GRAVITATIONAL COLLAPSE*

Peter G. Bergmann
Belfer Graduate School of Science, Yeshiva University, New York, New York
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The discovery of the so-called quasistellar objects,[1] with previously unheard-of rates of energy radiation over extended periods of time, has helped to sustain interest in the possibility of gravitational collapse of very large masses.[2] If such a collapse takes place, its early stages are represented quite adequately by Newtonian mechanics, which predicts the conversion of potential into kinetic energy if a large cloud of matter contracts. As the contraction proceeds, the amount of energy converted may become a significant fraction of the original rest mass of the contracting material; it may exceed the amount of energy that could be freed by thermonuclear reactions. When the kinetic energy approaches the original rest energy, the linear dimensions of the celestial body will be of the order of its Schwarzschild radius; the Schwarzschild radius is that distance from a point mass where the escape velocity of a test particle equals the speed of light.

From the point of view of general relativity, the metric and topological conditions prevailing at and about the Schwarzschild radius have been explored thoroughly.[3,4] As most of this work has been concerned with the specific spherically symmetric solution discovered by Schwarzschild,[5] or with other special solutions of Einstein's field equations, it is of interest to examine whether similar conditions may be expected to arise generally in the course of gravitational collapse, regardless of spherical symmetry. That is the purpose of this note.

In the most common coordinate system used, the metric at the Schwarzschild radius deviates from normal behavior in that the component g_{00} goes to 0, whereas the component g_{11} tends to ∞, so that the product of these two components remains -1.[6] We shall simulate this behavior by centering a coordinate system on a point exhibiting this peculiar behavior, by giving the two interesting components of the metric tensor the typical Schwarzschild behavior and by setting the remaining components constant,

$$d\tau^2 = \xi dt^2 - (1/\xi)d\xi^2 - (dy^2 + dz^2). \qquad (1)$$

The form (1) is to be assumed valid only in the vicinity of the coordinate origin. There, however, the form (1) may be achieved in general by appropriate scaling of the four coordinates involved, by rearranging the directions of the three coordinates ξ, y, z in relation to t and to each other, and by scaling of the proper time element $d\tau$. Appropriate scaling will also make the values of the derivatives of the metric components up to a designated finite order as small as desired provided they are finite to begin with. It is implicit in the assumed functional behavior of the coefficients of the line element (1) that the Schwarzschild radius, and any other characteristic intrinsic parameters having the dimensions of lengths, be large in terms of the unit of the proper length. As a matter of fact, the metric (1) is flat.

By the transformation $4\xi = x^2$ the line element (1) may be brought into the "isotropic" form[6]

$$d\tau^2 = \tfrac{1}{4}x^2 dt^2 - (dx^2 + dy^2 + dz^2). \qquad (2)$$

This line element lends itself to an intuitive interpretation: Its only deviation from the standard Minkowski-Lorentz form is that the coefficient of dt^2, the gravitational (Newtonian) potential, is variable, and the Schwarzschild radius is that region where the potential drops sufficiently to cause infinite red-shift. Conversely, whenever a large negative gravitational potential causes infinite red-shift, coordinates and units of length may be introduced so as to approximate locally the expression (1), or (2). Parenthetically, the Riemann-Christoffel curvature tensor has the dimension $(L)^{-2}$; it goes to zero with the square of the unit in which lengths are measured.

In a drawing in which the geodesics of the Minkowski manifold appear as straight lines, the (x, t)-coordinate grid of Eq. (2) has the appearance shown in Fig. 1. The y and z directions have been omitted. The curves x = constant are hyperbolas, all of which have the same asymptotes. All points with $x = 0$, $-\infty < t < \infty$, are really the same point, the one at which the two asymptotes intersect. The asymptotes themselves are light lines; their points can be reached from the interior of the region $x \geqslant 0$ only asymptotically, as they correspond to $x = 0$, $t = \pm\infty$. The transformation $t' = t + t_0$ is a Lorentz transformation about the Schwarzschild point on the left of the figure

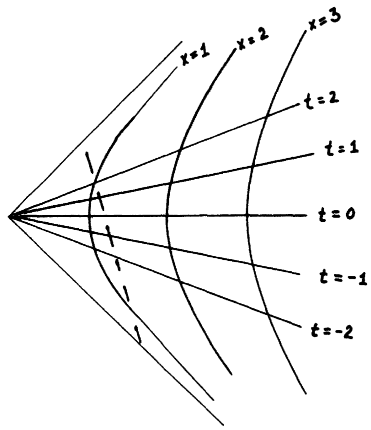

FIG. 1. Coordinate grid in the vicinity of a point where $g_{00}=0$.

as the axis. Each of the hyperbolas is mapped on itself, and so are the two limiting null lines, and their intersection. Increasing the value of t by a constant t_0 means on each hyperbola a disp.acement that is proportional both to x and to t_0. Hence the length of every hyperbola is infinite, not only in coordinate time but also in proper time.

The free fall of a test particle is illustrated by the dashed straight line of Fig. 1. In the vicinity of the Schwarzschild region, at least, such a free-fall curve will reach from null line to null line, and it will have a finite proper length. In terms of coordinate time, t, it extends from $-\infty$ to ∞. This coordinate time, however, is proportional to the natural time for an observer who maintains a constant distance from the Schwarzschild region, and this is so both at his own location and elsewhere (at constant x), where he times events with his own clock and by means of continuous optical observation. For such an observer the free fall from any location to the Schwarzschild region takes forever.

We may finally inquire what happens to an extended object in free fall. In view of the flatness of our model manifold, such an object may reach the Schwarzschild region in rigid motion, and in particular without changing its volume. In a curved manifold rigid motion may be impossible, but it will remain true that any change in volume and shape need only be finite.

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[1]Among recent papers, we mention C. Hazard, M. B. Mackey, and A. J. Shimmins, Nature 197, 1037 (1963); M. Schmidt, Nature 197, 1040 (1963); J. B. Oke, Nature 197, 1040 (1963); J. L. Greenstein and T. A. Matthews, Nature 197, 1041 (1963); H. J. Smith and D. Hoffleit, Nature 198, 650 (1963).

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