MAGNETIC FIELD OF A ROTATING SUPERCONDUCTOR*

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The generation of a small magnetic field upon rotation of a perfect conductor was first predicted by Becker and co-workers.[1] Their acceleration theory gave a result which depended on the state of rotation of the conductor when the resistance vanished. The same effect was later calculated by London[2] for a solid sphere, taking account of the Meissner effect.

The effect arises as follows: If one considers a multiply connected superconductor in the form of a cylindrical shell, then according to London,[2] the local mean value of the canonical momentum vector of the superelectrons $\vec{p}_s$, integrated about a closed path, is quantized:

$$\oint \vec{p}_s \cdot d\vec{l} = \oint \left(m\vec{v}_s + \frac{e}{c}\vec{A}\right) \cdot d\vec{l} = Kh, \qquad (1)$$

where m and e are the mass and charge of the electron, respectively. This expression is made consistent with the experiments of Deaver and Fairbank[3] and Doll and Näbauer[4] if K is taken to be $\frac{1}{2}n$ where n is an integer. Different n's are possible in a cylindrical shell depending on the initial conditions. For zero external field and for the shell initially at rest, n is equal to zero. Using London's argument regarding the Meissner effect in a rotating superconductor, it follows from Eq. (1) for $n=0$ that a superconducting shell rotating with angular velocity $\vec{\omega}$ produces a field given by

$$\vec{B} = -(2mc/e)\vec{\omega} = 1.137\times 10^{-7}\vec{\omega} \qquad (2)$$

in gauss. This is the same result found previously.[1]

The effect and the constant in this equation have now been verified experimentally. The experiment consisted of cooling a 5-cm diameter by 7.5-cm long lead shell to the superconducting transition in a very small steady field. In one case the sample consisted of a $5\text{-}\times 10^{-4}$-cm lead film electrodeposited on brass and in another case, a $2.5\text{-}\times 10^{-2}$-cm lead foil on a nonmetallic form. In both cases the shell was then rotated in liquid helium and the field in the central region was measured with a room-temperature flux-gate magnetometer probe. The angular velocity was measured with an optical tachometer and recorded along with the magnetometer reading on an x-y recorder. The relatively large cylinder size was chosen in order to avoid strong interaction with the probe. All parts were checked for magnetic impurities prior to assembly.

The experiment was performed in a Mu-metal enclosure with a steady field of about 0.7 milligauss. This was then canceled to the order of 10^{-5} gauss with Helmholtz coils. For a field of 10^{-4} gauss the accuracy of calibration was $\pm 0.5 \times 10^{-5}$ gauss or ±5%. The accuracy of the frequency measurement was ±2%. The over-all accuracy was thus ±7%.

When the experiments were first performed on the electrodeposited film, the expected effect was observed, but this was in the presence of a large field initially trapped. These results are tabulated in Table I. A careful check showed several minute magnetic impurities in the brass wall.

In an attempt to eliminate these large trapped fields, a separate experiment was performed on a pure lead foil. In approximately zero field it was found that the lead foil trapped less than 10^{-5} gauss on each of seven attempts.

The results of rotating such a foil are shown in Fig. 1. First it is noted that the field reverses upon changing direction of rotation. The direction of the observed field with respect to rotation is in agreement with Eq. (2). The small hysteresis is due to the tachometer. The negative intercept is attributed to a drift due to the venting of cold gases while transferring helium. A result not shown is that when the sample is not superconducting, a plot of zero slope is obtained.

The results of five experimental runs are shown in Table I. The large trapped fields are noted for the electroplated film. The data were analyzed

Table I. $\Delta\omega$ is the total change in revolutions per second for a change in field ΔB. B_i is the initially trapped field.

Material	Run	$B_i \times 10^5$ (gauss)	$\Delta\omega$ (rps)	$\Delta B \times 10^5$ (gauss)	$10^7 \Delta B/\Delta\omega$ (gauss sec)
Lead film	1	+ 24.5	290	18.0	0.99
	2	− 20.0	300	20.6	1.10
	3	+120	253	17.8	1.12
Lead foil	1	− 2.8	253	17.7	1.12
	2	− 3.0	266	18.4	1.10
				av	1.09

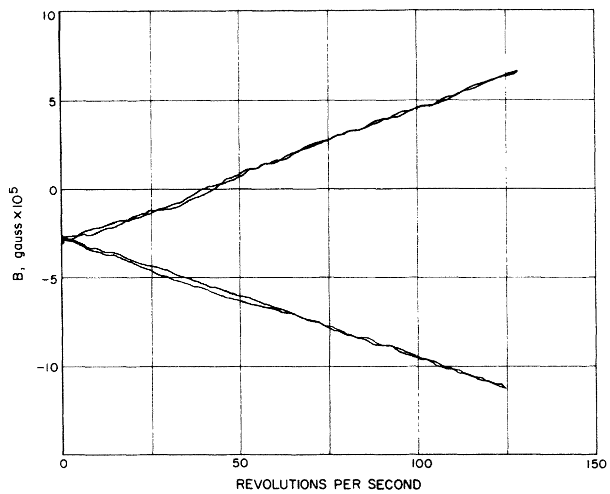

FIG. 1. Recording of magnetic field strength vs rotation rate in revolutions per second. The upper curve is for counterclockwise rotation and the lower curve for clockwise rotation. The material is a bulk lead foil.

by fitting curves such as those in Fig. 1. The agreement with Eq. (2) is good, especially if run 1 is omitted. This was the first experiment in which the effect was observed. The dependence of the effect on initial conditions for the shells considered here and for the simply connected case (where the original theories[1,2] gave different results) are being investigated.

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CONSERVATION OF THE FLUXOID

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As originally noted by London,[1] a superconductor is expected to conserve and quantize a quantity which he called the fluxoid. This principle applied to a thin superconducting cylinder leads to the expectation that a sum of the trapped magnetic flux and the mechanical angular momentum of the persistent current (in appropriate units) will be conserved and quantized[2] in units of $h/2e$. Although the contribution of the mechanical momentum to the conservation law has been too small to be particularly significant in most experiments,[3] its presence leads to a temperature dependence of the trapped flux. The mechanical momentum is a function of penetration depth λ and therefore varies with temperature. Consequently the amount of trapped flux would also be expected to show an oppositional temperature variation if it is presumed that the quantum number is an adiabatic invariant. The purpose of this Letter is to report experimental confirmation of such a temperature variation of the trapped flux consistent with adiabatic invariance for small temperature oscillations.

The presence of the mechanical angular momentum in the fluxoid leads to a modification[2] of the flux quantum $h/2e$ to

$$\Phi_0=(h/2e)(1+2\lambda^2/r\delta)^{-1}$$

for the modified flux unit (Φ_0) trapped by a long thin (δ) cylinder of radius r and penetration depth λ. Total flux trapped by such a cylinder must be an integral number n of these units, $n\Phi_0$. This trapped flux is then temperature dependent through the known temperature variation of the penetration depth. A temperature variation ΔT will be expected to cause a change $\Delta\Phi$ in trapped flux $n\Phi_0$ of

$$\Delta\Phi \simeq -n(h/2e)(2/r\delta)[d(\lambda^2)/dT]\Delta T.$$

This anticipated temperature variation of trapped flux depends only on the adiabatic invariance of the fluxoid and not on its quantization. Such an