[24]H. P. Noyes and D. Bailey (to be published).

[25]For a somewhat similar use of the Born approximation, see J. L. Gammel and R. M. Thaler, Progr. Cosmic Ray Phys. 5, 99 (1960).

[26]W. Ramsay (to be published); extension of the work presented by R. A. Bryan, C. R. Dismukes, and W. Ramsay, Nucl. Phys. 45, 353 (1963).

[27]P. Signell (private communication).

[28]K. E. Lassila, M. H. Hull, Jr., H. M. Ruppel, F. A. MacDonald, and G. Breit, Phys. Rev. 126, 881 (1962).

[29]T. Hamada and I. D. Johnston, Nucl. Phys. 34, 382 (1962).

UNITARY SYMMETRY AND THE SIGN OF $(K_1{}^0 - K_2{}^0)$ MASS DIFFERENCE IN POLE APPROXIMATION

S. N. Biswas and S. K. Bose
Tata Institute of Fundamental Research, Bombay, India
(Received 26 November 1963; revised manuscript received 15 January 1964)

The purpose of this Letter is to calculate the sign of the $(K_1{}^0 - K_2{}^0)$ mass difference in a model based on the following two assumptions: (1) that the mass difference arises from the one-particle "pole" terms in the $K_{1,2}{}^0$ propagator, and (2) that the relevant two-point coupling constants which occur in the expression for the mass difference can be related by means of the octet version of the unitary symmetry scheme due to Gell-Mann[1] and Ne'eman.[2] We find that these assumptions predict a heavier $K_1{}^0$. Other consequences of the model are discussed. These include the processes $K^+ \to \pi^+ + e^+ + e^-$, $K_2{}^0 \to 2\gamma$, and the question of the violation of the $\Delta T = \frac{1}{2}$ rule in the $K_2{}^0 \to \pi^+ + \pi^- + \pi^0$ decay.

The recently discussed[3,4] pole-approximation model consists in the assumption that the $(K_1{}^0 - K_2{}^0)$ mass difference arises from the self-mass contributions associated with the boson pole diagrams (Fig. 1). Let us first consider the contributions of particles belonging to the pseudoscalar octet, i.e., those of π^0 and η^0. We require the "weak" two-point vertices to satisfy CP invariance. These states can then contribute only to the self-mass of $K_2{}^0$, the resultant expression for the $K_2{}^0$ self-mass being

$$\delta m(K_2{}^0) = 2m_K{}^{-1}\left[\frac{f_\pi{}^2}{m_K{}^2 - m_\pi{}^2} + \frac{f_\eta{}^2}{m_K{}^2 - m_\eta{}^2}\right]. \qquad (1)$$

In (1) f_π and f_η are suitably normalized constants which measure the strength of weak vertices $K_2{}^0 \to \pi^0$ and $K_2{}^0 \to \eta^0$, respectively. m_K, m_π, and m_η denote the masses of the corresponding particles. To relate f_π and f_η, we now assume that the weak vertices transform as the matrix elements of a component $(T_3{}^2)$ of a rank-2 tensor. Following Okubo[5] we may then write

$$T_3{}^2 = a_1 A_3{}^2 + a_2 (AA)_3{}^2, \qquad (2)$$

where $A_3{}^2$'s are the generators of SU_3 and the term (2) satisfies the $\Delta T = \frac{1}{2}$ rule. The constants a_1 and a_2 occurring above are not independent, but related by CP invariance as[6]

$$a_1 = \tfrac{3}{2} a_2. \qquad (3)$$

From (2) and (3) we easily obtain the desired relation[7]

$$f_\eta = (\sqrt{3})^{-1} f_\pi. \qquad (4)$$

Equations (1) and (4) together with the insertion of observed mass values predict a negative self-mass of $K_2{}^0$ and hence a heavier $K_1{}^0$.

We must now emphasize that the above conclusion regarding the sign of the $(K_1{}^0 - K_2{}^0)$ mass difference will be meaningful only if the π^0 and η^0 contributions are indeed dominant. From the observed rates of $K_{\mu 2}$ decay, $\pi_{\mu 2}$ decay, and muon decay and with the neglect of certain strong interaction effects, Baker and Glashow[8] have estimated f_π to be $f_\pi \simeq 4 \times 10^{-8} m_K{}^2$. If we accept this estimate, then from (1) and (4) we find $\delta m \simeq 10^{-7}$ eV in contrast to the experimental value[9] $\delta m \simeq 10^{-5}$ eV. However, this estimate of f_π could be in error by a factor of 10 as already discussed by Oneda et al.[4] If this is indeed so, the π^0 and η^0 contributions would then be of the right order. The effect of such a large f_π will have other observable consequences. These are as follows:

(1) $K^+ \to \pi^+ + e^+ + e^-$ decay.—This decay mode of the K meson is possible even without the existence

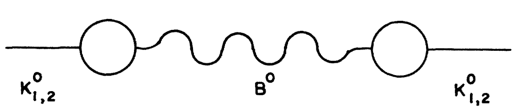

FIG. 1. Boson pole contribution to $K_{1,2}{}^0$ self-mass.

of a "primitive" neutral current, as this mode may be induced by the electromagnetic correction to the weak current. Using this idea, Baker and Glashow[8] have calculated the rate of this process in pole approximation. With $f_\pi \simeq 4\times10^{-8} m_K^2$ they find a branching ratio of $\sim10^{-6}$ for this process. However, with a f_π ten times larger, as is necessary to explain the $(K_1^0 - K_2^0)$ mass difference on the basis of π^0 and η^0, this rate will be enhanced, leading to a branching ratio of $\sim10^{-4}$. It might thus be possible to observe this mode.

(2) Electromagnetic violation of $\Delta T = \frac{1}{2}$ rule in $K_2^0 \to \pi^+ + \pi^- + \pi^0$ decay. —The $\Delta T = \frac{1}{2}$ rule relates the rate of $K_2^0 \to \pi^+ + \pi^- + \pi^0$ decay to that of $K^+ \to \pi^+ + \pi^0 + \pi^0$ decay as

$$\omega(K_2^0 \to \pi^+ + \pi^- + \pi^0) = 1.30\times 2\omega(K^+ \to \pi^0 + \pi^0 \pi^+),$$
$$= (2.87\pm0.23)\times10^6\ \mathrm{sec}^{-1}, \quad (5)$$

which differs considerably from the experimental[9] rate

$$\omega(K_2^0 \to \pi^+ + \pi^- + \pi^0) = (1.4\pm0.43)\times10^6\ \mathrm{sec}^{-1}. \quad (6)$$

One way of explaining this anomaly without invoking the presence of a $\Delta T = \frac{3}{2}$ amplitude is to ascribe this to electromagnetic correction through the sequence

$$K_2^0 \to \eta^0 \to \pi^+ + \pi^- + \pi^0. \quad (7)$$

The inclusion of this contribution changes Eq. (5) into[10]

$$\omega(K_2^0 \to \pi^+ + \pi^- + \pi^0)$$
$$= 1.03\times 2(1-x)^2 \omega(K^+ \to \pi^+ + \pi^0 + \pi^0), \quad (8)$$

with x related to f_η as[11]

$$x^2 = 0.53\times10^{-5}(f_\eta^{\ 2}/2m_K^{\ 4})\omega(\eta^0 \to \pi^+ + \pi^- + \pi^0); \quad (9)$$

$\omega(\eta^0 \to \pi^+ + \pi^- + \pi^0)$ is the rate of this process in sec^{-1}. From the observed[12-14] branching ratios of η,

$$\omega(\eta^0 \to \text{all neutrals})/\omega(\eta^0 \to \pi^+ + \pi^- + \pi^0) = 3.0,$$
$$\omega(\eta^0 \to \pi^0 + \pi^0 + \pi^0)/\omega(\eta^0 \to \pi^+ + \pi^- + \pi^0) = 1.68,$$

one can infer that

$$\omega(\eta^0 \to 2\gamma)/\omega(\eta^0 \to \pi^+ + \pi^- + \pi^0) = 1.32. \quad (10)$$

Using unitary symmetry one can estimate $\omega(\eta^0 \to 2\gamma)$ from the observed rate of $\pi^0 \to 2\gamma$. In this way, using (10), one concludes[11]

$$\omega(\eta^0 \to \pi^+ + \pi^- + \pi^0) = 1.7\times10^{17}\ \mathrm{sec}^{-1}. \quad (11)$$

With $f_\eta = \frac{4}{3}\times10^{-7} m_K^2$, we obtain from (9) and (11)

$$x \simeq 0.15, \quad (12)$$

while the experimental rate of $K_2^0 \to \pi^+ + \pi^- + \pi^0$ given by (6) corresponds to $0.19 \leq x \leq 0.41$.

(3) $K_2^0 \to 2\gamma$ decay. —The rate of this process can be calculated in pole approximation by taking the contributions of π^0 and η^0. This gives for the matrix element

$$F(K_2^0 \to 2\gamma) = \frac{f_\pi}{m_K^{\ 2} - m_\pi^{\ 2}} F(\pi^0 \to 2\gamma)$$
$$- \frac{f_\eta}{m_\eta^{\ 2} - m_K^{\ 2}} F(\eta^0 \to 2\gamma). \quad (13)$$

Using (4) and the unitary symmetry result $F(\eta^0 \to 2\gamma) = (\sqrt{3})^{-1} F(\pi^0 \to 2\gamma)$, this becomes

$$F(K_2^0 \to 2\gamma) = f_\pi[(m_K^{\ 2} - m_\pi^{\ 2})^{-1} - \tfrac{1}{3}(m_\eta^{\ 2} - m_K^{\ 2})]$$
$$\times F(\pi^0 \to 2\gamma). \quad (14)$$

With $f_\pi \simeq 4\times10^{-7} m_K^2$, (14) yields

$$\omega(K_2^0 \to 2\gamma) = 6.86\times10^{-13}\omega(\pi^0 \to 2\gamma),$$
$$\simeq 6.5\ \times10^3\ \mathrm{sec}^{-1}. \quad (15)$$

This corresponds to a branching ratio of $\simeq 4\times10^{-4}$.

Let us now consider the contributions of particles belonging to the vector octet, i.e., those of ρ^0 and ω^0. From CP invariance these states again contribute[15] to the K_2^0 self-mass. The ρ^0 contribution is

$$\delta m(K_2^0) = \frac{f_\rho^{\ 2}}{2m_K} k_\mu \frac{\delta_{\mu\nu} - k_\mu k_\nu/m_\rho^{\ 2}}{m_K^{\ 2} - m_\rho^{\ 2}} k_\nu,$$

$$= -(f_\rho^{\ 2}/2m_K) m_K^{\ 2}/m_\rho^{\ 2}; \quad (16)$$

f_ρ denotes the strength of the $K_2^0 \to \varphi^0$ vertex. The ω^0 contributes a similar term. Lastly,[16] we might consider the unitary singlet φ meson. The φ^0 contribution to K_2^0 self-mass is again of similar structure. We must emphasize that we are unable to estimate f_ρ, f_ω, etc., so as to compute the magnitude of the vector contributions to $\delta m(K_2^0)$. The crucial point to remember, however, is that the vector-meson contributions to the K_2^0 self-mass being negative, these contributions together with those of pseudoscalar mesons unambiguously predict a heavier K_1^0.

Finally, a word about possible other pole contributions. A scalar octet heavier than 500 MeV will depress the K_1^0 mass, thus threatening our conclusion about the sign of the $(K_1^0 - K_2^0)$ mass difference. The evidence for such an octet, at present, is weak, however. Lately, there seems to be some evidence[17] for a scalar particle at 380 MeV with 0^{++}, which was first suggested by Brown and Singer.[18] This particle can contribute a positive self-mass to K_1^0, thus reinforcing our conclusion regarding the sign of the mass difference. An assumption regarding the transformation property of weak vertices similar to ours has recently been applied to the problem of nonleptonic hyperon decays in pole approximation by Sugawara[19] who finds it possible to correlate presently known experimental results on the basis of such a model. We are aware of the limitations of a restrictive model like the pole approximation for the $(K_1^0 - K_2^0)$ mass difference, whose understanding might eventually turn out to require a consideration of more complicated intermediate states than those considered here. This possibility is underlined by our inability to convincingly calculate the magnitude of the mass difference. We feel, however, that since the present model, apart from being simple and giving finite results, leads to definite predictions which relate the sign of $(K_1^0 - K_2^0)$ mass difference to other effects, it might still be of some interest to check its conclusions with experimental results.

We are thankful to Professor G. Feinberg and Professor B. M. Udgaonkar for several discussions.

---

[1]M. Gell-Mann, California Institute of Technology Report No. CTSL-20, 1961 (unpublished); Phys. Rev. 125, 1067 (1962).

[2]Y. Ne'eman, Nucl. Phys. 26, 222 (1961).

[3]S. K. Bose, Phys. Letters 2, 92 (1962).

[4]S. Oneda, S. Hori, M. Nakagawa, and A. Toyoda, Phys. Letters 2, 243 (1962).

[5]S. Okubo, Progr. Theoret. Phys. (Kyoto) 27, 949 (1962).

[6]This may be derived from the condition $\langle\pi^0|H_\omega|K_1^0\rangle = 0$ which is a consequence of CP invariance. The condition $\langle\eta^0|H_\omega|K_1^0\rangle = 0$ leads to the same conclusion.

[7]Equation (4) has been obtained independently by S. Okubo (private communication).

[8]M. Baker and S. Glashow, Nuovo Cimento 25, 857 (1962).

[9]G. Alexander, S. Almeida, and F. Crawford, Jr., Phys. Rev. Letters 9, 69 (1962).

[10]C. Bouchiat, J. Nuyts, and J. Prentki, Phys. Letters 3, 156 (1962).

[11]S. Oneda and S. Hori, Phys. Rev. 132, 1800 (1963).

[12]M. Chrētien, F. Bulos, H. Crouch, Jr., R. Lanou, Jr., and J. Massimo, Phys. Rev. Letters 9, 127 (1962).

[13]C. Alff et al., Phys. Rev. Letters 9, 325 (1962).

[14]E. Fowler, F. Crawford, Jr., L. Lloyd, R. Grossman, and L. Price, Phys. Rev. Letters 10, 111 (1963).

[15]This may be easily seen by writing the phenomenological two-particle interaction as $(\partial_\mu K)B_\mu$, K and B_μ denoting the field amplitudes for the K meson and vector boson, respectively, and remembering the parity property of B_μ, i.e., $PB_iP^{-1} = -B_i$, $i = 1,2,3$, and $PB_4P^{-1} = B_4$. We would like to acknowledge the benefit of an illuminating conversation regarding this point with Professor G. Feinberg.

[16]We have not included the recently reported [M. Abolins, R. Lander, W. Mehlhop, N. Xuong, and P. Yager, Phys. Rev. Letters 11, 381 (1963)] B particle into consideration as its quantum numbers are yet to be determined. In any case, since the B particle has no natural place in the SU_3 scheme, the possibility of its belonging to some octet other than the vector octet does not seem to be ruled out. In particular, it could be a Regge recurrence of one of the known vector mesons. The spin of B will then be 3.

[17]F. Crawford, Jr., R. Grossman, L. Lloyd, L. Price, and E. Fowler, Phys. Rev. Letters 11, 564 (1963).

[18]L. Brown and P. Singer, Phys. Rev. Letters 8, 460 (1962).

[19]H. Sugawara (to be published).

ARE THE f^0 AND B MESONS TWO DECAY MODES OF THE SAME PARTICLE?*

William R. Frazer,† Sharashchandra H. Patil, and Nguyen-huu Xuong
University of California, San Diego, La Jolla, California
(Received 19 December 1963)

The striking fact that the recently discovered B meson has the same mass and width (within statistics) as the f^0 suggests the hypothesis that these two resonances are, in fact, different decay modes of the same particle.[1-4] In this Letter we examine the consequences of this hypothesis and find that it is not incompatible with the currently available experimental data. In fact, it seems to be easier to reconcile the data to this viewpoint than to the usual one in which the f^0 is assigned $I = 0$.

A particle which decays strongly, as does the B, into a π and an ω, must have $I = 1$. If it also decays strongly into two pions, as does the f^0, it must have odd J and negative parity. We shall confine our attention largely to the 1^- assignment, and refer to this hypothetical particle as the ρ' meson. We shall use the name B to designate the