It is a pleasure to acknowledge the contribution of Michiyuki Nakamura, Richard Mendez, John La-Pierre, and Sam Nolan in connection with the multidimensional analyzer and stabilization systems. We wish to thank Claudette Rugge for help with the computer programs and Joan Phillips for help with this paper.

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†Work done under the auspices of the U. S. Atomic Energy Commission.

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EXCITATION OF ISOBARIC ANALOG STATES IN ^{89}Y AND ^{90}Zr†

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(Received 16 October 1963; revised manuscript received 17 January 1964)

We wish to report the excitation of compound nucleus resonances in ^{89}Y and ^{90}Zr by proton bombardment of ^{88}Sr and ^{89}Y, respectively. These states are interpreted as isobaric analogs of the corresponding low-lying states of ^{89}Sr and ^{90}Y. Thin evaporated targets of natural strontium (83 % ^{88}Sr) and yttrium (100 % ^{89}Y) on evaporated carbon backings were bombarded with protons and the resulting neutrons detected with a Hanson-McKibben long counter placed at 50° to the beam direction and at 65 cm from the target. The same targets were also used for measurements of (p,p) elastic-scattering angular distributions using junction counters in a scattering chamber.

The excitation functions observed for ^{88}Sr(p,n) and ^{89}Y(p,n) are shown in Fig. 1. Proton elastic-scattering data are shown in Fig. 2(a) for ^{88}Sr $+p$ at 90° and 125.5° (laboratory angles) and in Fig. 2(b) for ^{89}Y $+p$ at the same angles. The targets were less than 10 keV thick to 5-MeV protons and the tandem Van de Graaff beam has an energy spread of approximately 3 keV. Typical bombarding times led to integrated beams of 50 to 100 microcoulombs.

Analysis of the (p,p) data indicates that the strong resonances are due to d-wave proton capture and leads to spin-parity assignments of $5/2^+$ and 2^- and 3^- for the anomalies in ^{88}Sr $+p$ at 5.08 MeV and ^{89}Y $+p$ at 4.82 and 5.02 MeV, respectively. Typical theoretical fits to the data are shown for ^{89}Y(p,p) in Fig. 2(b). The theoretical curves are calculated using the conventional single-level approximation.[1] The widths used in both cases are about 10 keV and the proton partial widths are about half the total widths.

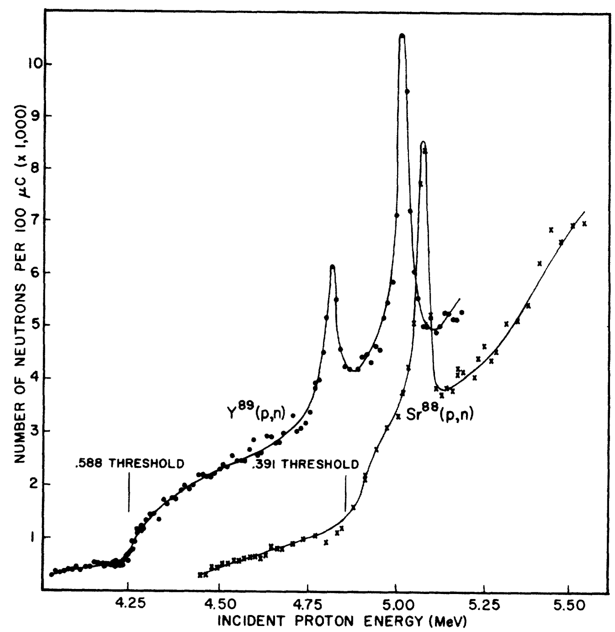

FIG. 1. Neutron yields for ^{88}Sr(p,n) and ^{89}Y(p,n) near threshold.

The experimental results and their interpretation on the basis of shell-model configurations are given in Table I. Also given in Table I are the known low-lying levels of the nuclei formed by (d,p) reactions[2] on the same target nuclei. Comparing the relative energies of the corresponding pairs of configurations (e. g. ^{89}Sr and ^{89}Y*), we

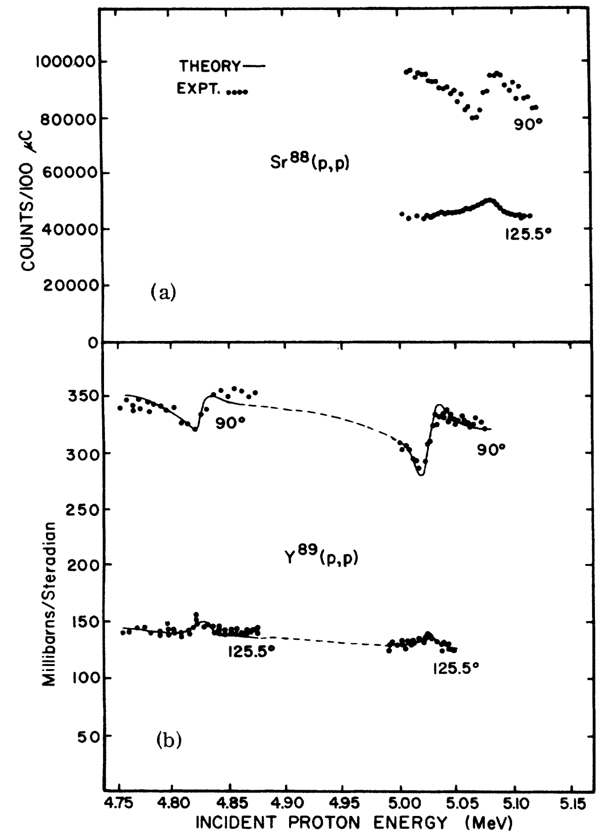

FIG. 2. (a) Proton elastic-scattering yield from ^{88}Sr + p measured at 90° and 125.5°. (b) Proton elastic-scattering cross sections for ^{89}Y + p measured at 90° and 125°. The solid curves are theoretical fits to the data.

find that they are separated by $(E_C - \delta)$ MeV, where δ is the neutron-proton mass difference (0.78 MeV) and E_C is given in the last column of Table I. If the corresponding pairs of states are isobaric analogs, then E_C should be very nearly equal to the Coulomb energy of the added proton. The values of E_C given in Table I are in good agreement with the Coulomb displacement energies measured for similar nuclei.[3] The requirement that isobaric analog pairs have the same quantum numbers except for T_z and energy is clearly satisfied for the states listed in Table I. We conclude therefore that the compound nucleus resonances observed here by proton-induced reactions should be interpreted as the isobaric analogs of the low-lying levels of the corresponding nuclei formed by neutron capture.

One must be careful not to confuse the experiments reported here with the work of Anderson, Wong, and McClure.[3] Here, we are apparently observing analog states as compound nucleus resonances, whereas the analog states investigated in reference 3 are observed as states in the residual nucleus. The important features of the present work is that the resonances observed in proton-induced reactions are closely related to the spectra observed in (d,p) reactions on the same target nucleus.

The authors wish to acknowledge the help in data taking of Dr. J. A. Becker, P. Richard, C. E. Watson, and D. D. Long. Mrs. Theresa Park

Table I. Comparison of analog states observed in $A = 89$ and $A = 90$ nuclei.

Nucleus	Configuration p	Configuration n	J^π	Excitation energy (MeV)	Doublet splitting (MeV)	E_C (MeV)
^{88}Sr	(38)	(50)	0^+	Gnd. State	...	...
^{89}Sr	(38)	$(50)d_{5/2}$	$5/2^+$	Gnd. State	...	11.80 ± 0.08
^{89}Y*	$(38)d_{5/2}$	(50)	$5/2^+$	12.47 ± 0.05	...	
^{89}Y	$(38)p_{1/2}$	(50)	$1/2^-$	Gnd. State	...	...
^{90}Y$_A$	$(38)p_{1/2}$	$(50)d_{5/2}$	2^- 3^-	Gnd. State 0.202	0.202	11.63 ± 0.05
^{90}Zr$_A$*	$(38)p_{1/2}d_{5/2}$	(50)	2^- 3^-	13.12 ± 0.03 13.32 ± 0.03	0.204	
^{90}Y$_B$	$(38)p_{1/2}$	$(50)s_{1/2}$	0^- 1^-	1.214 1.374	0.160	11.56 ± 0.05
^{90}Zr$_B$*	$(38)p_{1/2}s_{1/2}$	(50)	0^- 1^-	14.27 ± 0.03 14.40 ± 0.03	0.13	
^{90}Y$_C$	$(38)p_{1/2}$	$(50)d_{3/2}$	2^- 1^-	2.479 2.627	0.148	11.58 ± 0.05
^{90}Zr$_C$*	$(38)p_{1/2}d_{3/2}$	(50)	2^- 1^-	15.55 ± 0.03 15.70 ± 0.03	0.15	

assisted in the computations.

*Work supported in part by the U. S. Air Force Office of Scientific Research and the U. S. Office of Naval Research.

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PHOTON-PROTON SCATTERING AT 16.7 BeV/c AND INTERACTION WITH THE NUCLEON CORE*

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(Received 17 January 1964)

The detailed experimental data for small-angle elastic scatterings in proton-proton collisions at 7-20 BeV/c were reported by Foley et al.[1] In addition, Cocconi et al.[2] examined recently the large-angle proton-proton elastic scatterings at high energy. On the basis of these experimental data we study in this paper the proton-proton interaction at[3] 16.7 BeV/c by taking into account the effect of Fermi statistics.

In a previous study[4] of the pion-proton interaction we adopted the following expression for the scattering amplitude[5]:

$$f(\theta)=i\{\exp\tfrac{1}{2}(A_0+A_1t)+C \pm \exp\tfrac{1}{2}[B_0+B_1(u-u_0)]\}\ (\text{mb})^{1/2}, \qquad (1)$$

where the first and third terms are responsible, respectively, for the forward peak and the backward peak. The second term iC depends strongly on the incident energy and may be interpreted as an effect due to inelastic scattering which might be described in terms of the statistical model. Since the experimental data[1] are given by the form $|d\sigma/dt|=(\pi/k^2)d\sigma/d\Omega$, it is better to reexpress the scattering amplitude as follows:

$$f(\theta)=(ik/\sqrt{\pi})[\exp\tfrac{1}{2}(A_0+A_1t)+C \pm \exp\tfrac{1}{2}(B_0+B_1u)]\ (\text{mb})^{1/2}, \qquad (1')$$

where $t=-2k^2(1-\cos\theta)$ and $u=-2k^2(1+\cos\theta)$.[6] An exchange $t\rightleftarrows u$ in Eq. (1′) leads to the expression of $f(\pi-\theta)$. Note that, if the upper (lower) sign in the double sign ($\pm$) is taken, one must take the upper (lower) sign in all the double signs which will appear throughout this paper.[7] By taking into account the effect of Fermi statistics, the differential cross section for p-p elastic scattering is given by

$$d\sigma/d\Omega=\tfrac{3}{4}|f(\theta)-f(\pi-\theta)|^2+\tfrac{1}{4}|f(\theta)+f(\pi-\theta)|^2. \qquad (2)$$

The factors $\frac{3}{4}$ and $\frac{1}{4}$ indicate the statistical weight factors for the spin triplet and singlet states, respectively.

So far as elastic scattering in the forward direction (0°-90°) is concerned, $d\sigma/dt$ can be expressed approximately as follows:

$$\begin{aligned}|d\sigma/dt| &\cong \tfrac{3}{4}[\exp\tfrac{1}{2}(A_0+A_1t)\mp\exp\tfrac{1}{2}(B_0+B_1t)]^2 \\ &\quad+\tfrac{1}{4}[\exp\tfrac{1}{2}(A_0+A_1t)\pm\exp\tfrac{1}{2}(B_0+B_1t)+2C]^2, \\ &=\exp(A_0+A_1t)+\exp(B_0+B_1t) \\ &\quad\mp\exp\tfrac{1}{2}[A_0+B_0+(A_1+B_1)t] \\ &\quad+C[\exp\tfrac{1}{2}(A_0+A_1t)\pm\exp\tfrac{1}{2}(B_0+B_1t)] \\ &\quad+C^2\ \text{mb}/(\text{BeV}/c)^2. \end{aligned} \qquad (3)$$

In our study of $d\sigma/dt$, the scattering angles from 0° to 90° are divided into the following three regions: (I) the region of the diffraction peak; $|t|=0-1$ (BeV/c)^2, (II) the intermediate region; $|t|=1-8$ (BeV/c)^2, and (III) the region in which a statistical model is available and in which there is no effect of diffraction scattering; $|t|=8-14.81$ (BeV/c)^2. In region (III), $d\sigma/dt$ is nearly equal to C^2 [see Eq. (3)].[8] Using the experimental data given by Cocconi et al.,[2] we estimate the value of C as

$$C=\pm 0.8\times10^{-3}\ (\text{mb})^{1/2}/(\text{BeV}/c). \qquad (4)$$

The behavior of $d\sigma/dt$ in region (I) can be approximately described by the first, second, and third terms in Eq. (3), because the contribution from the C term can be neglected. We now try to determine the parameters A_0, A_1, B_0, and B_1 so that the experimental results given by Foley et al.[1] may be reproduced, although the results have been fitted by an expression of the form[1]

$$|d\sigma/dt|=\exp(a+bt+ct^2), \qquad (5)$$

with

$$a=4.52,\ b=9.79\ (\text{BeV}/c)^{-2},$$