periments show it to be quite, but not entirely isotropic. Since a number of poorly resolved A-K signals are found with similar mass values, no assignment within the Lomer model is possible at this time.

It should be pointed out that the subharmonic spectrum given by Eq. (3) has previously been derived by Noziéres[14] for trigonal orbits in graphite[3] under classical skin-effect conditions and for spatially flat orbits. The mechanism in that case is the noncircular nature of the orbits, a situation similar to that found in the valence bands of Si which also leads to observable subharmonics in cyclotron resonance.[15] Under anomalous skin-effect conditions, cyclotron resonance due to flat orbits would be Doppler-shifted beyond our experimental reach.[5] The observation of the predicted signals implies that the orbits are considerably less flat than the anomalous microwave skin depth.

The subharmonic relation of Eq. (3) may be easily generalized for an arbitrary number η of equivalent, equally spaced skin-depth traversals which may occur on other types of skipping orbits:

$$\tau_c/\tau_\mu = 1 \pm \eta m, \quad m = 0, 1, 2, \cdots. \tag{4}$$

The single-contact case, $\eta = 1$, is just the A-K resonance condition generalized to tilted orbits. For $\eta = 2$, all even subharmonics are suppressed but discrimination between senses of circular polarization occurs only for $\eta \geq 3$. While cases in which $\eta > 3$ may well be found, it should be noted that a necessary condition for the occurrence of nonflat real-space orbits is that the extremal plane in crystal-momentum space fail to have mirror symmetry. Combined with the overall inversion symmetry of Fermi surfaces, this severely restricts the type of nonflat extremal orbits which may be expected on pieces of Fermi surface centered on high symmetry points in momentum space. Such restrictions do not apply to pieces located at general positions in the zone, though the number of such pieces and their relative orientations must satisfy the lattice symmetry.

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MULTIPHONON PROCESSES IN THE PHOTOCONDUCTIVITY OF InSb†

H. J. Stocker, C. R. Stannard, Jr., H. Kaplan, and H. Levinstein
Department of Physics, Syracuse University, Syracuse, New York
(Received 10 December 1963)

Oscillations periodic in longitudinal optical (LO) phonon energy have been observed in the photoconductivity spectrum of acceptor-valence band transitions in Cu-, Au-, and Ag-doped InSb[1] and in Cu- and Zn-doped Ge.[2] We find similar oscillations in the photoconductivity due to valence-conduction band transitions in p-type InSb doped with Au, Ag, and Cu as well as with fewer than 10^{13} cm^{-3} residual impurities. The spacing of the oscillations is characteristic of both the

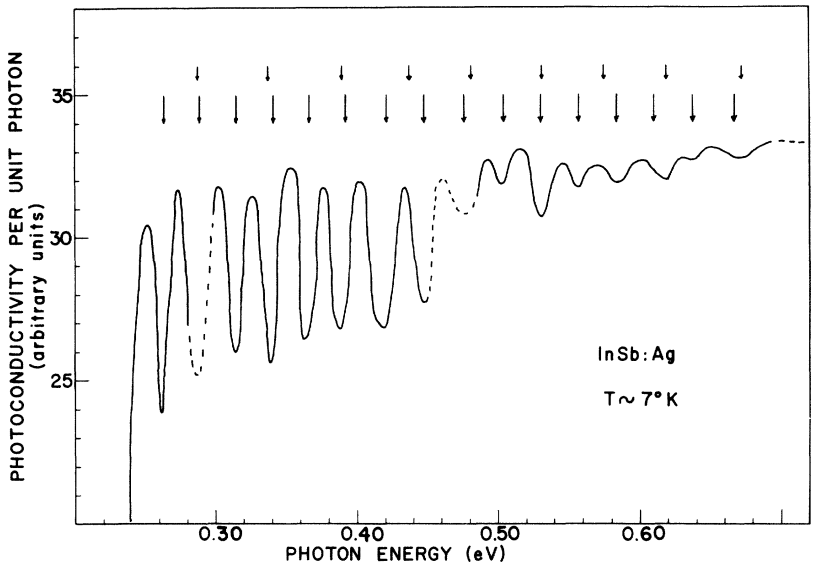

FIG. 1. Photoconductive response per unit photon vs incident photon energy for Ag-doped InSb sample at $T \sim 7°K$. Large arrows indicate the predicted minima due to heavy-hole-conduction band transitions, according to the mechanism proposed; small arrows those due to light-hole-conduction band transitions.

LO phonon energy and the band structure of InSb. A more involved interpretation than previously given[1] has to be adopted to account for the lack of oscillatory structure in the optical absorption.

Figure 1 shows the relative photoconductive response per unit photon vs incident photon energy for an Ag-doped sample at $T \sim 7°K$. The spacing of the first few minima is ~0.024 eV, equal to the energy of the long-wavelength LO phonon in InSb.[3,4] As the photon energy increases, the spacing between successive minima increases and the dips have a more spread-out appearance. As a function of temperature, the "threshold energy" decreases from 0.236 eV at $T \sim 7°K$ to about 0.215 eV at $T \sim 60°K$ in agreement with the expected shift of the energy gap. The minima of the oscillations also shift towards lower energies by approximately the same amount, but the relative spacing between minima remains the same. At temperatures above $T \sim 25°K$ the oscillations become weaker and at $T \sim 60°K$, they are barely recognizable.

While the photoconductive response shows these oscillations, no such structure is observed in the optical absorption. Transmission measurements performed on single-crystal slices of 10- to 20-micron thickness prepared from materials which exhibit oscillatory photoconductivity show that the optical absorption coefficient (within experimental error of about 3%) increases monotonically with increasing photon energy.

A mechanism responsible for the oscillations can be viewed as follows: Suppose there exist a number of impurity states with energies very close to the bottom of the conduction band. Electrons injected into the conduction band (e.g., by photon absorption) can make transition to these impurity states by emitting optical phonons, acoustical phonons, or photons. If coupling to LO phonons is strongest, as is to be expected in InSb, and if the dispersion of the LO phonons involved is small (energy ~ const $= \hbar\omega_0$), the lifetime of electrons in the conduction band will depend strongly on their energy. Conditions for a transition to the impurity states, with the emission of n phonons, will be most favorable if the electron in its initial state in the conduction band has an energy $n\hbar\omega_0$ larger than in its final, impurity-bound state. Since photoconductivity depends on the time the electrons are able to stay in the conduction band (we ignore the hole contribution), the photoconductive response will depend on photon energy. It reaches a minimum whenever the electron transferred to the conduction band is at energies $n\hbar\omega_0$ above the impurity states. A transition, corresponding to a minimum in the photoconductive response, according to this model, is illustrated in Fig. 2. Apart from the phonon energy the position of the minima in photon energy will depend on the band structure. The shape of the oscillations depends on the following factors: (1) the nonspherical nature of the heavy-hole band,

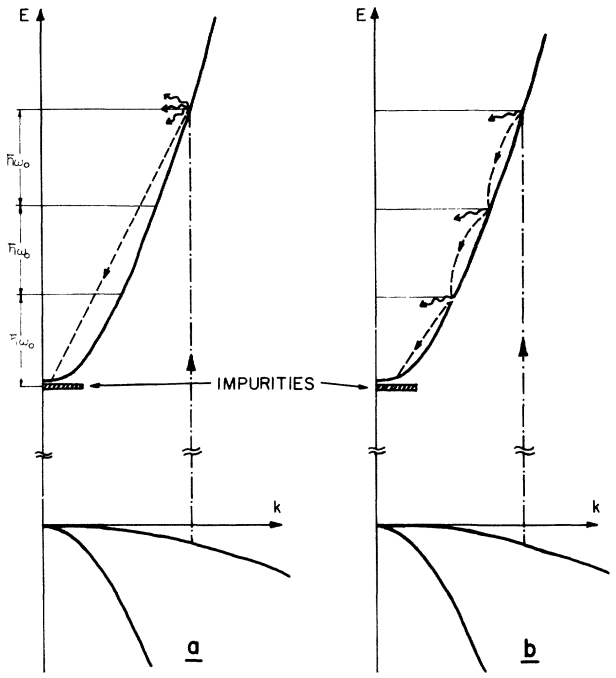

FIG. 2. Illustration of the mechanism proposed. The valence-conduction band transition is indicated by a dash-dot line. The transition of conduction electrons to shallow donor impurities with emission of LO phonons can occur either by direct capture (a), or the "cascade" process (b). See text.

(2) the spread in energies of the impurity states, (3) the dispersion of the phonons involved, and (4) lifetime broadening. Since the heavy-hole band of InSb happens to be fairly flat compared with the conduction band, the first few dips are separated by almost exactly the phonon energy, but at higher photon energies the separation becomes larger because of the curvature of the heavy-hole band. We have calculated the energies of the minima expected from this model, using the band structure of InSb calculated by Kane[5] and confirmed by the absorption measurements of Gobeli and Fan.[6] These are indicated in Fig. 1 by large arrows for transitions originating in the heavy-hole bands and by small arrows for those from the light-hole band. The "predicted minima" at low energies agree well with the experiment. At higher energies they are sensitive to small changes in the band-structure parameters. While it can thus be concluded that the spacing between oscillations is consistent with the present knowledge of the band structure of InSb, amplitudes of the oscillations obtained from this model are larger than might be expected. Quantitative estimates of the electron-phonon coupling constants using the theory of Huang and Rhys[7] and Gumme and Lax[8] give far too low values to account for the experimental indication that a, say, 13-phonon process still gives a 2% change in signal.

Thus we are led to consider a somewhat different view of the n-phonon process: Suppose that the electrons can lose energy within the conduction band only by emitting LO phonons. (Although various scattering processes are no doubt present in these InSb samples, one would expect energy loss primarily by LO phonons.) Then an electron excited with energy $n\hbar\omega_0$ above the impurity states will "cascade" down rapidly in energy, successively emitting $n-1$ LO phonons until it is captured by an impurity, emitting one more phonon. This process is illustrated in Fig. 2(b). If, for example, the electron initially has an energy of $(n+1/2)\hbar\omega_0$ above the impurity energy, it will similarly "cascade" down in energy, emitting n phonons but it will remain in a conduction-band state with energy $1/2\hbar\omega_0$ above the impurity state. Since capture by the impurity is now much less probable, the electron is able to conduct current until it is finally removed by some other process or until it has gained enough energy from the small dc electric field to emit one more LO phonon. The minima in the photoconductive response according to this "cascade" process clearly would occur at the same photon energies as for direct capture by impurities. A realistic theoretical treatment of the "cascade" process proposed should include the influence of all other scattering, trapping, and recombination processes, but unfortunately little is known about their relative magnitudes.

A crucial role in the above discussion is played by impurity states with energies close to the bottom of the conduction band. Such shallow donor impurities are actually present in concentrations of 10^{13} cm^{-3} or larger in the purest crystals available. Their ground state is separated by less than 10^{-3} eV from the conduction band and their wave function is so extended that they likely form an "impurity band." This, however, is not an argument against the mechanism considered, because the mobility in the "impurity band" will be significantly lower than in conduction-band states, sufficiently removed from it.

The lack of oscillatory structure in the absorption spectrum is consistent with the mechanism discussed. The modification of the optical absorption, due to the electron-phonon interaction, is expected to be small, probably unobservable,

since absorption is primarily governed by the densities of states in the valence and conduction bands.

An interpretation similar to the one proposed here seems feasible for the case of impurity-valence band transitions,[1,2] with shallow acceptors or perhaps excited states of the "deep" acceptors playing the role of capturing centers.

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DETERMINATION OF EFFECTIVE MASSES FROM GIANT QUANTUM OSCILLATIONS IN ULTRASONIC ABSORPTION

Yaacov Shapira*
National Magnet Laboratory,† Massachusetts Institute of Technology, Cambridge, Massachusetts

and

Benjamin Lax
National Magnet Laboratory,† Massachusetts Institute of Technology, Cambridge, Massachusetts and
Lincoln Laboratory,‡ Massachusetts Institute of Technology, Lexington, Massachusetts
(Received 6 January 1964)

We have observed "giant quantum oscillations" in the absorption of ultrasonic waves in gallium and have determined the effective mass from the width of the absorption peaks. While quantum oscillations in the attenuation of longitudinal sound waves in a parallel magnetic field have been observed in metals,[1] semimetals,[2] and semiconductors,[3] this is the first time that the sharp spikelike character of the oscillations has been observed experimentally and used to determine effective masses. The so-called "giant quantum oscillations" have been first predicted theoretically by Gurevich et al.[4] When

$$(2kT/m^*)^{1/2}q\tau \gg 1, \tag{1}$$

where m^* is the effective mass, q is the wave number of the ultrasonic wave, and τ is the electron-scattering time, the attenuation coefficient Γ of a longitudinal ultrasonic wave in a parallel magnetic field is given by[4]

$$\Gamma = \Gamma_0 \frac{\beta H}{8kT} \sum_{n, s_z} \cosh^{-2}\left[\frac{\beta H(n+\frac{1}{2}) + s_z \mu H - \zeta}{2kT}\right], \tag{2}$$

where Γ_0 is the attenuation coefficient at zero magnetic field, $\beta = e\hbar/m^*c$, μ is the magnetic moment of the electron, and ζ is the Fermi energy. n and s_z are the Landau level number and spin quantum number, respectively. We assume initially that a given absorption peak corresponds either to a single term in the double summation of of Eq. (2), or two terms having maxima at the same field. The latter case corresponds to an effective g value which is an exact even multiple of the ratio of the free-electron mass to the cyclotron mass. Under this assumption the line shape of a single absorption peak i is given by

$$\Gamma_i = A \cosh^{-2}[(B_i\beta H - \zeta)/2kT], \tag{3}$$

where A and B_i are constants. Here we have neglected the slight variation across the absorption peak of the amplitude factor $\beta H/8kT$ in Eq. (2). The field H_i at the center of the peak satisfies the relation

$$B_i = \zeta/\beta H_i = [(\Delta H^{-1})H_i]^{-1}, \tag{4}$$

where ΔH^{-1} is the usual de Haas–van Alphen period. From Eqs. (3) and (4),

$$\Gamma_i = A \cosh^{-2}[\beta(H - H_i)/2kTH_i(\Delta H^{-1})]. \tag{5}$$

The full width $(\delta H)_i$ at half-height is then given