (k/\sqrt{\pi})[\exp(a_0 + a_1 x) \mp \exp(b_0 + b_1 x) - \exp(a_0 - a_1 x) \pm \exp(b_0 - b_1 x)]\ (\text{mb})^{1/2},$$
$$\sum_{l=\text{even}} (2l+1)\xi_l P_l(x)$$
$$= (k/\sqrt{\pi})[\exp(a_0 + a_1 x) \pm \exp(b_0 + b_1 x) + \exp(a_0 - a_1 x) \pm \exp(b_0 - b_1 x) + 2C]\ (\text{mb})^{1/2}, \quad (11)$$

where $a_0 = A_0/2 - A_1 k^2$, $a_1 = A_1 k^2$, $b_0 = B_0/2 - B_1 k^2$, and $b_1 = B_1 k^2$. It must be noted that the numerical values in the bracket on the right-hand side of Eq. (11) are given in units of $(\text{mb})^{1/2}/(\text{BeV}/c)$. From Eq. (11) we get the expression of ξ_l.

$$\xi_l = (k/2\sqrt{\pi})[\xi_{l,1} \mp \xi_{l,2} - \xi_{l,3} \pm \xi_{l,4}]$$
$$= (k/\sqrt{\pi})(\xi_{l,1} \mp \xi_{l,2}) \text{ for } l = \text{odd}, \quad (12)$$

$$\xi_l = (k/2\sqrt{\pi})[\xi_{l,1} \pm \xi_{l,2} + \xi_{l,3} \pm \xi_{l,4} + 2\xi_{l,5}],$$
$$= (k/\sqrt{\pi})(\xi_{l,1} \pm \xi_{l,2} + \xi_{l,5}) \text{ for } l = \text{even}; \quad (13)$$

where

$$\xi_{l,1} = \int_{-1}^{1} \exp(a_0 + a_1 x) P_l(x) dx,$$
$$\xi_{l,2} = \int_{-1}^{1} \exp(b_0 + b_1 x) P_l(x) dx,$$
$$\xi_{l,3} = \int_{-1}^{1} \exp(a_0 - a_1 x) P_l(x) dx,$$
$$\xi_{l,4} = \int_{-1}^{1} \exp(b_0 - b_1 x) P_l(x) dx,$$

and

$$\xi_{l,5} = \int_{-1}^{1} C P_l(x) dx.$$

These integrals can easily be done analytically as was shown in the previous paper.[4] Values of $(1 - \eta_l)$ thus obtained are shown in Table II. In case (II), the $\xi_{2l,2}$ and $\xi_{2l,5}$ $(= 2C\delta_{2l,0})$ terms with negative sign contribute to $(1 - \eta_{2l})$. This is the reason why the value of $(1 - \eta_0)$ in case (II) is not so large. Making use of the values of $(1 - \eta_l)$ mentioned in Table II, we can easily estimate both elastic and inelastic cross sections due to the lth wave. Their detailed values are omitted in the interest of brevity.

Table II. Values of $(1 - \eta_l)$.

	$1 - \eta_l$			$1 - \eta_l$	
l	Case (I)	Case (II)	l	Case (I)	Case (II)
0	1.0938	0.7173	1	0.8450	1.0320
2	1.0142	0.7253	3	0.8095	0.9200
4	0.8806	0.6978	5	0.7315	0.7678
6	0.7211	0.6264	7	0.6144	0.6124
8	0.5616	0.5189	9	0.4791	0.4692
10	0.4156	0.3987	11	0.3486	0.3429
12	0.2905	0.2876	13	0.2379	0.2383
14	0.1919	0.1946	15	0.1525	0.1567
16	0.1194	0.1243	17	0.0920	0.0975
18	0.0700	0.0750	19	0.0523	0.0573
20	0.0386	0.0427	21	0.0280	0.0321
22	0.0202	0.0227	23	0.0141	0.0172
24	0.0100	0.0110	25	0.0066	0.0095

Finally we state the conclusions which can be derived from our analysis.

(i) As was emphasized before,[4] interaction with the nucleon core is responsible for the backward peak whose character is expressed by the third term in Eq. (1′). In the case of p-p scattering, an effect due to this interaction is also reflected on the forward peak [see Eq. (3)]. Its property can be seen through the behavior of $\xi_{l,2}$ in Eqs. (12) and (13). Here we should like to point out that the $\xi_{2l,2}$ and $\xi_{2l+1,2}$ terms with opposite sign contribute, respectively, to the $(1-\eta_{2l})$ and $(1-\eta_{2l+1})$. If the effects due to Fermi statistics are neglected and $d\sigma/dt$ in the forward direction is expressed by a form[10] $[\exp\frac{1}{2}(A_0+A_1t)+\exp\frac{1}{2}(B_0+B_1t)]^2$ instead of the expression (3), then all the $\xi_{l,2}$ terms contribute with positive sign to the scattering amplitude. Recently Krisch[11] analyzed the experimental data for p-p scattering. His results for the effects of the core term are quite different from ours in that his results are obtained using the same sign for all the $\xi_{l,2}$ terms. In our previous study[4] of pion-proton interactions at high energy, we have also pointed out that $\xi_{2l,4}$ and $\xi_{2l+1,4}$ have opposite signs. Such results may give us an important clue for solving the divergence difficulty inherent in field theory.

(ii) We have shown two kinds of solutions for phase shifts (η_l's) corresponding to cases (I) and (II). The terms $\exp\frac{1}{2}(A_0+A_1t)$ and $\exp\frac{1}{2}(B_0+B_1t)$ interfere destructively and constructively in cases (I) and (II), respectively. Therefore, in the region $[|t|=0.7-1.5\ (\mathrm{BeV}/c)^2]$ where both terms have almost the same magnitude, $d\sigma/dt$ values in the former case are much smaller than those in the latter case. This makes it possible to decide which solution should be chosen. When we discuss the shrinkage of diffraction peak for p-p scattering, it is necessary to examine the energy dependence not only of the contribution from core term but also of the interference between the terms $\exp\frac{1}{2}(A_0+A_1t)$ and $\exp\frac{1}{2}(B_0+B_1t)$, although this problem has not been treated in this paper.

(iii) Needless to say, there is a condition $0 \leq 1-\eta_l \leq 1$ owing to the unitarity of the S matrix. However, our results show that $1-\eta_0$ and $1-\eta_2$ in case (I) $[1-\eta_1$ in case (II)$]$ are slightly larger than unity. This is probably due to the following reason: In our treatment, the real part of the scattering amplitude is neglected and its effect is covered by the imaginary part of scattering amplitude. Since the shift from unity is very small, particularly in case (II), it can be said that this approximation is fairly good. Through the estimated values of parameters A_0, A_1, B_0, and B_1, we can see the following: The height of the peak due to the core term (backward peak) is of the order of $10^{-2}-10^{-3}$ times that of the peak due to the pion term (forward peak). The width of the former peak is much larger than that of the latter peak.

The author would like to express his sincere thanks to Professor T. Kinoshita for his kindly giving the latest information about p-p scattering.

---

*Work supported by the National Science Foundation and by a grant-in-aid from the University Council on Research.

†On leave of absence from Osaka City University, Osaka, Japan.

[1]K. J. Foley et al., Phys. Rev. Letters 11, 425 (1963).

[2]G. Cocconi et al., Phys. Rev. Letters 11, 499 (1963).

[3]The reason why we focus our attention on this energy is the following: In our description we assume that the real part of elastic-scattering amplitude is negligibly small. This assumption seems to be consistent with the experiment at 16.7 BeV/c, that is, the experimental results for the forward scattering at this energy could almost be explained in terms of the imaginary part of scattering amplitude (see reference 1).

[4]S. Minami, Phys. Rev. (to be published).

[5]Nambu and Sugawara have shown that the scattering amplitude for the nonshrinking diffraction peak has an expression similar to this: Y. Nambu and M. Sugawara, Phys. Rev. Letters 10, 304 (1963); M. Sugawara and Y. Nambu, Phys. Rev. 131, 2335 (1963).

[6]The value of u_0 (u at 180°) is equal to zero in the p-p scattering.

[7]Note that the sign of C will be determined later.

[8]Recently, Jones has pointed out that the observed data outside the diffraction peak might be in agreement with the prediction of a statistical model: G. Fast and R. Hagedorn, Nuovo Cimento 27, 208 (1963); G. Fast, R. Hagedorn, and L. W. Jones, Nuovo Cimento 27, 856 (1963); L. W. Jones (to be published).

[9]In order to explain the experimental results for p-p scattering with large momentum transfer, Serber proposed a Yukawa-shaped, purely absorptive optical model: R. Serber, Phys. Rev. Letters 10, 357 (1963).

[10]This corresponds to the expression in which the positive signs of all the (±) or (∓) signs in Eq. (3) are taken. As was mentioned before it is necessary to take the upper (lower) signs of all the double signs in order to satisfy the condition imposed by Fermi statistics.

[11]A. D. Krisch, Phys. Rev. Letters 11, 217 (1963).