is weak, however. Lately, there seems to be some evidence[17] for a scalar particle at 380 MeV with 0^{++}, which was first suggested by Brown and Singer.[18] This particle can contribute a positive self-mass to K_1^0, thus reinforcing our conclusion regarding the sign of the mass difference. An assumption regarding the transformation property of weak vertices similar to ours has recently been applied to the problem of nonleptonic hyperon decays in pole approximation by Sugawara[19] who finds it possible to correlate presently known experimental results on the basis of such a model. We are aware of the limitations of a restrictive model like the pole approximation for the $(K_1^0 - K_2^0)$ mass difference, whose understanding might eventually turn out to require a consideration of more complicated intermediate states than those considered here. This possibility is underlined by our inability to convincingly calculate the magnitude of the mass difference. We feel, however, that since the present model, apart from being simple and giving finite results, leads to definite predictions which relate the sign of $(K_1^0 - K_2^0)$ mass difference to other effects, it might still be of some interest to check its conclusions with experimental results.

We are thankful to Professor G. Feinberg and Professor B. M. Udgaonkar for several discussions.

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[6]This may be derived from the condition $\langle\pi^0|H_\omega|K_1^0\rangle = 0$ which is a consequence of CP invariance. The condition $\langle\eta^0|H_\omega|K_1^0\rangle = 0$ leads to the same conclusion.

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[14]E. Fowler, F. Crawford, Jr., L. Lloyd, R. Grossman, and L. Price, Phys. Rev. Letters 10, 111 (1963).

[15]This may be easily seen by writing the phenomenological two-particle interaction as $(\partial_\mu K)B_\mu$, K and B_μ denoting the field amplitudes for the K meson and vector boson, respectively, and remembering the parity property of B_μ, i.e., $PB_iP^{-1} = -B_i$, $i = 1,2,3$, and $PB_4P^{-1} = B_4$. We would like to acknowledge the benefit of an illuminating conversation regarding this point with Professor G. Feinberg.

[16]We have not included the recently reported [M. Abolins, R. Lander, W. Mehlhop, N. Xuong, and P. Yager, Phys. Rev. Letters 11, 381 (1963)] B particle into consideration as its quantum numbers are yet to be determined. In any case, since the B particle has no natural place in the SU_3 scheme, the possibility of its belonging to some octet other than the vector octet does not seem to be ruled out. In particular, it could be a Regge recurrence of one of the known vector mesons. The spin of B will then be 3.

[17]F. Crawford, Jr., R. Grossman, L. Lloyd, L. Price, and E. Fowler, Phys. Rev. Letters 11, 564 (1963).

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ARE THE f^0 AND B MESONS TWO DECAY MODES OF THE SAME PARTICLE?*

William R. Frazer,† Sharashchandra H. Patil, and Nguyen-huu Xuong
University of California, San Diego, La Jolla, California
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The striking fact that the recently discovered B meson has the same mass and width (within statistics) as the f^0 suggests the hypothesis that these two resonances are, in fact, different decay modes of the same particle.[1-4] In this Letter we examine the consequences of this hypothesis and find that it is not incompatible with the currently available experimental data. In fact, it seems to be easier to reconcile the data to this viewpoint than to the usual one in which the f^0 is assigned $I = 0$.

A particle which decays strongly, as does the B, into a π and an ω, must have $I = 1$. If it also decays strongly into two pions, as does the f^0, it must have odd J and negative parity. We shall confine our attention largely to the 1^- assignment, and refer to this hypothetical particle as the ρ' meson. We shall use the name B to designate the

$\pi\omega$ decay mode of the ρ' meson, and f to designate the $\pi\pi$ decay mode, just as θ and τ designated different decay modes of the K. We now examine the implications of the ρ' hypothesis.

(I) Angular distribution in f^0 decay.—The f^0 is observed in the reaction

$$\pi^- + p \rightarrow \pi^+ + \pi^- + n. \tag{1}$$

Distributions have been measured in $\cos\theta_{\pi\pi}$, the angle between incoming and outgoing π^- in the barycentric system of the final pions. It seems quite plausible to assume that for low momentum transfers to the nucleon the above reaction proceeds primarily through one-pion exchange. Then the angular distribution should be $[P_J(\cos\theta_{\pi\pi})]^2$, where J is the spin of the f^0. The data on $\cos\theta_{\pi\pi}$ of Guiragossián[4] and of Bondår et al.[3] are reproduced in Figs. 1(a) and 1(b).[5] In Fig. 1(c) are plotted the combined data for both experiments. We also plot the normalized curves predicted by $J=1$ (solid line) and $J=2$ (dashed line). It is apparent that the best fit is obtained with $J=1$. A χ^2 test of the $J=1$ curve fitted to the distribution in Fig. 1(c) has an expected χ^2 of 4.0; we find 4.1, which corresponds to a probability of 40%. The $J=2$ curve gives a χ^2 of 105; i.e., a very small probability.[6] The curve predicted by $J=3$ also does not fit the data; we get $\chi^2=40$.

Thus the angular distribution in f^0 decay strongly favors the ρ' hypothesis. The scarcity of events at $\cos\theta_{\pi\pi}=0$ is a serious obstacle to the assignment of $I=0$, and therefore J even, to the f^0.

(II) Branching ratios.—The most obvious objection to the ρ' hypothesis is the fact that the decay $f^\pm \rightarrow \pi^\pm + \pi^0$ has not been observed. In the reaction

$$\pi^- + p \rightarrow \pi^- + \pi^0 + p, \tag{2}$$

no peak has been seen in the invariant mass spectrum of the final pions in the neighborhood of 1250 MeV. On this basis Selove et al.[2] made the assignment $I=0$ for the f^0. Moreover, no such peak was seen by Alff et al.[7] and by Carmony et al.[8] in the reaction

$$\pi^+ + p \rightarrow \pi^+ + \pi^0 + p. \tag{3}$$

The ρ' hypothesis, however, assigns $I=1$ to the f^0 and therefore implies a triangle inequality

$$\sqrt{\sigma_+} + \sqrt{\sigma_-} \geq (2\sigma_0)^{1/2}, \tag{4}$$

where $\sigma_\pm$ are the total cross sections for $\pi^\pm + p \rightarrow f^\pm + p$, and σ_0 is the total cross section for $\pi^- + p \rightarrow f^0 + n$. This apparent contradiction of the ρ' hypothesis cannot be explained by arguments invoking violation of isospin conservation, because the large width of the ρ' insures that its decay is via strong interactions. We propose that the decays $f^\pm \rightarrow \pi^\pm + \pi^0$ do indeed exist, but that the corresponding peaks in Reactions (2) and (3) are much harder to resolve from the background than is the f^0 peak in Reaction (1). Due to the existence of the ρ peak, detection of the f peak depends critically on the depth of the valley between the two peaks. The valley between the $\rho^\pm$ and $f^\pm$ peaks could be much shallower than the one be-

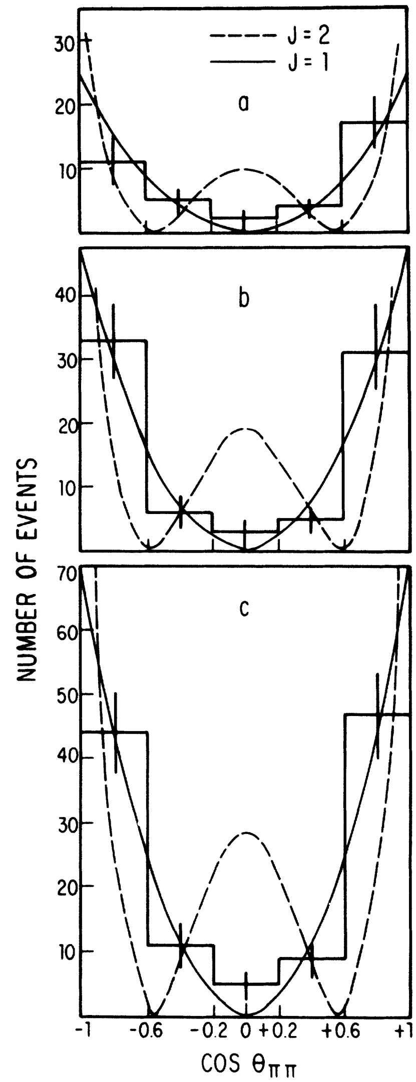

FIG. 1. $\cos\theta_{\pi\pi}$ distribution of peripheral events from the reaction $\pi^- + p \rightarrow n + \pi^+ + \pi^-$ with $M_{\pi\pi}$ in the f^0 peak: (a) from Guiragossián (reference 4), (b) from Bondår et al. (reference 3), (c) from both experiments. Also are plotted the normalized curves predicted by $J=2$ and $J=1$.

tween the ρ^0 and f^0. We shall attempt to make this plausible by considering a simple model of ρ' production which exhibits this feature.

(III) One-pion and one-omega exchange model. In an earlier Letter by Watson and two of the present authors,[9] the reaction in which the B was observed,

$$\pi^+ + p \rightarrow \pi^+ + \omega + p, \tag{5}$$

was analyzed by the one-pion-exchange model, Fig. 2(a). This model, by assuming that the B is produced from a $\pi\pi$ state, implies that its quantum numbers are 1^- (or 3^-, etc.) and that it has a strong $\pi\pi$ decay mode; therefore, it is consistent with the ρ' hypothesis. The left-hand vertex in Fig. 2(a) involves the reaction

$$\pi + \pi \rightarrow \pi + \omega, \tag{6}$$

whose amplitude in the $J=1$ state we call M_{12}. In reference 9 this amplitude, along with the amplitudes M_{11} and M_{22} for the processes

$$\pi + \pi \rightarrow \pi + \pi \tag{7}$$

and

$$\pi + \omega \rightarrow \pi + \omega \tag{8}$$

in the 1^- state, was parametrized by a multichannel effective-range theory based on the ND^{-1} method. The parameters were adjusted to fit the positions and widths of the B and the ρ. A rough and very conservative prediction of the width of the ω was made, $\Gamma_\omega \geq 1$ MeV,[10] which was subsequently confirmed by a direct measurement of the ω width.[11] Despite this confirmation, it is apparent that some extension of the one-pion-exchange (OPE) model is necessary. Although the Treiman-Yang test was reasonably well satisfied in Reaction (5), other tests for one-pion exchange in this reaction did not seem to be satisfied.[10] The most natural extension is to include one-omega exchange (OOE), Fig. 2(c). The left-hand vertex involves M_{22}, which is determined by our effective-range model. For the $N\bar{N}\omega$ vertex we take the form and strength found by Scotti and Wong to give a good fit to the nucleon-nucleon scattering data.[12] In addition, form factors, which are slowly varying functions of the momentum transfer to the nucleon, are included for both OPE and OOE. The Ferrari-Selleri[13] result is used for the OPE form factor, and the extra parameter introduced in the OOE form factor is used to fit the distribution in $\cos\theta_{\pi\pi}$ observed at the ρ peak in Reaction (2). Further discussion of the model is relegated to the Appendix, and to a more detailed subsequent publication, since the essential arguments of this Letter depend only on the qualitative features of the model.

Having used all the parameters of the model to fit the ρ and the B decay mode of the ρ', we are able to extract from the model predictions about the f-decay mode. These are calculated from the diagrams of Figs. 2(b) and 2(d). We emphasize that no experimental information about the f has been used as input. The resulting distributions in $M_{\pi\pi}$ are shown in Fig. 3 for an incident pion momentum of 4 BeV/c, as in the experiment of Bondâr et al.[3] [dashed line for Reaction (1) and

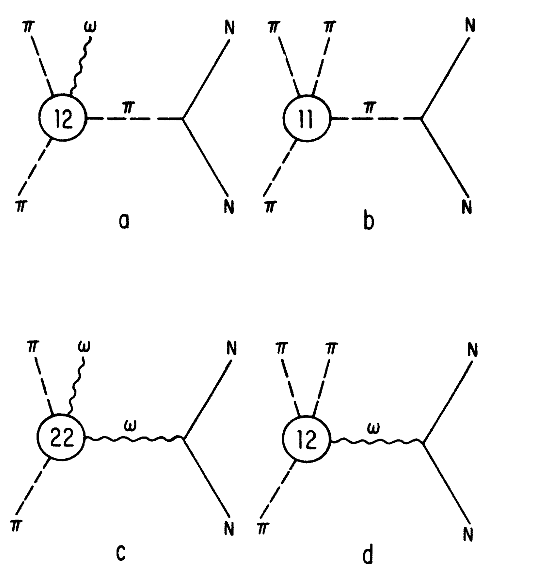

FIG. 2. Diagrams included in the O(P+O)E model (see text).

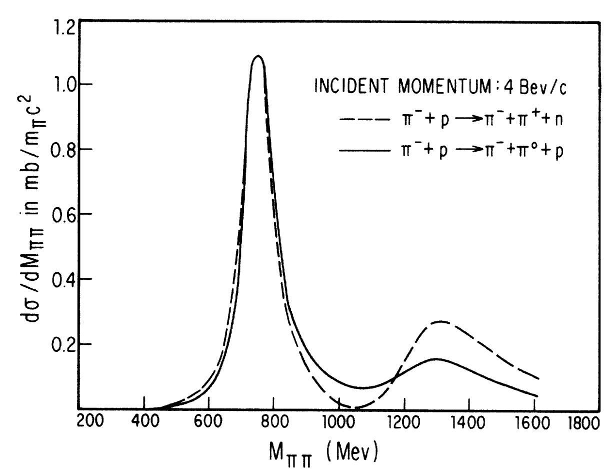

FIG. 3. Effective-mass distribution of the pion pair from the reaction $\pi N \rightarrow N\pi\pi$ as predicted by the O(P+O)E model for an incident momentum of 4 BeV/c.

solid curve Reactions (2) and (3)]. The predictions for Reaction (1) are in good agreement with the experimental data; those for Reaction (2) do not seem to be inconsistent.

We call the reader's attention to the striking difference between f^0 and $f^\pm$ production. The actual values of the cross sections at the f peak are, of course, consistent with the triangle inequality, Eq. (4). In f^0 production, however, there is a much more pronounced valley between the f and the ρ' than there is in $f^\pm$ production. This effect, which makes the $f^\pm$ peak much more difficult to resolve from the background, provides a possible explanation of why the $f^\pm$ has escaped detection. We estimate from our model that the ratio of the apparent (above the valley) cross sections for $f^\pm$ production in Reaction (2) or (3) and f^0 production in Reaction (1) is 0.20 at 3 BeV/c and 0.25 at 4 BeV/c. The experimental ratio is ≤ 0.3 at 3 BeV/c and ≤ 0.25 at 4 BeV/c.

The difference between the predictions of the model for f^0 and $f^\pm$ production arises from the fact that in $f^\pm$ production via Reaction (2) or (3) both OPE and OOE contribute, whereas for f^0 production via Reaction (1) only OPE is possible (the exchanged particle must be charged). Thus the solid curve in Fig. 3 is a sum of the two contributions (the interference term vanishes): The OOE term, and the OPE term which for Reaction (2) is $\frac{1}{2}$ times the dashed curve. The OOE term differs in two important respects from the OPE term: (a) Because of the spin of the omega it has a kinematical factor in the numerator which suppresses high values of $M_{\pi\pi}$. Its contribution is large around the ρ peak, but decreases rapidly to become almost negligible at the f peak. (b) It does not vanish between the two peaks, as does the OPE term.[14]

Our model gives a branching ratio $(\rho' \to \pi + \omega)/(\rho' \to \pi + \pi) = \frac{1}{3}$, consistent with the experimental upper limit of 1.0 found by Carmony et al.[8]

(IV) Some experimental tests. – The ρ' hypothesis can be tested in many ways. We conclude by listing some which have occurred to us or have been suggested to us.

(A) Determination of the spin and parity of the B: Our hypothesis, of course, requires 1^- for the B. Some tests for its spin and parity have been proposed by Zemach.[15] Using these tests, Carmony et al. found evidence for $J \geq 1$ but could not distinguish between odd and even parity.[8]

(B) Search for $f^0 \to \pi^0 + \pi^0$: This is allowed if the f^0 has $I = 0$, forbidden if it has $I = 1$. This search can be done by comparing the missing-mass distribution M_x of the reaction

$$\pi^+ + D \to (p) + p + x$$

with the $\pi^+\pi^-$ effective-mass distribution of the reaction

$$\pi^+ + D \to (p) + p + \pi^+ + \pi^-.$$

Because ρ^0 does not decay neutrally, a peak in the M_x distribution at the f^0 mass would be easily observable. This peak must be $\frac{1}{2}$ of the corresponding $\pi^+\pi^-$ peak to satisfy the $I = 0$ hypothesis. With the ρ' hypothesis one would expect a ratio $\leq \frac{1}{10}$ (due to the B^0 decay mode with the ω decaying neutrally).

(C) Comparison of $n + p \to D + f^0$ and $p + p \to D + f^+$: If the f is the ρ', with $I = 1$, these reactions occur in the ratio 1 to 2.

(D) Search for $f^\pm$: One can, of course, keep looking for a small bump in the cross sections of Reactions (2) or (3). Alternatively, one could investigate the distribution in $\cos\theta_{\pi\pi}$ as a function of $M_{\pi\pi}$. A plot of the forward-backward asymmetry in $\cos\theta_{\pi\pi}$ has been given by Bondâr et al.[3] The plot for Reaction (2), in which the f^- is expected, is remarkably similar to (the negative of) the plot for Reaction (1), in which the f^0 is observed. We have no theory of this asymmetry, since it involves interference with background, but the similarity lends some support to our hypothesis.

Appendix. – For the amplitudes $M_{ij}(s)$, we use the "effective-range" parametrization $M = ND^{-1}$, where

$$N_{ij} = s_0 n_{ij}/(s + s_0),$$

$$D_{ij} = \delta_{ij} - s_0 n_{ij} \frac{s}{\pi} \int_{s_i} \frac{\rho_i(s')ds'}{s'(s' + s_0)(s' - s)},$$

where $\rho_1 = 2q_1^3/s^{1/2}$, $\rho_2 = 2q_2^3 s^{1/2}$, and q_1, q_2 are c.m. momenta in the $\pi\pi$ and $\pi\omega$ states. Observing that the integral in D_{21} and D_{22} diverges, we introduce a cutoff in the form

$$\rho_2 \to \rho_2[\Lambda/(s + \Lambda)]^2.$$

We now can calculate M_{ij} as a function of five parameters: n_{11}, n_{12}, n_{22}, s_0, and Λ ($M_{12} = M_{21}$ implies $n_{12} = n_{21}$). We eliminate two of these by requiring the zero of the determinant of D to represent a ρ meson with the correct position and width. Another parameter is determined to give the experimental width of the ω meson as described in reference 9. The remaining two parameters are determined so as to fit the position and width of the B in the process (5) using the π,ω exchange model.

Expressions for differential cross sections including diagrams in Fig. 2 can be calculated in terms of M_{ij} elements and $N\bar{N}\pi, N\bar{N}\omega$ couplings (taken to be of the form $g_\omega \bar{N}\gamma_\mu N\omega_\mu$). They give the following expressions for the production cross sections of process (5) and the two-pion production:

$$\frac{d^2\sigma_{\pi\pi}}{ds\,d\Delta^2}=\frac{1}{8\pi^2m^2q_{1L}^{\;2}}\left[-\left(\frac{g_\pi^{\;2}}{4\pi}\right)\frac{\Delta^2}{(\Delta^2-m_\pi^{\;2})^2}\frac{q_i^{\;2}q_f^{\;3}}{48}|A_{11}|^2+\left(\frac{g_\omega^{\;2}}{4\pi}\right)\frac{(2\vec{p}_2^{\;2}\sin^2\theta'-\Delta^2)}{(\Delta^2-m_\omega^{\;2})^2}\frac{q_i^{\;2}q_f^{\;3}}{24}|A_{12}|^2\right], \quad (13)$$

$$\frac{d^2\sigma_{\pi\omega}}{ds\,d\Delta^2}=\frac{1}{8\pi^2m^2q_{1L}^{\;2}}\left[-\left(\frac{g_\pi^{\;2}}{4\pi}\right)\frac{\Delta^2}{(\Delta^2-m_\pi^{\;2})^2}\frac{q_i^{\;2}q_f^{\;3}}{24}|A_{12}|^2+\left(\frac{g_\omega^{\;2}}{4\pi}\right)\frac{(2\vec{p}_2^{\;2}\sin^2\theta'-\Delta^2)}{(\Delta^2-m_\omega^{\;2})^2}\frac{q_i^{\;2}q_f^{\;3}}{12}|A_{22}|^2\right], \quad (14)$$

where $A_{11}=48\pi M_{11}$, $A_{12}=24\sqrt{2}\pi M_{12}$, $A_{22}=24\pi M_{22}$, m = nucleon mass; q_i and q_f are the magnitudes of the incoming and outgoing pion momenta, and θ' is the angle between $\vec{q}_1$ and $\vec{p}_2$, all of these being evaluated in the c.m. system of outgoing $\pi\pi$ or $\pi\omega$. All kinematical quantities are evaluated with the exchanged particles off the mass shell.[13]

Formulas (13) and (14) are extended to off-shell values of Δ^2 by introducing form factors f_π, f_ω for the π and ω exchange amplitudes, respectively. We take

$$f_\pi=(m_1^{\;2}-m_\pi^{\;2})/(m_1^{\;2}-\Delta^2),$$

$$f_\omega=(m_2^{\;2}-m_\omega^{\;2})/(m_2^{\;2}-\Delta^2).$$

The analysis of Ferrari and Selleri gives $m_1^{\;2}\sim 15$, while a fit to the $\cos\theta_{\pi\pi}$ distribution in the ρ production gives $m_2^{\;2}\sim 46$.

In the curves shown in Fig. 3, the values of the parameters are $s_0=1.92\times10^6$, $\Lambda=120$, $g_\omega^{\;2}/4\pi=2.7$, $n_{11}=2.94\times10^{-2}$, $n_{22}=4.44\times10^{-3}$, and $n_{12}=5.37\times10^{-3}$.

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[3]L. Bondâr et al., Aachen-Birmingham-Bonn-Hamburg-London-München Collaboration, Phys. Letters 5, 153 (1963); I. Derado (private communication). There is an error of scale in Fig. 1(b). The scale of the "number of $\pi^-\pi^0$ combinations" must be doubled.

[4]Z. G. T. Guiragossián, Phys. Rev. Letters 11, 85 (1963).

[5]We do not include the data of Hagopian and Selove (reference 2) at 3 BeV/c because the marked asymmetry of their $\cos\theta_{\pi\pi}$ distributions, in contrast with the symmetric distributions found at higher energies, seems to indicate a larger contamination from processes other than OPE. If, however, we divide the Hagopian and Selove data in the f^0 region into five intervals in $\cos\theta_{\pi\pi}$ and fit it with $J=1$ and $J=2$, we find χ^2 of 66 and 95, respectively ($\chi^2=4$ expected).

[6]A note of caution is in order here. We cannot rule out the possibility that the f^0 has spin 2, and that the observed distribution in $\cos\theta_{\pi\pi}$ results from interference with a complicated background of some sort. The hypothesis of a ρ' meson produced by one-pion exchange does, however, provide a much simpler way of accounting for the distribution.

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[12]A. Scotti and D. Y. Wong, Phys. Rev. Letters 10, 142 (1963).

[13]See, for example, F. Selleri, Phys. Rev. Letters 3, 76 (1962).

[14]Between the two peaks $|M_{11}|$ very nearly vanishes, because the phase shift in this channel goes through π in rising from $\pi/2$ at the ρ to around $3\pi/2$ at the ρ'.

[15]A. C. Zemach (to be published).