ENHANCEMENT OF THE $\pi K\bar{K}$ INTERACTION*

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It has been suggested by Peierls[1] that the πN peak $N_{1/2}^*$(1512 MeV) in the $I=\frac{1}{2}$ channel might be understood in terms of the resonant $\frac{3}{2}$-$\frac{3}{2}$ πN state $N_{3/2}^*$(1238 MeV) through its interactions with an additional pion. The salient feature of Peierls's suggestion is that unstable vertices in a Feynman graph can lead to singularities near the physical region, thereby manifesting themselves in certain cases as an enhancement of the process at a reasonably well-defined energy, although not in a definite state of angular momentum and isotopic spin. The Peierls mechanism has been applied to vector-meson resonances by Nauenberg and Pais.[2] They show in detail how an enhancement of the 3π interaction might be understood in terms of a resonant 2π interaction, and briefly mention several other applications, including the $\pi K\bar{K}$ system.

Following Nauenberg and Pais[2] we wish to report here the detailed results of applying the Peierls mechanism to the $\pi K\bar{K}$ system and to point out the possible relevance to the recently reported peak observed in the $\pi K\bar{K}$ mass spectrum around 1410 MeV.[3]

Consider the contribution to the interaction in the $\pi K\bar{K}$ state coming from the graph shown in Fig. 1. (An equal contribution comes from the graph in which initial and final particles are interchanged.) The very strong πK interactions are approximated by replacing the πK and $\pi\bar{K}$ in the initial and final states, respectively, by the K^*(888 MeV) resonance. Regarding this P-wave resonance as a vector particle having the complex mass $M-i\Gamma/2$, where $M=888$ MeV and $\Gamma=50$ MeV, the corresponding amplitude is

$$A=(g^2/4\pi)(\epsilon_i k)(\epsilon_f k)/(k^2+\mu^2). \tag{1}$$

In Eq. (1) $g^2/4\pi$ is the effective $K^*K\pi$ coupling constant, ϵ_i and ϵ_f are the initial and final polarization four-vectors, $\mu=140$ MeV is the pion mass, and $k=p_i-q_f=p_f-q_i$ is the four-momentum transfer, the four-momenta being as shown in Fig. 1.

Squaring the amplitude, summing over final spins, and averaging over initial spins lead to[4]

$$\frac{1}{3}\sum_{\text{spins}}|A|^2=\frac{(g^2/4\pi)^2}{48M^4}\frac{\{[W^2-2q^2(1-\cos\theta)-M^2-m^2]^2-4M^2m^2\}^2}{[W^2-2q^2(1-\cos\theta)+\mu^2-2M^2-2m^2]^2+M^2\Gamma^2(1-\omega/E)^2}. \tag{2}$$

Here q is the momentum, $\cos\theta=\hat{p}_1\cdot\hat{p}_2$ is the cosine of the scattering angle, E and ω are the K^* and $\bar{K}$ (or $\bar{K}^*$ and K) energies, respectively, and $W=E+\omega$, all in the center-of-mass system; $m=496$ MeV is the kaon mass. Equation (2), integrated over all angles, is plotted[5] in Fig. 2 as a function of the center-of-mass energy W. In Fig. 2 we have taken $g^2/4\pi=2.5$, an estimate based on the width for the decay $K^*\to K+\pi$.

We wish to suggest the peak at 1450 MeV shown in Fig. 2 might be the origin of the recently reported bump in the $\pi K\bar{K}$ mass spectrum around 1410 MeV observed in antiproton-proton annihilations.[3] The shift in position could be due to the crudeness in approximating the rather broad resonance K^* by a vector particle of mass (888-25i) MeV. The location of the $\pi K\bar{K}$ peak depends sensitively on the effective mass of the K^*, which could be lower than the central value used here.

Several features peculiar to this mechanism can be investigated to test this suggestion.[1,2] These are the following:

(a) According to the present model, the $\pi K\bar{K}$ events in the region of enhancement should nearly all be of the type $\bar{K}K^*$ or $K\bar{K}^*$.

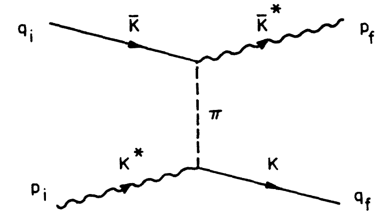

FIG. 1. One-pion-exchange scattering diagram.

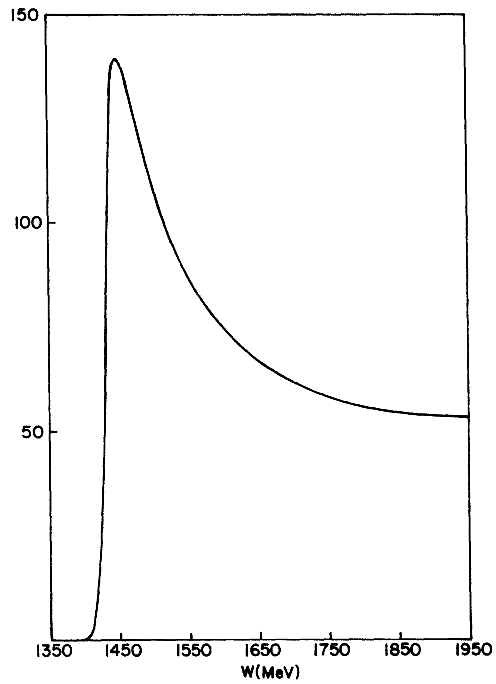

FIG. 2. Center-of-mass energy dependence of the one-pion-exchange matrix element for $\bar{K}+K^* \rightarrow \bar{K}^*+K$. Inital and final polarizations have been averaged and summed, respectively, and the scattering angle has been integrated over.

(b) The shape of the $\pi K\bar{K}$ peak in Fig. 2 is clearly asymmetric, rising considerably faster than falling, and therefore should not be consistent with the resonant Breit-Wigner form. In practice the total energy available to the $\pi K\bar{K}$ system will be limited by the conditions under which the particles are produced, which might complicate this test by cutting off the higher energies, thus causing a more rapid decline. In addition, unitarity, which is not included in this approximate calculation, might modify the shape of the peak at higher energies.

(c) The $\pi K\bar{K}$ bump shown in Fig. 2 does not occur in a definite isotopic spin state. However, a simple calculation shows that the possible isotopic spin states 0, 1, and 2 should occur with the relative weights 9, 1, and 0, respectively. This relative enhancement of the $I=0$ channel in practice might very well mask the indefiniteness of this quantum number.

(d) A very important feature of the $\pi K\bar{K}$ enhancement arising from the forces shown in Fig. 1 is that it does not occur in a definite angular momentum state. The nonresonating kaon [see (a)] should have an angular distribution in the $\pi K\bar{K}$

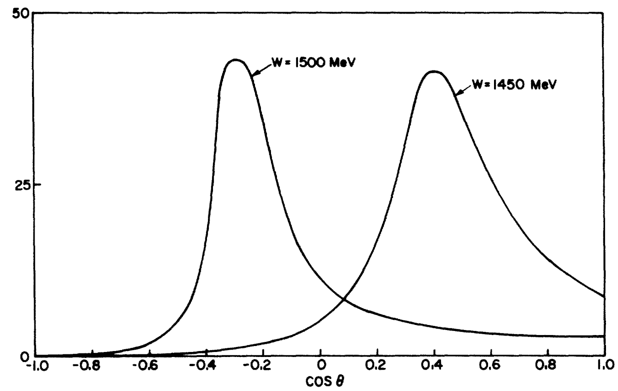

FIG. 3. Angular distribution in the center-of-mass system of the one-pion-exchange matrix element for $\bar{K}+K^* \rightarrow \bar{K}^*+K$ at center-of-mass energies of 1450 MeV and 1500 MeV.

center of mass which is given by Eq. (2). It is easy to see from Eq. (2) that the angular distribution depends on energy, the peak shifting from the forward to the backward direction as the energy increases. This point is illustrated in Fig. 3, where we have plotted[5] this angular distribution for two energies in the region of enhancement, again taking $g^2/4\pi = 2.5$.

Similar phenomena should occur in other final-state interactions.[2] We find the 3π final state should show an enhancement around 1125 MeV due to the 2π resonance ρ (750 MeV).

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[3]R. Armenteros, D. N. Edwards, T. Jacobsen, A. Shapira, J. Vandermeulen, Ch. d'Andlau, A. Astier, P. Baillon, H. Briand, J. Cohen-Ganouna, C. Defoix, J. Siaud, C. Ghesquière, and P. Rivet, Proceedings of the Siena Conference on Elementary Particles, Siena, Italy, 1962 (unpublished).

[4]Equation (2) does not reduce to Eq. (1) of reference 2 for $m=\mu$ as one might expect because here the conditions $(p_i\epsilon_i)=(p_f\epsilon_f)=0$ have been taken into account while they are not included in reference 2. These conditions have the effect of reducing the distributions shown in Figs. 2 and 3 by approximately a factor of 10 but do not seriously affect their shapes. I am grateful to Professor S. M. Berman and Professor C. K. Id-

dings for discussions on this point. In both derivations unimportant terms of order of the square of the K^* width have been neglected in the numerator.

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TIME-DEPENDENT SCATTERING EXPERIMENT

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The principle of superposition of states for position and momentum may be verified, for example, by observing the diffraction pattern built up by single photons scattered from a system whose characteristic space dimension is comparable with the wavelength of the photon.

The verification of the principle for energy and time requires the measurement of very short time intervals on the atomic scale. The time development of the state $\psi(t)$ of a quantum mechanical system with a Hamiltonian H is described by the Schrödinger equation

$$i\hbar d\psi/dt = H\psi. \tag{1}$$

The proposed experiment is a scattering experiment in which only the asymptotic form of the wave function in space and time is used and the development of the wave function from $t=-\infty$ to $t=+\infty$ is given by the S matrix without the use of the Schrödinger equation. Thus the time development of the wave function $\psi(t)$ is not observed in the vicinity of the scattering potential, as would be necessary to verify (1).[1]

The time interval characteristic of the scattering of a particle of energy E in the isolated resonance region by a many-body system is the reciprocal of the width Γ_S of the compound state $|s\rangle$ whose energy ϵ_S is closest to E. The case where more than one compound state contributes appreciably to the scattering amplitude in the vicinity of E will be treated in a subsequent publication.

Most compound nucleus states have lifetimes of the order of 10^{-15} sec. Times of the order of 10^{-10} sec are the smallest that can be experimentally resolved, so it is impossible to do a time-dependent experiment with such a system. Exceptions are the metastable states of nuclei, for example, the 14-keV state of ^{57}Fe which has a lifetime of 10^{-7} sec. Atomic states and particularly laser states of longer lifetime are well known. If a photon is scattered from such a resonance it would be easy to observe the effect of small time changes.

On a nuclear scale a time-dependent experiment has been performed by Holland et al.,[2] who essentially measured the spectrum of total elapsed time between the excitation of the 14-keV level in an ^{57}Fe source and the detection of the corresponding γ ray after it had passed through a resonant absorber. This may be regarded as a scattering experiment in which the incident wave packet has a half-exponential time spectrum of width $\hbar/\Gamma_S$. The time spectra of both the initial and final states must be known separately for a time-dependent scattering experiment.

The proposed experiment consists in following the time development of a wave packet with average energy E (near ϵ_S) and variable time width T (corresponding to an energy width $2\delta = \hbar/T$) which is scattered resonantly from the appropriate nucleus. The theory of the experiment is as follows.

The initial state is a wave packet which can be written as a superposition of the complete set of asymptotic states $\psi_{\vec{k}'}$ of the scattering problem:

$$\xi_{\vec{k}} = \int \omega(\vec{k}', \vec{k}) \psi_{\vec{k}'} d^3k', \tag{2}$$

where, if a Hamiltonian H for the problem exists,

$$H\psi_{\vec{k}} = E_{\vec{k}} \psi_{\vec{k}}. \tag{3}$$

The integral in (2) corresponds to a sum over discrete eigenstates and an integral over eigenstates with continuous eigenvalues.

The time development of $\psi_{\vec{k}}$ is given by inte-