DETERMINATION OF CARRIER SIGN BY SKIPPING ORBIT CYCLOTRON RESONANCE

Walter M. Walsh, Jr.
Bell Telephone Laboratories, Murray Hill, New Jersey
(Received 23 December 1963)

The use of microwave radiation circularly polarized about a static magnetic field H in determining the hole or electron character of cyclotron orbits has proven successful in semiconductors[1] and semimetals[2-4] under classical skin-effect conditions. The dependence of the surface impedance on H under anomalous skin-effect conditions has been calculated by Miller and Haering[5] and observed in Bi by Kirsch and Miller.[6] In true metals whose Fermi velocities greatly exceed the phase velocity of microwaves in the metal, one does not expect to observe the Miller-Haering type of Doppler-shifted resonance with presently available magnetic fields. The results of Galt et al. on Zn and Cd[7,8] indicate, however, that some discrimination between holes and electrons may still be obtained in particular situations, but interpretation is lacking. We wish to exhibit a new class of resonances in which circularly polarized radiation discriminates between holes and electrons but whose mechanism is closely related to the type of cyclotron resonance originally analyzed by Azbel' and Kaner.

Recently Koch and Kip[9] have observed multiple subharmonic cyclotron resonance of the Azbel'-Kaner (A-K) variety[10] in Sn with the magnetic field tilted out of the sample plane and even normal to it. These are tilted-orbit resonances[11] arising from extremal orbits which, while lying in the plane perpendicular to H in crystal-momentum space, are tilted in real space since the field has been applied along a low-symmetry direction of some piece or pieces of Fermi surface. The tilted real-space orbit causes carriers to pass in and out of the skin depth once in each cycle and thus provides the condition for A-K cyclotron resonance.

A natural extension of this idea is a class of nonflat extremal orbits, termed skipping orbits, such that the carriers interact more than once with the radiation field in the course of one cyclotron period. The possibility of such orbits has been mentioned by Koch and Kip. In particular, we shall discuss a set of extremal orbits in tungsten whose normal A-K spectrum is most clearly seen when H is applied parallel to a $\langle 111\rangle$ axis[12] where the effective mass ratio is $m^*/m_e = 0.83$ [see Fig. 2(a)]. A variety of evidence indicates that this signal arises from waist orbits around

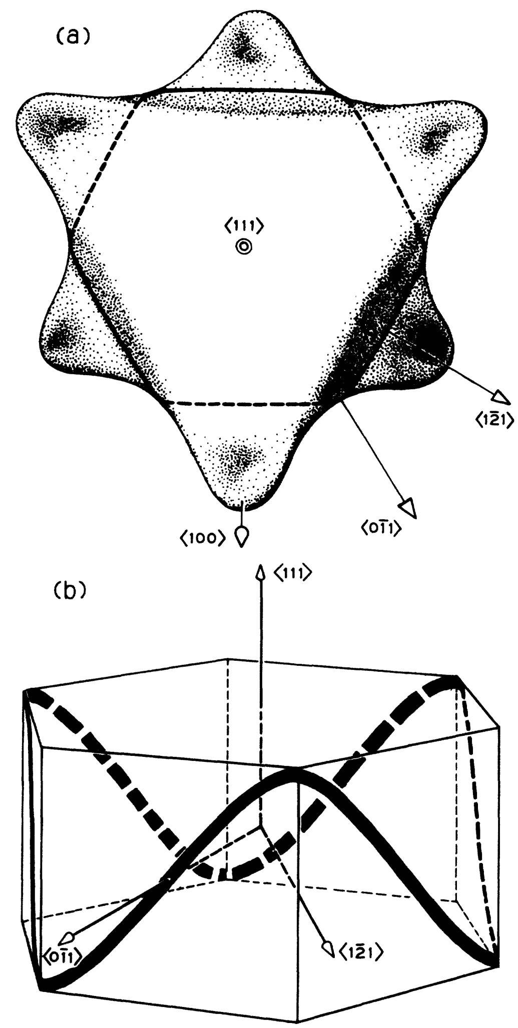

FIG. 1. (a) The qualitative aspect of a piece of Fermi surface in tungsten viewed along a threefold axis. The hexagonal curve indicates the waist orbit which is flat in k space. The carrier velocity vector, normal to the Fermi surface, rocks up and down three times with respect to that plane during a cyclotron period. (b) An isometric view of the real-space character of the waist orbit. Due to its alternately rising and falling motion, a carrier may "skip" in and out of the anomalous skin depth of a (111)-plane sample when the magnetic field is normal to that surface.

a distorted octahedral Fermi surface schematically shown in Fig. 1(a). This surface resembles the principal electron and hole surfaces predicted by Lomer[13] for the hexavalent, body-centered-cubic

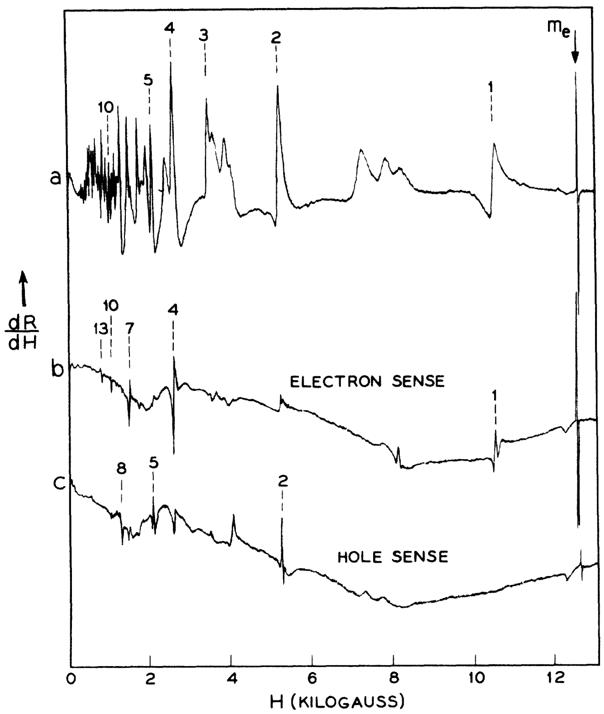

FIG. 2. Magnetic field derivatives, in arbitrary units, of the surface resistances of two tungsten samples ($\rho_{298°K}/\rho_{4.2°K} \simeq 28\,000$) observed at 4.2°K and ~35 kMc/sec. System gain in traces (b) and (c) was increased ~×50 over that used to record trace (a). (a) Usual Azbel'-Kaner cyclotron-resonance oscillations observed with $\vec{H}$ parallel to a ⟨111⟩ axis lying in a (110) plane. Linearly polarized microwave currents were excited nearly perpendicular to $\vec{H}$. The series corresponding to an effective-mass ratio $m^*/m_e = 0.83$ arises from extremal orbits of the type indicated in Fig. 1. (b) Here $\vec{H}$ was directed along ⟨111⟩ normal to a (111) plane and circularly polarized radiation was used. The field direction was that appropriate to excitation of free-electron cyclotron resonance. Note the near absence of the third-order subharmonic of $m^*/m_e = 0.83$ and the low amplitude of the second relative to those of the first and fourth. (c) Conditions identical to (b) save for reversal in sign of the magnetic field. Now the second subharmonic dominates the first and fourth while the third remains very weak. A second electron signal is also seen in traces (b) and (c) with $m^*/m_e = 0.63$. A free-radical spin resonance is seen at $m^*/m_e = 1.00$ on each trace.

transition metals. The real-space aspect of the waist orbit is indicated in Fig. 1(b). That the orbit has the particular character of rising and falling three times per revolution results from the threefold symmetry and absence of mirror-plane symmetry.

The utility of the nonflat nature of the orbit becomes apparent in an experiment where H is applied normal to a (111)-plane sample on which there flow circularly polarized currents of period τ_μ. Carriers executing the orbits in question pass through the skin depth three times per cyclotron period τ_c. The qualitative behavior of the surface resistance under such circumstances may be approximated by computing the power P delivered to a free-space particle of charge e, velocity $\vec{v}$ and mass m^* (whence $\tau_c = 2\pi m^* c/eH$) which is allowed to "feel" the rotating electric vector $\vec{E}$ during three short, equivalent intervals $\delta \ll \tau_c$. The energy transferred per interaction from the rotating field to the particle in the direction of motion of the latter is

$$W = e\vec{E}\cdot\vec{v}\delta. \tag{1}$$

For the three-point skipping orbit, the power transferred is

$$P = \frac{3eEv\delta}{\tau_c}\frac{1}{N}\sum_{n=1}^{N}\cos\tfrac{2}{3}\pi n\left(\frac{\tau_c}{\tau_\mu} - 1\right). \tag{2}$$

In the limit $N \to \infty$, nonzero values of P result only if

$$\tau_c/\tau_\mu = 1 \pm 3m, \quad m = 0, 1, 2, 3, \cdots, \tag{3}$$

where τ_c/τ_μ is just the order of a subharmonic which may take on all integer values in the usual, one-contact A-K experiment. The positive solutions, corresponding to rotation of the carrier and the microwave field in the same sense, occur for $\tau_c/\tau_\mu = 1, 4, 7, 10, \cdots$. The negative solutions, those for which the carrier and the electric field rotate in opposite senses, are $\tau_c/\tau_\mu = -2, -5, -8, \cdots$. Two salient features of this subharmonic pattern are the asymmetry with respect to sense of circular polarization which permits identification of the carrier sign and the absence of the subharmonics $|\tau_c/\tau_\mu| = 3, 6, 9, \cdots$ for either sense of polarization.

The results of the appropriate experiment are shown in Figs. 2(b) and 2(c). The signals are approximately 50 times weaker than in the usual Azbel'-Kaner geometry and have unusual "shapes" but may still be clearly identified by the sudden changes in the field derivative of the surface resistance. The third subharmonic is nearly absent for both senses of circular polarization. The first and fourth subharmonics dominate the second for the electron sense of rotation whereas the reverse is true in the hole sense. Another electron signal, corresponding to $m^*/m_e = 0.63$, is also seen on these traces. Normal A-K experiments show it to be quite, but not entirely isotropic. Since a number of poorly resolved A-K signals are found with similar mass values, no assignment within the Lomer model is possible at this time.

It should be pointed out that the subharmonic spectrum given by Eq. (3) has previously been derived by Nozières[14] for trigonal orbits in graphite[3] under classical skin-effect conditions and for spatially flat orbits. The mechanism in that case is the noncircular nature of the orbits, a situation similar to that found in the valence bands of Si which also leads to observable subharmonics in cyclotron resonance.[15] Under anomalous skin-effect conditions, cyclotron resonance due to flat orbits would be Doppler-shifted beyond our experimental reach.[5] The observation of the predicted signals implies that the orbits are considerably less flat than the anomalous microwave skin depth.

The subharmonic relation of Eq. (3) may be easily generalized for an arbitrary number η of equivalent, equally spaced skin-depth traversals which may occur on other types of skipping orbits:

$$\tau_c/\tau_\mu = 1 \pm \eta m, \quad m = 0, 1, 2, \cdots. \tag{4}$$

The single-contact case, $\eta = 1$, is just the A-K resonance condition generalized to tilted orbits. For $\eta = 2$, all even subharmonics are suppressed but discrimination between senses of circular polarization occurs only for $\eta \geq 3$. While cases in which $\eta > 3$ may well be found, it should be noted that a necessary condition for the occurrence of nonflat real-space orbits is that the extremal plane in crystal-momentum space fail to have mirror symmetry. Combined with the overall inversion symmetry of Fermi surfaces, this severely restricts the type of nonflat extremal orbits which may be expected on pieces of Fermi surface centered on high symmetry points in momentum space. Such restrictions do not apply to pieces located at general positions in the zone, though the number of such pieces and their relative orientations must satisfy the lattice symmetry.

We are grateful for the assistance of L. W. Rupp, Jr., in performing the experiments and that of P. H. Schmidt in preparing the samples from crystals obtained from Dr. H. Sell of Westinghouse Lamp Division. We have also benefited from discussions with Dr. E. Fawcett and Dr. J. K. Galt.

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MULTIPHONON PROCESSES IN THE PHOTOCONDUCTIVITY OF InSb[†]

H. J. Stocker, C. R. Stannard, Jr., H. Kaplan, and H. Levinstein
Department of Physics, Syracuse University, Syracuse, New York
(Received 10 December 1963)

Oscillations periodic in longitudinal optical (LO) phonon energy have been observed in the photoconductivity spectrum of acceptor-valence band transitions in Cu-, Au-, and Ag-doped InSb[1] and in Cu- and Zn-doped Ge.[2] We find similar oscillations in the photoconductivity due to valence-conduction band transitions in p-type InSb doped with Au, Ag, and Cu as well as with fewer than 10^{13} cm^{-3} residual impurities. The spacing of the oscillations is characteristic of both the